**Problem 1 - (25 points)**

Find an algebraic expression for the voltage gain, \(v_{out}/v_{in}\), and the output resistance, \(R_{out}\), of the source follower shown in terms of the small-signal model parameters, \(g_m\) and \(R_L\) (ignore \(r_{ds}\)). If the bias current is 1mA find the numerical value of the voltage gain and the output resistance. Assume that \(K_{N'} = 110\mu A/V^2\), \(V_{TN} = 0.7V\), and \(K_{P'} = 50\mu A/V^2\), \(V_{TP} = -0.7V\).

**Solution**

A small-signal model for this circuit is shown below neglecting \(r_{ds}\) of the transistors.

![Small-signal model diagram](image)

Summing currents at the output node gives,

\[ g_m v_{gs3} = g_m v_{gs1} + G_L v_{out} \]

Also, \(v_{gs3} = -g_m v_{gs1}(1/g_m) \)

\[ g_m v_{gs1} = g_m \left( -\frac{g_m}{g_m} \right) v_{gs1} + G_L v_{out} \]

\[ g_m v_{gs1} \left( 1 + \frac{g_m}{g_m} \right) = G_L v_{out} \]

Setting \(v_{in} = 0\) and applying \(i_t\) and solving for \(v_{out}\) and ignoring \(R_L\) gives,

\[ i_t = g_m v_{gs3} + g_m v_{out} = g_m \left( \frac{g_m}{g_m} \right) v_{out} + g_m v_{out} \]

\[ \frac{v_{out}}{i_t} = \frac{1}{g_m \left( 1 + \frac{g_m}{g_m} \right)} \]

Note that the 1mA splits between M1(M2) and M3 in a ratio of 1 to 100. Therefore, \(I_{D1} = I_{D2} = 9.9\mu A\) and \(I_{D3} = 990.1\mu A\).

\[ g_m = \sqrt{2\cdot110\cdot100\cdot9.9} = 466.71\mu S, \quad g_m = \sqrt{2\cdot50\cdot1\cdot9.9} = 31.47\mu S \]

and \(g_m = \sqrt{2\cdot110\cdot100\cdot990.1} = 3146.7\mu S\)

\[ \frac{v_{out}}{v_{in}} = \frac{466.71\cdot101}{466.71\cdot101 + 1750} = \frac{47.137}{47.137 + 20} = 0.702 \text{ V/V} \]

\[ R_{out} = \frac{1000}{47.137} = 21.2\Omega \]
**Problem 2 - (25 points)**

Using the open-circuit and short-circuit time constant methods, find the two poles of the circuit shown below (assuming that the two poles are far apart from each other). $\beta_o=100$, $I_C=0.5\,mA$, $f_T=1\,GHz$, $C_\mu=0.2\,pF$, $V_T=26\,mV$.

![Circuit Diagram]

**Solution**

$$r_\pi = \beta_o/g_m = 100 \times \frac{26}{0.5} = 5.2\,k\Omega$$

$$\tau_T = \frac{1}{2\pi f_T} = 159ps$$

$$C_\pi = g_m \tau_T - C_\mu = (0.5/26) \times 159\,pF - 0.2\,pF = 2.86\,pF$$

$$R_i = R_\parallel r_\pi = 1.04\,k\Omega$$

**Open circuit time constant method:**

$$P_1 \equiv \frac{1}{\Sigma \tau} = \left[ R_i C_\pi + (R_f + R_L + g_m R_f R_L)C_\mu \right]^{-1} = 22.13 \times 10^6 \,rad/s = 3.52\,MHz$$

**Short circuit time constant method:**

$$P_2 \equiv \Sigma (1/\tau) = \left[ (R_\parallel/\mu) \left[ R_f \parallel R_L \right] C_\pi \right]^{-1} + \left[ R_L C_\mu \right]^{-1} = 7.224 \times 10^9 \,rad/s = 1.15\,GHz$$
Problem 3 - (25 points)

a) For the emitter follower output stage shown below, find the value of $R_i$ for maximum efficiency and find the value of that efficiency. $V_{CC} = -V_{EE} = 2.5\text{V}$, $V_{CE(sat)} = 0.2\text{V}$, $R_L = 10k\Omega$, $V_{BE(on)} = 0.7\text{V}$.

b) A load capacitor of 100pF is attached to the output voltage. If the input voltage suddenly drops from 2.5V to -2.5V, explain what happens at the output and accurately sketch the output voltage as a function of time, specifying its initial and final values and times.

Solution

The $I_Q$ for maximum efficiency is found as,

$$I_Q = \left( \frac{V_{CC} - V_{CE(sat)}}{R_L} \right) = 230\mu\text{A}$$

$$R_i = \left( \frac{-V_{EE} - V_{BE}}{I_Q} \right) = 7.826k\Omega$$

$$P_{L(max)} = \left( \frac{V_{CC} - V_{CE(sat)}}{\sqrt{2}} \right) \left( \frac{I_Q}{\sqrt{2}} \right) = 0.5(2.3\text{V})(0.23\text{mA}) = 0.2645\text{mW}$$

$$P_{supply} = 2V_{CC}I_Q = 2(2.5)(0.23\text{mA}) = 1.15\text{mW}$$

$$\eta = \frac{P_{L(max)}}{P_{supply}} = \frac{1}{4} \left( 1 - \frac{V_{CE(sat)}}{V_{CC}} \right) = 23\%$$

b) The output would slew under such condition. The current will be limited by the bias current:

Slew rate = 0.23mA/100pF = 2.3V/\mu s

[Diagram showing output voltage vs. time with initial and final values and times]
**Problem 4 - (25 points)**

Find the numerical values of all roots and the midband gain of the transfer function $v_{out}/v_{in}$ of the differential amplifier shown. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $\lambda_N = 0.04V^{-1}$. The values of $C_{gs} = 0.2pF$ and $C_{gd} = 20fF$.

**Solution**

A small-signal model appropriate for this circuit is shown.

Summing the currents at the output nodes gives,

$$g_m v_{gs1} + sC_{gd}(v_{out} - v_{in}) + (g_{ds1} + G_L)v_{out} + sC_L v_{out} = 0$$

(Note: we are ignoring the fact that $v_{out}$ and $v_{in}$ should be divided by two since it makes no difference in the results and is easier to write.) Replacing $v_{gs1}$ by $v_{in}$ gives

$$-(g_m - sC_{gd})v_{in} = [(g_{ds1} + G_L) + sC_L + sC_{gd}] v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{-(g_m - sC_{gd})}{s(C_L + C_{gd}) + (g_{ds1} + G_L)} = \frac{-g_m}{(g_{ds1} + G_L)} \left( \frac{1 - \frac{sC_{gd}}{g_m}}{1 + s \left( \frac{C_L + C_{gd}}{g_{ds1} + G_L} \right)} \right)$$

∴  $\text{MGB} = -g_m(r_{ds1}||R_L)$,  $\text{Zero} = \frac{g_m}{C_{gd}}$  and  $\text{Pole} = -\frac{g_{ds1} + G_L}{C_{gd} + C_L}$

$g_m = \sqrt{2\cdot110\cdot100\cdot500} = 3316.7\mu S$  and  $r_{ds} = \frac{1}{\lambda I_D} = \frac{25}{500\mu A} = 50\ k\Omega$

∴  $\text{MGB} = -3.3167mS\cdot(10k\Omega||50k\Omega) = -27.64\ V/V$

$\text{Zero} = \frac{3.3167\times10^{-3}}{20\times10^{-15}} = 1.658\times10^{11}\text{ radians/sec.}$

$\text{Pole} = \frac{-1}{1.02\times10^{-12}(10k\Omega||50k\Omega)} = -1.1176\times10^8\text{ radians/sec.}$