Problem 1 - (20 points - This problem is required)

An open-loop comparator has a gain of $10^4$, a dominant pole of $10^5$ radians/sec., a slew rate of $5V/\mu s$ and an output swing of $1V$. (a.) If $V_{in} = 1mV$ find the propagation delay time of this comparator (the time for the output to go halfway from one state to the other). (b.) Repeat part (a.) if $V_{in} = 10mV$. (c.) Repeat part (a.) if $V_{in} = 100mV$.

Solution

a.) We know that the linear output voltage of a single-pole comparator can be written as,

$$v_{out}(t) = A(1-e^{-t/\tau_1})V_{in}$$

which can be solved for the propagation delay time as

$$t_p = \tau_1 \ln\left(\frac{2k}{2k-1}\right)$$

where $k = \frac{V_{in}}{V_{in}(\text{min})}$ and $V_{in}(\text{min}) = \frac{(V_{OH}-V_{OL})}{A} = \frac{1V}{10^4} = 0.1mV$

We must first determine if the comparator is linear or slewing. The maximum slope occurs at $t = 0$ and is given as

$$\frac{dv_{out}(t)}{dt} = A \cdot p_1 \cdot V_{in} \cdot e^{-t/\tau_1} \cdot \frac{dv_{out}(0)}{dt} = A \cdot p_1 \cdot V_{in} = 10^4 \cdot 10^5 \cdot 10^{-6} = 10^6 = 1V/\mu s$$

∴ The comparator is not slewing and $t_p$ is given as

$$t_p = \tau_1 \ln\left(\frac{2k}{2k-1}\right) = 10^{-5} \ln\left(\frac{20}{20-1}\right) = 0.513\mu s$$

b.) We see the maximum slope for the linear response is $10V/\mu s$ which means that the comparator is slewing. Slewing at a rate of $5V/\mu s$ requires $0.1\mu s$ to go $0.5V$. Therefore,

$$t_p = 0.100\mu s$$

c.) The answer is the same as b.) namely $t_p = 0.100\mu s$
Problem 2 - (20 points - This problem is optional)
Assume the capacitors connected to the drains of M1 and M2 (C1 and C2) are initially discharged. Express \( \Delta V_{out} = v_{o2} - v_{o1} \) as a function of the applied input, \( \Delta V_{in} = v_{i1} - v_{i2} \), in the time domain assuming \( \Delta V_{in} \) is a step input. If \( g_{m1} = g_{m2} = 1\, \text{mS}, \, g_{m3} = g_{m4} = 100\, \mu\text{S}, \) and \( C1 = C2 = 1\, \text{pF} \), what is the propagation delay time (\( \Delta V_{out} = 0.5(V_{OH} - V_{OL}) \)) for a step input of \( \Delta V_{in} = 0.01( V_{OH} - V_{OL} ) \)?

**Solution**

Small-signal model:

The nodal equations corresponding two these two circuits are:

\[
g_{m1}v_{gs1} + g_{ds1}v_{o1} + g_{d3}v_{o1} + sC_1v_{o1} + g_{m3}v_{gs3} = 0
\]

and

\[
g_{m2}v_{gs2} + g_{ds2}v_{o2} + g_{d4}v_{o2} + sC_2v_{o2} + g_{m4}v_{gs4} = 0
\]

We can write that,

\[
(g_{ds1} + g_{ds3} + sC_1)v_{o1} = - g_{m1}v_{gs1} - g_{m3}v_{gs3}
\]

and

\[
(g_{ds2} + g_{ds4} + sC_2)v_{o2} = - g_{m1}v_{gs2} - g_{m4}v_{gs4}
\]

Assuming matching, we get

\[
(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1) = g_{m1}(v_{gs1} - v_{gs2}) + g_{m3}(v_{gs3} - v_{gs4})
\]

or

\[
(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1) = g_{m1}(v_{i1} - v_{i2}) + g_{m3}(v_{o2} - v_{o1})
\]

\[
(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1 - g_{m3}) = (v_{o2} - v_{o1})( sC_1 - g_{m3}) = g_{m1}(v_{i1} - v_{i2})
\]

\[
(v_{o2} - v_{o1}) = \Delta V_{out}(s) = \frac{g_{m1}}{sC_1 - g_{m3}} \Delta V_{in}(s) = \frac{g_{m1}/ (g_{m3}/C_1)}{s - (g_{m3}/C_1)} \frac{\Delta V_{in}}{s}
\]

\[
\Delta V_{out}(s) = \frac{g_{m1}}{C_1} \Delta V_{in} \left[ \frac{k_1}{s} + \frac{k_2}{s - (g_{m3}/C_1)} \right] = \frac{g_{m1}/ (g_{m3}/C_1)}{s - (g_{m3}/C_1)} \frac{\Delta V_{in}}{s}
\]

Solving for \( k_1 \) and \( k_2 \) gives \( k_1 = -(C_1/g_{m3}) \) and \( k_2 = (C_1/g_{m3}) \). Thus,

\[
\Delta V_{out}(s) = \frac{g_{m1}}{g_{m3}} (s - (g_{m3}/C_1)) \frac{1}{s} \Delta V_{in} \rightarrow \Delta V_{out}(t) = \frac{g_{m1}}{g_{m3}} \Delta V_{in} \left[ e^{(g_{m3}/C_1)t} - 1 \right]
\]

If \( \Delta V_{in} = 0.01( V_{OH} - V_{OL} ) \), then

\[
0.5(V_{OH} - V_{OL}) = 0.1(V_{OH} - V_{OL})[e^{10^8 t_p - 1}] \rightarrow 5 = e^{10^8 t_p - 1}
\]

or

\[
e^{10^8 t_p} = 6 \rightarrow t_p = \frac{1}{10^8} \ln(6) = \frac{1.792}{10^8} = 19.92 \, \text{ns}
\]

\[
\]
**Problem 3** - (20 points - This problem is optional)

An internally-compensated, cascode op amp is shown. (a) Derive an expression for the common-mode input range. (b) Find $W_1/L_1$, $W_2/L_2$, $W_5/L_5$, and $W_6/L_6$ when $I_{BIAS} = 80$ A and the input CMR is $-1.25$ V to $+2$ V. Use $K'N = 110$ A/V$^2$, $K'p = 50$ A/V$^2$ and $|V_T| = 0.6$ to $0.8$ V. (c.) Develop an expression for the small-signal differential-voltage gain and output resistance of the cascode op amp.

![Circuit Diagram](image)

**Solution**

(a.) 

$V_{icm}(min) = V_{SS} + V_{DS7}(sat) + V_{DS1}(sat) + V_{T1}(max)$

and

$V_{icm}(max) = V_{DD} - V_{SD5}(sat) - V_{T5}(max) + V_{T1}(min)$ (we will ignore that M8 and M14 cause a more severe upper ICM limit)

$\therefore ICMR = V_{icm}(max) - V_{icm}(min)$

$ICMR = (V_{DD} - V_{SS}) - [V_{DS7}(sat) + V_{DS1}(sat) + V_{T1}(max)] - [V_{SD5}(sat) + V_{T5}(max) - V_{T1}(min)]$

(b.) 

$I_7 = \frac{1}{4} I_{BIAS} = 20$ A

Using $V_{DS}(sat) = \sqrt{\frac{2I}{K'(W/L)}}$ and $\frac{W}{L} = \frac{2I}{K'[V_{DS}(sat)]^2}$ gives,

$V_{DS7}(sat) = 0.191$ V $\rightarrow V_{icm}(min) = -2.5$ V + $0.191$ V + $V_{DS1}(sat)$ + $0.8$ V $\rightarrow V_{DS1}(sat) = 0.259$ V

$\therefore \frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{20}{110 \times 0.259^2} = 2.70$

$V_{icm}(max) = 2.0$ V = $2.5$ V - $V_{SD5}(sat)$ - $0.6$ V + $0.8$ V $\rightarrow V_{SD5}(sat) = 0.3$ V

$\therefore \frac{W_5}{L_5} = \frac{W_6}{L_6} = \frac{20}{50 \times 0.3^2} = 4.44$

(c.) By inspection, we can write:

$^{*}A_v = \left( -g_{m1}r_{ds2} \right) \left( \frac{g_{m14}}{g_{m14} + g_{ds13} + g_{ds14}} \right) \left( \frac{-g_{m8}}{g_{ds8} + g_{ds9}} \right)$
Problem 4 - (20 points - This problem is optional)

George P. Burdell has submitted the following input stage for the design challenge problem in ECE 6412. Assuming that the transistor model parameters are $K_N' = 110 \mu A/V^2$, $V_{TN} = 0.7 V$, $\lambda_N = 0.04 V^{-1}$, $K_P' = 50 \mu A/V^2$, $V_{TP} = -0.7 V$, $\lambda_P = 0.05 V^{-1}$, your job is to check this op amp out. In particular, what is the upper and lower input common mode voltages, what is the minimum power supply that gives zero input common mode range, what is the small-signal voltage gain, and compare this input stage with the classical differential input stage (list advantages and disadvantages).

![Input stage diagram]

**Solution**

GPB Differential Input Stage:

$V_{icm^+} = V_{DD} - V_{SD3}(sat) + V_{T1} - V_{GS6}(50 A) = V_{DD} - V_{SD3}(sat) + V_{T1} - V_{GS6}(sat) - V_{T6}$

$V_{icm^-} = V_{DD} - V_{SD3}(sat) - V_{GS6}(sat)$

$V_{DS1}(sat) = V_{DS6}(sat) = \sqrt{\frac{2E50}{110E10}} = 0.301 V$, $V_{DS5}(sat) = \sqrt{\frac{2E100}{110E10}} = 0.426 V$,

and $V_{SD3}(sat) = \sqrt{\frac{2E50}{50E22}} = 0.301 V$

$\therefore V_{icm^+} = V_{DD} - 0.602 V$ and $V_{icm^-} = 0.426 + 0.301 - 0.301 = 0.426 V$

Note that $V_{DS6} = V_{GS6} - V_{GS7} = 1.001 - 0.73 = 0.271 < V_{DS6}(sat)$ so that M6 is slightly in the active region but this is not a problem.

The minimum $V_{DD}$ is found by letting $V_{icm^+} = V_{icm^-}$ which gives $V_{DD}(min) = 1.028 V$

The small-signal voltage gain is $A_{vd} = \frac{V_{out}}{V_{i^+} - V_{i^-}} = \frac{-g_{m1}}{g_{ds1} + g_{ds3}} = \frac{-g_{m2}}{g_{ds2} + g_{ds4}}$

Classical Differential Input Stage:

$V_{icm^+} = V_{DD} - V_{SD3}(sat) + V_{T1} = V_{DD} - 0.301 + 0.7 = V_{DD} + 0.4 V$

$V_{icm^-} = V_{DS5}(sat) + V_{GS1}(50 A) = 1.427 V$

$V_{DD}(min) = 1.027 V$ and the small-signal voltage gain is the same.
Problem 4 – Continued

Comparison between the two approaches:

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<tr>
<td>$V_{icm}^-$</td>
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<td>1.427V</td>
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<td>$V_{DD}(\text{min})$</td>
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