LECTURE 010 – ECE 4430 REVIEW I
(READING: GHLM - Chap. 1)

Objective
The objective of this presentation is:
1.) Identify the prerequisite material as taught in ECE 4430
2.) Insure that the students of ECE 6412 are adequately prepared

Outline
• Models for Integrated-Circuit Active Devices
• Bipolar, MOS, and BiCMOS IC Technology
• Single-Transistor and Multiple-Transistor Amplifiers
• Transistor Current Sources and Active Loads

MODELS FOR INTEGRATED-CIRCUIT ACTIVE DEVICES

PN Junctions - Step Junction

Barrier potential-
\[ \psi_0 = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) = V_t \ln \left( \frac{N_A N_D}{n_i^2} \right) = U_T \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Depletion region widths-
\[ W_1 = \sqrt{ \frac{2 \varepsilon_s q (\psi_0 - V_D) N_D}{q N_A (N_A + N_D)}} \]
\[ W_2 = \sqrt{ \frac{2 \varepsilon_s q (\psi_0 - V_D) N_A}{q N_D (N_A + N_D)}} \]

Depletion capacitance-
\[ C_j = A \sqrt{ \frac{\varepsilon_s q N_A N_D}{2 (N_A + N_D)}} \sqrt{ \frac{1}{\psi_0 - V_D}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\psi_0}}} \]

Fig. 010-01
**PN-Junctions - Graded Junction**

Graded junction:

\[
\begin{align*}
W_1 &= \frac{2\varepsilon_s q (\psi_o - V_D) N_D}{N_D (N_A + N_D)^m} \\
W_2 &= \frac{2\varepsilon_s q (\psi_o - V_D) N_A}{N_D (N_A + N_D)^m}
\end{align*}
\]

Depletion capacitance:

\[
C_j = A \left( \frac{\varepsilon_s q N_A N_D}{2(N_A + N_D)} \right)^m \frac{1}{(\psi_o - V_D)^m} = \frac{C_{j0}}{\left( 1 - \frac{V_D}{\psi_o} \right)^m}
\]

where \(0.33 \leq m \leq 0.5\).

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**Large Signal Model for the BJT in the Forward Active Region**

Large-signal model for a *npn* transistor:

\[
i_B = \frac{I_s}{\beta_F} \exp \left( \frac{V_{BE}}{V_t} \right)
\]

Assumes \(V_{BE}\) is a constant and \(i_B\) is determined externally.

Large-signal model for a *pnp* transistor:

\[
i_B = -\frac{I_s}{\beta_F} \exp \left( \frac{V_{BE}}{V_t} \right)
\]

Assumes \(V_{BE}\) is a constant and \(i_B\) is determined externally.

---

**Early Voltage:**

Modified large signal model becomes

\[
i_C = I_S \left( 1 + \frac{V_{CE}}{V_A} \right) \exp \left( \frac{V_{BE}}{V_t} \right)
\]

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The Ebers-Moll Equations

The reciprocity condition allows us to write,

\[ \alpha_F I_{EF} = \alpha_R I_{CR} = I_S \]

Substituting into the previous form of the Ebers-Moll equations gives,

\[ i_C = I_S \left( \exp \frac{v_{BE}}{V_t} + 1 \right) - \frac{I_S}{\alpha R} \left( \exp \frac{v_{BC}}{V_t} + 1 \right) \]

and

\[ i_E = -\frac{I_S}{\alpha F} \left( \exp \frac{v_{BE}}{V_t} + 1 \right) + I_S \left( \exp \frac{v_{BC}}{V_t} + 1 \right) \]

These equations are valid for all four regions of operation of the BJT.

Also:

- Dependence of \( \beta_F \) as a function of collector current
- The temperature coefficient of \( \beta_F \) is,

\[ TCF = \frac{1}{\beta_F} \frac{\partial \beta_F}{\partial T} \approx +7000\text{ppm/°C} \]

Simple Small Signal BJT Model

Implementing the above relationships, \( i_c = g_m v_i + g_o v_{ce} \), and \( v_i = r_\pi i_b \), into a schematic model gives,

\[ g_m v_i + g_o v_{ce} \]

Note that the small signal model is the same for either a \( npn \) or a \( pnp \) BJT.

Example:

Find the small signal input resistance, \( R_{in} \), the output resistance, \( R_{out} \), and the voltage gain of the common emitter BJT if the BJT is unloaded (\( R_L = \infty \)), \( v_{out}/v_{in} \), the dc collector current is 1mA, the Early voltage is 100V, and \( \beta_0 \) at room temperature.

\[ g_m = \frac{I_C}{V_t} = \frac{1\text{mA}}{26\text{mV}} = \frac{1}{26} \text{mhos or Siemans} \]

\[ R_{in} = r_\pi = \frac{\beta_0}{g_m} = 100 \cdot 26 = 2.6k\Omega \]

\[ R_{out} = r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{1\text{mA}} = 100k\Omega \]

\[ \frac{v_{out}}{v_{in}} = -g_m r_o = -26\text{mS} \cdot 100\text{kΩ} = -2600\text{V/V} \]
Complete Small Signal BJT Model

The capacitance, \( C_\pi \), consists of the sum of \( C_{je} \) and \( C_b \).

\[
C_\pi = C_{je} + C_b
\]

Example 1

Derive the complete small signal equivalent circuit for a BJT at \( I_C = 1\,mA \), \( V_{CB} = 3\,V \), and \( V_{CS} = 5\,V \). The device parameters are \( C_{je}0 = 10\,fF \), \( n_e = 0.5 \), \( \psi_{0e} = 0.9\,V \), \( C_\mu 0 = 10\,fF \), \( n_c = 0.3 \), \( \psi_{0c} = 0.5\,V \), \( C_{cs0} = 20\,fF \), \( n_s = 0.3 \), \( \psi_{0s} = 0.65\,V \), \( \beta_0 = 100 \), \( \tau_F = 10\,ps \), \( V_A = 20\,V \), \( r_b = 300\,\Omega \), \( r_c = 50\,\Omega \), \( r_{ex} = 5\,\Omega \), and \( r_\mu = 10\beta_0 r_0 \).

Solution

Because \( C_{je} \) is difficult to determine and usually an insignificant part of \( C_\pi \), let us approximate it as 2\( C_{je}0 \).

\[
C_{je} = 20\,fF
\]

\[
C_\mu = \frac{C_{\mu 0}}{1 + \frac{V_{CB}}{\psi_{0c} n_e}} = \frac{10\,fF}{1 + \frac{3}{0.5} 0.3} = 5.6\,fF
\]

\[
C_{cs} = \frac{C_{cs0}}{1 + \frac{V_{CS}}{\psi_{0s} n_s}} = \frac{20\,fF}{1 + \frac{5}{0.65} 0.3} = 10.5\,fF
\]

\[
g_m = \frac{I_C}{V_t} = \frac{1\,mA}{26\,mV} = 38\,mA/V
\]

\[
C_b = \tau_F g_m = (10\,ps)(38\,mA/V) = 0.38\,pF
\]

\[
r_\pi = \frac{\beta_0}{g_m} = \frac{100\cdot 26\,\Omega = 2.6\,k\Omega}{2.6\,k\Omega}
\]

\[
r_\mu = \frac{V_A}{I_C} = \frac{20\,V}{1\,mA} = 20\,k\Omega
\]

\[
r_\mu = 10 \beta_0 r_0 = 20\,M\Omega
\]
**Transition Frequency, \( f_T \)**

\( f_T \) is the frequency where the magnitude of the short-circuit, common-emitter current = 1.

Circuit and model:

Assume that \( r_c \approx 0 \). As a result, \( r_o \) and \( C_{cs} \) have no effect.

\[
V_1 \approx \frac{r_\pi}{1 + r_\pi(C_\pi + C_\mu)s} I_i \quad \text{and} \quad I_o \approx g_m V_1 \quad \Rightarrow \quad \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{\beta_o}{1 + \beta_o} \frac{g_m r_\pi}{g_m} \times \frac{C_\pi + C_\mu}{g_m}
\]

Now,

\[
\beta(j\omega) = \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{\beta_o}{1 + \beta_o} \frac{g_m}{j\omega(C_\pi + C_\mu)}
\]

At high frequencies,

\[
\beta(j\omega) \approx \frac{g_m}{j\omega(C_\pi + C_\mu)} \quad \Rightarrow \quad \text{When } |\beta(j\omega)| = 1 \text{ then } \omega_T = \frac{g_m}{C_\pi + C_\mu} \text{ or } f_T = \frac{1}{2\pi} \frac{g_m}{C_\pi + C_\mu}
\]

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**JFET Large Signal Model**

Large signal model:

Incorporating the channel modulation effect:

\[
i_D = I_{DSS} \left(1 - \frac{v_{GS}}{V_p}\right)^2 (1 + \lambda v_{DS}), \quad v_{DS} \geq v_{GS} - V_p
\]

Signs for the JFET variables:

<table>
<thead>
<tr>
<th>Type of JFET</th>
<th>( V_p )</th>
<th>( I_{DSS} )</th>
<th>( v_{GS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-channel</td>
<td>Positive</td>
<td>Negative</td>
<td>Normally positive</td>
</tr>
<tr>
<td>( n )-channel</td>
<td>Negative</td>
<td>Positive</td>
<td>Normally negative</td>
</tr>
</tbody>
</table>
Frequency Independent JFET Small Signal Model

Schematic:

Parameters:
\[ g_m = \frac{di_D}{dv_{GS}} = -\frac{2I_{DSS}}{V_p} \left(1 - \frac{V_{GS}}{V_p}\right) = g_{m0} \left(1 - \frac{V_{GS}}{V_p}\right) \]

where
\[ g_{m0} = -\frac{2I_{DSS}}{V_p} \]
\[ r_o = \frac{di_D}{dv_{DS}} = \lambda I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2 \approx \frac{1}{\lambda I_D} \]

Typical values of \( I_{DSS} \) and \( V_p \) for a p-channel JFET are -1mA and 2V, respectively. With \( \lambda = 0.02V^{-1} \) and \( I_D = 1mA \) we get \( g_m = 1mA/V \) or 1mS and \( r_o = 50k\Omega \).

Frequency Dependent JFET Small Signal Model

Complete small signal model:

All capacitors are reverse biased depletion capacitors given as,
\[ C_{gs} = \frac{C_{gs0}}{1 + \frac{V_{GS}}{\psi_o}^{1/3}} \] (capacitance from source to top and bottom gates)
\[ C_{gd} = \frac{C_{gd0}}{1 + \frac{V_{GD}}{\psi_o}^{1/3}} \] (capacitance from drain to top and bottom gates)
\[ C_{gss} = \frac{C_{gss0}}{1 + \frac{V_{GSS}}{\psi_o}^{1/2}} \] (capacitance from the gate (p-base) to substrate)

\[ f_T = \frac{1}{2\pi C_{gs} + C_{gd} + C_{gss}} = 30MHz \] if \( g_m = 1mA/V \) and \( C_{gs} + C_{gd} + C_{gss} = 5pF \)
**Simple Large Signal MOSFET Model**

**N-channel reference convention:**

Non-saturation:

\[
i_D = \frac{W \mu_0 C_{ox}}{L} \left[ (v_{GS} - V_T) v_{DS} - \frac{v_{DS}^2}{2} \right] (1 + \lambda v_{DS}) , \ 0 < v_{DS} < v_{GS} - V_T
\]

Saturation:

\[
i_D = \frac{W \mu_0 C_{ox}}{2L} \left( v_{GS} - V_T \right)^2 (1 + \lambda v_{DS}), \ 0 < v_{GS} - V_T < v_{DS}
\]

where:

- \( \mu_0 \) = zero field mobility (cm²/volt·sec)
- \( C_{ox} \) = gate oxide capacitance per unit area (F/cm²)
- \( \lambda \) = channel-length modulation parameter (volts⁻¹)
- \( V_T = V_{T0} + \gamma \left( 2|\phi_f| + |v_{BS}| - 2|\phi_f| \right) \)
- \( V_{T0} \) = zero bias threshold voltage
- \( \gamma \) = bulk threshold parameter (volts⁻⁰·⁵)
- \( 2|\phi_f| \) = strong inversion surface potential (volts)

For p-channel MOSFETs, use n-channel equations with p-channel parameters and invert current.

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**MOSFET Small-Signal Model**

**Complete schematic model:**

\[
g_m = \frac{di_D}{dv_{GS}} = \beta (V_{GS} - V_T) = \sqrt{2} \beta i_D \quad \text{and} \quad g_{ds} = \frac{di_D}{dv_{DS}} = \frac{\lambda i_D}{1 + \lambda v_{DS}} = \lambda i_D
\]

\[
g_{mbs} = \frac{\partial i_D}{\partial v_{BS}} = \left( \frac{\partial i_D}{\partial v_{GS}} \right) \left( \frac{\partial v_{GS}}{\partial v_{BS}} \right) = \beta \left( - \frac{\partial i_D}{\partial v_{T}} \right) \left( \frac{\partial v_{T}}{\partial v_{BS}} \right) = \frac{g_m \gamma}{2 \sqrt{2|\phi_f|} - V_{BS}} = \eta g_m
\]

**Simplified schematic model:**

\[
g_m \approx 10 g_{mbs} \approx 100 g_{ds}
\]

Extremely important assumption:
**MOSFET Depletion Capacitors - \( C_{BS} \) and \( C_{BD} \)**

Model:

\[
C_{BS} = \frac{CJ \cdot AS}{1 - \frac{v_{BS}}{PB}} + \frac{CJSW \cdot PS}{1 - \frac{v_{BS}}{MJSW}}, \quad v_{BS} \leq FC \cdot PB
\]

and

\[
C_{BS} = \frac{CJ \cdot AS}{1 - (1 + MJ)FC + MJ \frac{v_{BS}}{PB}} + \frac{CJSW \cdot PS}{1 - (1 + MJSW)FC + MJSW \frac{v_{BS}}{PB}},
\]

\( v_{BS} > FC \cdot PB \)

where

- \( AS \) = area of the source
- \( PS \) = perimeter of the source
- \( CJSW \) = zero bias, bulk source sidewall capacitance
- \( MJSW \) = bulk-source sidewall grading coefficient

For the bulk-drain depletion capacitance replace "\( S \)" by "\( D \)" in the above equations.

**MOSFET Intrinsic Capacitors - \( C_{GD}, C_{GS} \) and \( C_{GB} \)**

**Cutoff Region:**

\[
C_{GB} = 2C_5 = C_{ox}(W_{eff})(L_{eff}) + 2CGBO(L_{eff})
\]

\[
C_{GS} = C_1 = C_{ox}(LD)W_{eff} = CGSO(W_{eff})
\]

\[
C_{GD} = C_3 = C_{ox}(LD)W_{eff} = CGDO(W_{eff})
\]

**Saturation Region:**

\[
C_{GB} = 2C_5 = CGBO(L_{eff})
\]

\[
C_{GS} = C_1 + (2/3)C_2 = C_{ox}(LD + 0.67L_{eff})(W_{eff}) = CGSO(W_{eff}) + 0.67C_{ox}(W_{eff})(L_{eff})
\]

\[
C_{GD} = C_3 = C_{ox}(LD)W_{eff} = CGDO(W_{eff})
\]

**Active Region:**

\[
C_{GB} = 2C_5 = 2CGBO(L_{eff})
\]

\[
C_{GS} = C_1 + 0.5C_2 = C_{ox}(LD + 0.5L_{eff})(W_{eff}) = (CGSO + 0.5C_{ox}L_{eff})W_{eff}
\]

\[
C_{GD} = C_3 + 0.5C_2 = C_{ox}(LD + 0.5L_{eff})(W_{eff}) = (CGDO + 0.5C_{ox}L_{eff})W_{eff}
\]
**Small-Signal Frequency Dependent Model**

The depletion capacitors are found by evaluating the large signal capacitors at the DC operating point. The charge storage capacitors are constant for a specific region of operation.

Gainbandwidth of the MOSFET:

Assume $V_{SB} = 0$ and the MOSFET is in saturation,

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{1}{2\pi} \frac{g_m}{C_{gs}}$$

Recalling that

$$C_{gs} \approx \frac{2}{3} C_{ox}WL \quad \text{and} \quad g_m = \mu_o C_{ox} \frac{W}{L} (V_{GS}-V_T)$$

gives

$$f_T = \frac{3}{4\pi L^2} (V_{GS}-V_T)$$

**Subthreshold MOSFET Model**

Weak inversion operation occurs when the applied gate voltage is below $V_T$ and pertains to when the surface of the substrate beneath the gate is weakly inverted.

Regions of operation according to the surface potential, $\phi_S$.

- $\phi_S < \phi_F$: Substrate not inverted
- $\phi_F < \phi_S < 2\phi_F$: Channel is weakly inverted (diffusion current)
- $2\phi_F < \phi_S$: Strong inversion (drift current)

Drift current versus diffusion current in a MOSFET:
Large-Signal Model for Subthreshold

Model:
\[ i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t}(1 - e^{-v_{DS}/V_t})(1 + \lambda v_{DS}) \]

where
\[ K_x \] is dependent on process parameters and the bulk-source voltage
\[ n \approx 1.5 - 3 \]

and
\[ V_t = \frac{kT}{q} \]

If \( v_{DS} > 0 \), then
\[ i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t}(1 + \lambda v_{DS}) \]

Small-signal model:
\[ g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_Q = \frac{qI_D}{nkT} \]
\[ g_{ds} = \left. \frac{\partial i_D}{\partial v_{DS}} \right|_Q \approx \frac{I_D}{V_A} \]

SUMMARY

- Models
  - Large-signal
  - Small-signal
- Components
  - pn Junction
  - BJT
  - MOSFET
    - Strong inversion
    - Weak inversion
  - JFET
- Capacitors
  - Depletion
  - Parallel plate