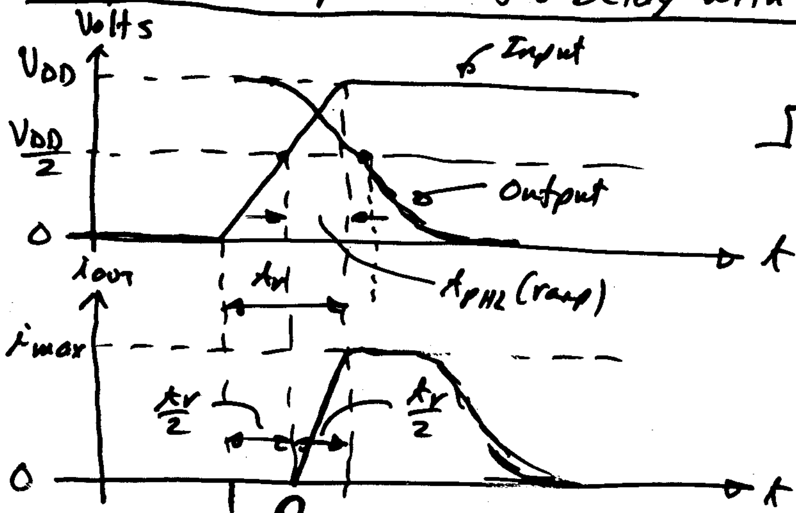
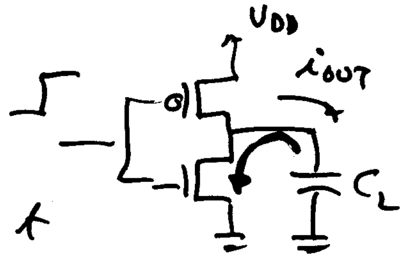
Exam #2 – Friday 11th
Prob. Session – Thursday 5pm–6pm (C240?)

Analytical Expression for Delay with a Ramp





$i = C\frac{dv}{dt}$

$$i_{OUT} = C_L \frac{dv_{OUT}}{dt} \rightarrow \int i_{OUT}\,dt = \int C_L\,dv_{OUT}$$

$$\int_0^{t_r/2} i_{out}\,dt + \int_{t_r/2}^{t_{PHL}(ramp)} i_{out}\,dt = C_L \int_0^{\frac{V_{DD}}{2}} dv_{OUT}$$

$i_{max}\left(\frac{t}{t_r/2}\right)$

$$\Downarrow$$

$$i_{max}\left[\frac{t_r}{4} + t_{PHL}(ramp) - \frac{t_r}{2}\right] = \frac{C\,V_{DD}}{2}$$

$$t_{PHL}(ramp) = \frac{t_r}{4} + \frac{C\,V_{DD}}{2\,i_{max}} = \frac{t_r}{4} + t_{PHL}(step)$$

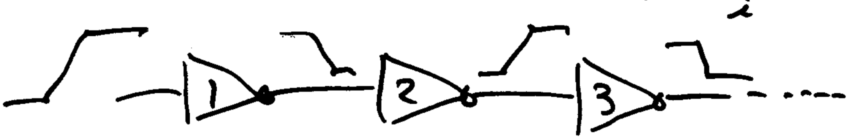
Generalization:

If t_{in} is the input rise/fall time and is approximated at $\frac{t_r}{2}$, then

$$t_{ramp} \simeq \frac{t_{in}}{2} + t_{step} = \frac{t_{in}}{2} + 0.7RC$$

Taking these concepts to an inverter chain we can develop a general formula –

$$\text{Total delay} \cong \sum_i R_i C_i$$



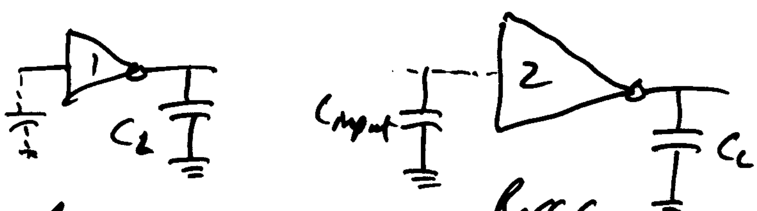
$$\text{Total delay} = \frac{t_{in}}{2} + t_{PHL1} + \frac{t_{in1}}{2} + t_{PHL2} + \frac{t_{in2}}{2} + t_{PHL3} + \cdots$$

$$= \frac{t_{in}}{2} + \underbrace{\underbrace{0.7R_1C_1 + \frac{0.7R_1C_1}{2}}_{\approx R_1C_1} + \underbrace{0.7R_2C_2 + \frac{0.7R_2C_2}{2}}_{\approx R_2C_2} + \cdots 0.7R_nC_n}_{R_nC_n}$$

$$\approx \sum_{i=1}^{n} R_i C_i$$

How Do you Optimally Size the Inverters for Min Delay?

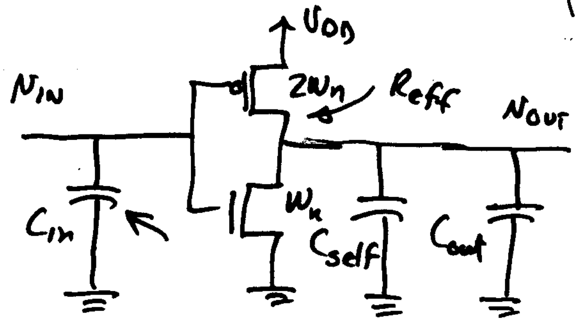
Minimize the delay globally –



$R_{eff1} C_L$ $R_{eff2} C_L$

If $R_{eff2} \downarrow$, the C_{input} goes up
(Move the delay to the previous stage)

Inverter delay –



Gate time constant, τ_{inv}:

$$\tau_{inv} \equiv R_{eff} C_{in} = R_{eqn}\left(\frac{L_n}{W_n}\right) C_g (3W_n)$$

$$= 3 R_{eqn} C_g L_n$$

$\therefore \; t_{delay} = R_{eff}\left[C_{out} + C_{self}\right] = R_{eff} C_{in}\left[\frac{C_{out}}{C_{in}} + \frac{C_{self}}{C_{in}}\right]$

$= \tau_{inv}\left[\frac{C_{out}}{C_{in}} + \gamma_{inv}\right] \qquad \gamma_{inv} \equiv \frac{C_{self}}{C_{in}} =$

Note: C_{self} is strongly dependent on the layout

Fanout of the inverter $= f = \frac{C_{out}}{C_{in}}$

$\therefore \; t_{delay} = \tau_{inv}\left[f + \gamma_{inv}\right]$

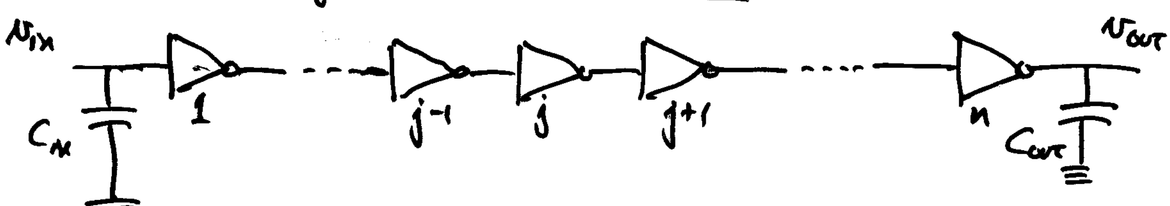
Ex. 6.8

Find τ_{inv} and γ_{inv} for 0.18 µm technology

$\tau_{inv} = 3 R_{eqn} C_g L_n = 3(12.5k\Omega)(2fF/\mu m)(0.2\mu m) = 15ps$

$\gamma_{inv} = \frac{C_{self}}{C_{in}} = \frac{C_{eff}(3W)}{C_g(3W)} = \frac{1fF}{2fF} = \frac{1}{2}$

Optimal Sizing of ~~an~~ Inverter Chain



$$\text{Total Delay} = \sum_{j=1}^{n} \tau_{inv}\left(\frac{C_{j+1}}{C_j} + \gamma_{inv}\right) = \sum_{j=1}^{n} \tau_{inv}\left(\frac{\cancel{C_g} W_{j+1}}{\cancel{C_g} W_j} + \gamma_{inv}\right)$$

$$= \sum_{j=1}^{n} \tau_{inv}\left(\frac{W_{j+1}}{W_j} + \gamma_{inv}\right)$$

$$\text{Consecutive Delay} \equiv D_j = \tau_{inv}\left(\frac{W_j}{W_{j-1}} + \gamma_{inv}\right) + \tau_{inv}\left(\frac{W_{j+1}}{W_j} + \gamma_{inv}\right)$$

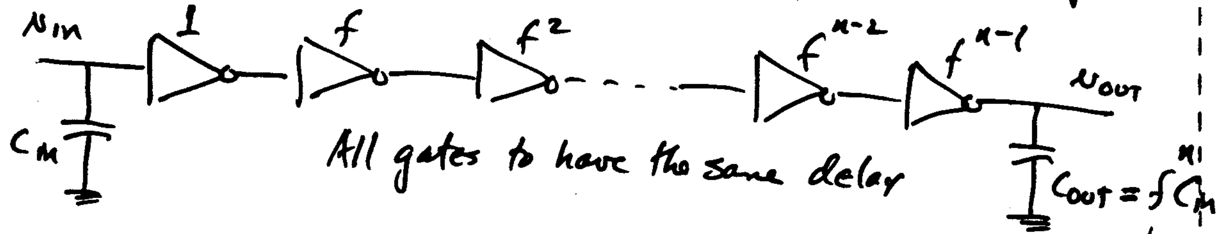
$$\frac{\partial D_j}{\partial W_j} = \tau_{inv}\frac{1}{W_{j-1}} - \tau_{inv}\frac{W_{j+1}}{(W_j)^2} = 0 \rightarrow \underline{W_j = \sqrt{W_{j+1} W_{j-1}}}$$

In general,

$$W_j = \left(W_1 W_2 W_3 \cdots W_{j-1} W_{j+1} \cdots W_{n-1} W_n\right)^{1/n}$$

How do we achieve the corresponding geometrical sizing?

Increase the size of each inverter by a fanout, $f = \frac{W_{j+1}}{W_j}$



Total delay $= n \times$ gate delay $= n \times \tau_{inv}\left(\frac{C_j}{C_{j+1}} + \gamma_{inv}\right)$ $\quad n = \frac{\ln\left(\frac{C_{out}}{C_{in}}\right)}{\ln f}$

$$\boxed{\text{Total delay} = \frac{\ln\left(\frac{C_{out}}{C_{in}}\right)}{\ln f}\,\tau_{inv}\,(f + \gamma_{inv})}$$

What is γ_{inv}?

$$\left.\begin{array}{l}\gamma_{inv} = 0 \rightarrow f = e = 2.72 \\ 0.5 < \gamma_{inv} < 2 \rightarrow 2.5 < f < 4\end{array}\right\} \text{Fig. 6.23}$$

Ex 6.9

1.) Find the optimal fanout for $n = N = 3$ and the total delay if $C_{out} = 200fF$ and $C_{in} = 1fF$. Assume $\tau_{inv} = 7.5ps$ and $\gamma_{inv} = \frac{1}{2}$

$$\ln f = \frac{\ln(C_{out}/C_{in})}{n} = \frac{\ln(200)}{3} = 1.766 \rightarrow f = 5.85$$

Total delay $= 3\,(7.5ps)\,(5.85 + \frac{1}{2}) = \underline{\underline{143ps}}$

2.) Find the optimal n and the delay for this case.

$N = 4 \rightarrow 123ps$

From Fig. 6.23 for $\gamma_{inv} = 0.5$ we see that $f \approx 3.6$

$$\therefore N = \frac{\ln(200)}{\ln(3.6)} = 4.13 \rightarrow \underline{\underline{N = 4}}$$

Total delay $= 4\,(7.5ps)\,[3.6 + 0.5] = \underline{\underline{123ps}}$