**LECTURE 070 – SINGLE-STAGE FREQUENCY RESPONSE - I**

*(READING: GHLM – 488-504)*

**Objective**

The objective of this presentation is:

1.) Illustrate the frequency analysis of single stage amplifiers
2.) Introduce the Miller technique and the approximate method of solving for two poles

**Outline**

- Differential and Common Frequency Response of the Differential Amplifier
- Emitter/Source Follower Frequency Response
- Common Base/Gate Frequency Response
- Summary

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**FREQUENCY RESPONSE OF THE DIFFERENTIAL AMPLIFIER**

**Differential Mode**

Differential Amplifiers:

![Differential Amplifier Circuit](image)

Half-Circuit Concept:

![Half-Circuit Concept](image)

Note that the following analysis is applicable to the CE and CS configurations.
**Differential Mode Analysis – Miller Approach**

Small Signal Model:

\[
\begin{align*}
\text{For MOS: } & \quad r_b = 0 \quad \text{and} \quad r_\pi = \infty \\
\end{align*}
\]

**Miller Approach:**

Assume that \( R_L < (1/\omega C_f) \), then \( v_{out} \approx -g_m R_L v_1 \)

Therefore, the small-signal model can be approximated as,

\[
\begin{align*}
\text{For MOS: } & \quad r_b = 0 \quad \text{and} \quad r_\pi = \infty
\end{align*}
\]

where

\[
C_m = C_f (1 + g_m R_L)
\]

**Differential Mode Analysis – Continued**

The small-signal analysis of the previous circuit defining \( C_t = C_i + C_m \) is,

\[
\frac{v_{out}}{v_{in}} = \left( \frac{v_{out}}{v_1} \right) \left( \frac{v_1}{v_{in}} \right) = (-g_m R_L) \left( \frac{r_\pi}{1 + r_\pi C_f s} \right) = -g_m R_L \left( \frac{r_\pi}{1 + r_\pi C_f s + R_I + r_b} \right) \left( \frac{1}{1 + \frac{s r_\pi C_f (R_I + r_b)}{R_I + r_b}} \right)
\]

Therefore we see that the gain \((K)\), pole \((p_1)\), and -3dB frequency \((\omega_{-3dB})\) is given as,

<table>
<thead>
<tr>
<th></th>
<th>(K)</th>
<th>(p_1)</th>
<th>(\omega_{-3dB})</th>
</tr>
</thead>
<tbody>
<tr>
<td>BJT</td>
<td>(-g_m R_L \left( \frac{r_\pi}{r_\pi + R_I + r_b} \right))</td>
<td>(r_\pi + R_I + r_b)</td>
<td>(r_\pi + R_I + r_b)</td>
</tr>
<tr>
<td>MOS</td>
<td>(-g_m R_L)</td>
<td>(\frac{1}{-C_i R_I})</td>
<td>(\frac{1}{C_i R_I})</td>
</tr>
</tbody>
</table>
Example 1
If $R_I = 1\,\text{k}\Omega$, $r_b=200\,\Omega$, $I_C=I_D=1\,\text{mA}$, $\beta_o=100$, $K_N'=100\,\mu\text{A/V}^2$, $f_T = 400\,\text{MHz}$ ($I_C=I_D=1\,\text{mA}$), $C_\mu=0.5\,\text{pF}$, $C_{gd}=0.5\,\text{pF}$, $W/L=1000$, $R_L=5\,\text{k}\Omega$, find the gain and $-3\text{dB}$ frequency of the BJT and MOS differential amplifier.

Solution
BJT:

\[
\begin{align*}
\beta_o &= 100 \Rightarrow r_\pi = \frac{\beta_o}{g_m} = 100(26)=2.6\,\text{k}\Omega, \quad \tau_T = \frac{1}{2\pi f_T} = 398\,\text{ps} \Rightarrow C_\pi = g_m \tau - C_\mu = 15.3\,\text{pF} - 0.5\,\text{pF} = 14.8\,\text{pF} \\
K &= \frac{-5000}{26} \left( \frac{2.6}{1+0.2+2.6} \right) = -131.6V/V \\
C_t &= C_\pi + C_\mu(1+g_m R_L) = 14.8\,\text{pF} + 0.5\,\text{pF} \left( 1 + \frac{5000}{26} \right) = 14.8\,\text{pF} + 96.7\,\text{pF} = 111.5\,\text{pF} \\
\omega_{-3\text{dB}} &= \frac{r_\pi + R_I + r_b}{r_\pi C_t (R_I+r_b)} = \frac{2600+1000+200}{2600(1000+200)111.5\,\text{pF}} = 10.92 \times 10^6 \Rightarrow f_{-3\text{dB}} = 1.74\,\text{MHz}
\end{align*}
\]

MOS:

\[
\begin{align*}
g_m &= \sqrt{2 \cdot 1000 \cdot 100 \cdot 1000} = 14.1\,\text{mS} \quad \text{and} \quad g_{gd}+g_{gs} = g_m/\omega_T = 14.1 \times 10^{-3} / 800\,\text{pMHz} = 5.6\,\text{pF} \\
C_g &= 5.6\,\text{pF} - 0.5\,\text{pF} = 5.1\,\text{pF}, \quad C_t = 5.1\,\text{pF} + 0.5\,\text{pF}(1+14.1 \cdot 5) = 5.1\,\text{pF} + 35.7\,\text{pF} = 40.8\,\text{pF} \\
K &= -14.1 \cdot 5 = -70.5V/V \quad \text{and} \quad \omega_{-3\text{dB}} = \frac{1}{40.8\,\text{pF}(1000)} = 24.5 \times 10^6 \Rightarrow f_{-3\text{dB}} = 3.90\,\text{MHz}
\end{align*}
\]

Differential Amplifier – Exact Frequency Response
The second method solves for the poles without using the Miller approximation.

Small-signal model:

\[
\begin{align*}
\text{For MOS:} \quad r_b &= 0 \quad \text{and} \quad r_\pi = \infty
\end{align*}
\]

Nodal Equations:

\[
\begin{align*}
I_{in} &= [G_1 + s(C_i + C_f)]V_1 - [sC_f]V_{out} \quad \text{and} \quad 0 = [g_m - sC_f]V_1 + [G_L + sC_f]V_{out}
\end{align*}
\]

Solving using Cramer’s rule gives,

\[
\begin{align*}
\frac{v_{out}(s)}{v_{in}(s)} &= \frac{-g_m - sC_f}{G_1 G_L + s[G_1 C_f G_L C_f + G_1 C_f + g_m C_f] + s^2[C_f C_i]} \\
\frac{v_{out}(s)}{v_{in}(s)} &= \left( \frac{-g_m R_L R_1}{R_I + r_b} \right) \left[ 1 + s \left( C_f / g_m \right) \right] \\
\text{or} \quad \frac{v_{out}(0)}{v_{in}(0)} &= -g_m R_L \left( \frac{r_\pi}{R_I + r_b + r_\pi} \right)
\end{align*}
\]

Note that the gain is

\[
\begin{align*}
\frac{v_{out}(s)}{v_{in}(s)} &= \frac{-g_m - sC_f}{G_1 G_L + s[G_1 C_f G_L C_f + G_1 C_f + g_m C_f] + s^2[C_f C_i]} \\
\text{or} \quad \frac{v_{out}(s)}{v_{in}(s)} &= \left( \frac{-g_m R_L R_1}{R_I + r_b} \right) \left[ 1 + s \left( C_f / g_m \right) \right] \\
\text{or} \quad \frac{v_{out}(0)}{v_{in}(0)} &= -g_m R_L \left( \frac{r_\pi}{R_I + r_b + r_\pi} \right)
\end{align*}
\]
**Differential Amplifier – Exact Frequency Response**

In general, \( D(s) = \left(1 - \frac{s}{p_1}\right) \left(1 - \frac{s}{p_2}\right) = 1 - s \left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2} \rightarrow D(s) = 1 - \frac{s}{p_1} + \frac{s^2}{p_1 p_2} \), if \(|p_2| >> |p_1|

\[
\begin{align*}
p_1 &= \frac{-1}{R_L C_f + R_1 C_f + R_1 R_L C_f} \
p_2 &= \frac{-[R_f R_1 C_f + R_1 R_L C_f] - g_m R_f}{R_1 R_2 C_f C_c} \approx \frac{g_m}{C_f} \end{align*}
\]

where \( g_m R_L > 1 \)

The Miller approximation gave,

\[
\begin{align*}
p_1 &\approx \frac{r_f + R_f + r_b}{r_f C_f (R_f + r_b)} \approx c \quad \text{and} \quad p_1 \approx \frac{1}{C_c R_f} \approx \frac{1}{g_m R_L C_f R_f}
\end{align*}
\]

which verifies the two methods.

**Common-Mode Analysis of the Differential Amplifier**

Assumptions: Tail capacitance is dominant and self-resistance is negligible.

\[
\begin{align*}
A_{cm} &= \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{Z_T} \quad \text{where} \quad Z_T = \frac{2 R_T}{1 + s R_T C_T} \\
A_{cm} &= \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{2 R_T (1 + s R_T C_T)} \\
CMRR &= \frac{A_{dm}}{A_{cm}} = \frac{\left(\frac{g_m 2 R_T r_f}{r_f + R_f + r_b} \frac{1}{1 + s R_T C_T}\right)}{\left(1 + s / \omega_f\right) (1 + s / p_1)}
\end{align*}
\]
**CMRR Frequency Response**

\[ |A_{cm}| \text{ dB} \]
\[ |A_{dm}| \text{ dB} \]
\[ \text{CMRR dB} \]

-6dB/octave
-12dB/octave

High Frequency Common Mode Poles

\[ |p_1|/2\pi \]

\[ \frac{1}{2\pi R_T C_T} \]

**Voltage Buffers**

Define \( R'_I = R_I + r_p \) and write,

\[ V_{in} = I_i R'_I + V_1 + V_{out}, \quad I_i = \frac{V_1}{z \pi}, \quad \text{and} \quad I_i + g_m V_1 = \frac{V_{out}}{R_L} \]

Combining the last three terms gives,

\[ V_1 = \frac{V_{out}}{R_L} \left( \frac{1}{g_m + \frac{1}{r_\pi} (1 + s C_i r_\pi)} \right) + V_{out} \]

Finally,

\[ V_{in} = \left( \frac{R'_I}{z \pi} + 1 \right) \frac{V_{out}}{R_L} \frac{1}{g_m + \frac{1}{r_\pi} (1 + s C_i r_\pi)} + V_{out} \]

where

\[ z_1 = \frac{g_m + (1/r_\pi)}{C_i}, \quad p_1 = \frac{1}{R_1 C_i}, \quad \text{and} \quad R_1 = r_\pi \frac{R'_I + R_L}{1 + g_m R_L} \]
Frequency Response of the Emitter Follower

Assume that $g_m R_L >> 1$ and $g_m R_L >> (R'_I + R_L)/r_\pi$, then

$$z_1 = \frac{g_m (1/r_\pi)}{C_i} \approx \frac{g_m}{C_\pi} \quad \text{and} \quad p_1 = -\frac{1}{R_1 C_\pi}$$

Example

Calculate the transfer function for an emitter follower with $C_\pi = 10\text{pF}$, $C_\mu = 0$, $R_L = 2\text{k}\Omega$, $R_I = 50\Omega$, $r_b = 150\Omega$, $\beta = 100$, and $I_C = 1\text{mA}$.

From the data, $g_m = 1/26S = 38.5\text{mS}$, $r_\pi = 2.6k\Omega$, and $R'_I = R_I + r_b = 200\Omega$.

$$z_1 \approx \frac{g_m}{C_\pi} = -\frac{3.85 \times 10^9}{10^{-11}} = -3.85 \times 10^9 \text{ rad/s} = |\omega_1|$$

$$R_1 = r_\pi || (R'_I + R_L) = 2.6k || 1+76.9 = 27.9\Omega$$

$$p_1 = -\frac{1}{R_1 C_\pi} = -\frac{1}{27.9 \times 10^{-11}} = -3.58 \times 10^9 \text{ rad/s} (570\text{MHz})$$

Note the pole and zero are closely spaced. Should consider the influence of $C_\mu$ for $\omega_{3dB}$.

$$\frac{v_{out}}{v_{in}} = \frac{g_m R_L + R_L}{1 + g_m R_L + \frac{R'_I + R_L}{r_\pi}} \approx \frac{76.9 + 2000}{2600} = 0.986$$

Emitter Follower Frequency Response-Continued

Include the influence of $C_\mu$.

The pole due to $C_\mu$ is approximately, $p_2 = -1/R'_I C_\mu$. For $C_\mu = 1\text{pF}$, $p_2 \approx 2\pi(795\text{MHz})$

The emitter follower bandwidth is still quite good even considering $C_\mu$.

Next, we will consider the input and output impedances of the emitter follower in the next lecture.