LECTURE 090 – MULTIPLE-STAGE FREQUENCY RESPONSE - I
(READING: GHLM – 516-527)

Objective

The objective of this presentation is:

1.) Develop methods for the frequency analysis of multiple stage amplifiers

2.) Illustrate by examples

Outline

- Dominant Pole Approximation
- Zero-Value (Open-circuit) Time Constant Analysis
- Examples
- Short-Circuit Time Constants
- Examples
- Summary

Dominant Pole Approximation

If one of the poles is significantly closer to the origin of the complex frequency plane, its magnitude is a good approximation to the –3dB frequency.

Consider the following general transfer function:

$$A(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} = \frac{K}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\cdots\left(1 - \frac{s}{p_n}\right)}$$

Equating denominator terms gives,

$$b_1 = -\frac{1}{p_1} - \frac{1}{p_2} \cdots - \frac{1}{p_n} = \sum_{i=1}^{n}\left(-\frac{1}{p_i}\right)$$

If $|p_1| << |p_2|, |p_3|, \ldots$ then $b_1 \approx -\frac{1}{p_1}$ and $\omega_{-3dB} \approx |p_1| = \frac{1}{b_1}$

Complex frequency plane:

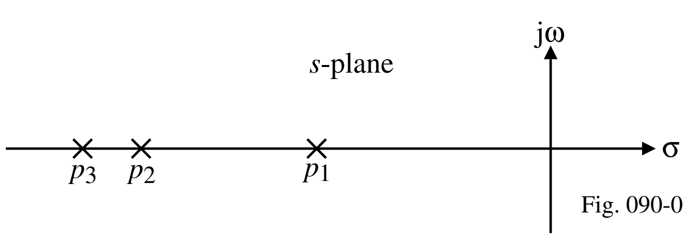


Fig. 090-01

Open-Circuit Time Constant Analysis

This method is suitable for finding the dominant pole of a circuit with multiple capacitors.

Consider the following n-port network,

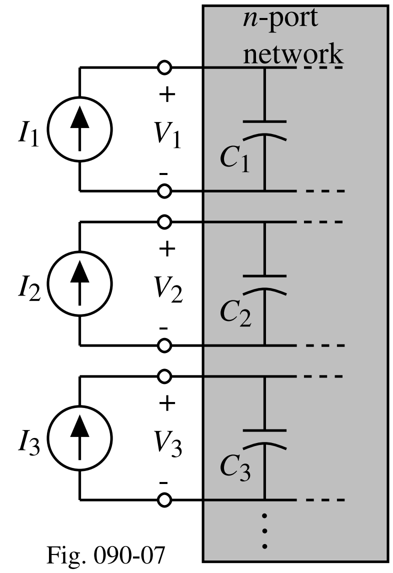


Fig. 090-07

We may express the nodal equations as,

$$\begin{aligned}
I_1 &= (g_{11}+sC_1)V_1 + g_{12}V_2 + g_{13}V_3 + \cdots \\
I_2 &= g_{21}V_1 + (g_{22}+sC_2)V_2 + g_{23}V_3 + \cdots \\
I_3 &= g_{31}V_1 + g_{32}V_2 + (g_{33}+sC_3)V_3 + \cdots \\
&\vdots
\end{aligned}$$

The determinant of the above can be expressed as

$$\Delta(s) = K_0 + K_1 s + K_2 s^2 + K_3 s^3 = K_0(1 + b_1 s + b_2 s^2 + b_3 s^3)$$

where $K_0 = \Delta(C_i = 0)$ for all $i \equiv \Delta_0$

Consider now the K_1 which involves a single capacitor and is given as

$$K_1 = h_1 sC_1 + h_2 sC_2 + h_3 sC_3$$

The h_i terms can be evaluated by expanding the determinant about each row:

First row: $\Delta(s) = (g_{11}+sC_1)\Delta_{11} + g_{12}\Delta_{12} + g_{13}\Delta_{13} \Rightarrow h_1 = \Delta_{11}(C_i{=}0,\ i{\neq}1)$

Open-Circuit Time Constant Analysis - Continued

Second row: $\Delta(s) = g_{21}\Delta_{12} + (g_{22}+sC_2)\Delta_{22} + g_{23}\Delta_{23} \Rightarrow h_2 = \Delta_{22}(C_i{=}0,\ i{\neq}2)$

Third row: $\Delta(s) = g_{31}\Delta_{31} + g_{32}\Delta_{32} + (g_{33}+sC_1)\Delta_{33} \Rightarrow h_3 = \Delta_{33}(C_i{=}0,\ i{\neq}3)$

$\therefore \quad K_1 = C_1\Delta_{11}(C_i{=}0,\ i{\neq}1) + C_2\Delta_{22}(C_i{=}0,\ i{\neq}2) + C_3\Delta_{33}(C_i{=}0,\ i{\neq}3)$

Finally,

$$b_1 = \frac{K_1}{K_0} = \frac{\Delta_{11}(C_i{=}0,\ i{\neq}1)}{\Delta_0}C_1 + \frac{\Delta_{22}(C_i{=}0,\ i{\neq}2)}{\Delta_0}C_2 + \frac{\Delta_{33}(C_i{=}0,\ i{\neq}3)}{\Delta_0}C_3$$

If we realize that the driving-point impedance of the i-th port are expressed as $\dfrac{V_i}{I_i} = \dfrac{\Delta_{11}}{\Delta(s)}$

then $R_{1o} = \dfrac{\Delta_{11}(C_i{=}0,\ i{\neq}1)}{\Delta_0}$, $R_{2o} = \dfrac{\Delta_{22}(C_i{=}0,\ i{\neq}2)}{\Delta_0}$, $R_{3o} = \dfrac{\Delta_{33}(C_i{=}0,\ i{\neq}3)}{\Delta_0}$, $\cdots$

are the driving-point impedances at ports 1, 2, 3,… with all capacitors set equal to zero.

$\therefore \quad b_1 = R_{1o}C_1 + R_{2o}C_2 + R_{3o}C_3 + \cdots$

where $R_{io}C_i$ are called the *open-circuit time constants*.

If there are no dominant zeros, then the dominant pole, p_1, is given as

$$\omega_{-3\text{dB}} \approx |p_1| = \frac{1}{b_1} = \frac{1}{R_{1o}C_1 + R_{2o}C_2 + R_{3o}C_3 + \cdots} = \frac{1}{\sum_{i=1}^{n}(R_{io}C_i)}$$

Example 1

A more exact model of the common-emitter BJT is shown below. Using the open-circuit time constant approach, find an expression for the –3dB frequency.

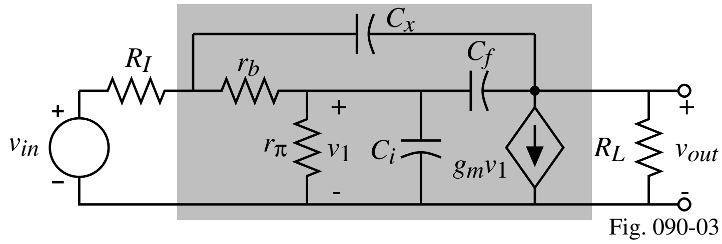


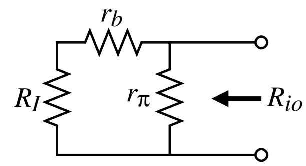
Fig. 090-03

Solution

The procedure involves finding the open-circuit time constants.

R_{io}(Corresponds to C_i):

$$\therefore \quad R_{io} = r_\pi || (r_b + R_I)$$



R_{fo}(Corresponds to C_f):

$$V_t = I_t r_\pi || (r_b + R_I) + (I_t + g_m V_1) R_L$$

$$V_1 = I_t r_\pi || (r_b + R_I) = I_t R_{io}$$

$$\therefore \quad V_t = I_t R_{io} + I_t R_L + I_t g_m R_{io} R_L$$

$$R_{fo} = R_{io} + R_L + g_m R_{io} R_L$$

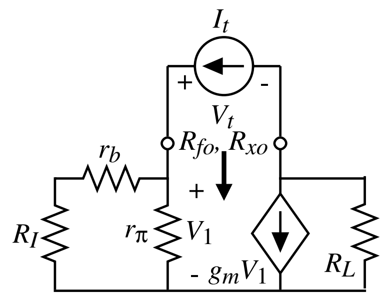


Fig. 090-04

Example 1 - Continued

R_{xo}(Corresponds to C_x):

$$V_t = I_t (r_\pi + r_b) || R_I + (I_t + g_m V_1) R_L$$

$$V_1 = I_t [R_I r_\pi / (R_I + r_b + r_\pi)]$$

$$\therefore \quad V_t = I_t (r_\pi + r_b) || R_I + I_t R_L + I_t g_m R_L [(r_\pi + r_b) || R_I]$$

$$R_{xo} = (r_\pi + r_b) || R_I + R_L + g_m [R_I r_\pi / (R_I + r_b + r_\pi)] R_L$$

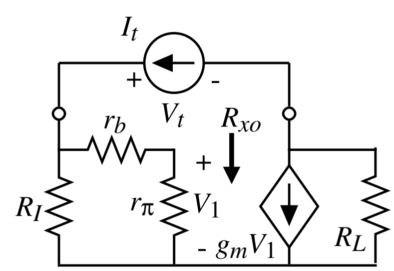


Fig. 090-05

$$\omega_{-3dB} \approx \frac{1}{R_{io}C_i + R_{fo}C_f + R_{xo}C_x}$$

$$\omega_{-3dB} = \frac{1}{r_\pi || (r_b+R_I) C_i + (R_{io}+R_L+g_m R_{io} R_L) C_f + \{(r_\pi + r_b) || R_I + R_L + g_m R_L [R_I r_\pi / (R_I + r_b + r_\pi)]\} C_x}$$

$$\omega_{-3dB} \approx \frac{1}{r_\pi || (r_b+R_I) C_i + [g_m R_L (r_\pi || R_I)](C_f + C_x)} \quad \text{if } r_b \to 0$$

Let $R_I = 1\text{k}\Omega$, $r_b = 200\Omega$, $I_C = 1\text{mA}$, $\beta_o = 100$, $f_T = 400\text{MHz}$ ($I_C = 1\text{mA}$), $C_\mu = 0.5\text{pF}$, $C_x = 0$, $R_L = 5\text{k}\Omega$, and find the –3dB frequency.

$$r_\pi = \frac{\beta_o}{g_m} = 100(26) = 2.6\text{k}\Omega, \quad \tau_T = \frac{1}{2\pi f_T} = 398\text{ps} \Rightarrow C_\pi = g_m \tau_T - C_\mu = 15.3\text{pF} - 0.5\text{pF} = 14.8\text{pF}$$

$$\therefore \ \omega_{-3dB} \approx \frac{10^{-9}}{2.6||1.2(14.8) + (5000/26)(2.6||1)0.5} = \frac{10^{-9}}{12.2 + 69.4} = 12.3\text{x}10^6 \text{ rad/s (1.95MHz)}$$

Example 2 - Cascade Voltage-Amplifier Frequency Response

Calculate the –3dB frequency of the cascade voltage amplifier shown which has the following parameters:

$R_I = 10\text{k}\Omega$	$R_{L1} = 10\text{k}\Omega$	$R_{L2} = 5\text{k}\Omega$
$C_{gs1} = 5\text{pF}$	$C_{gs2} = 10\text{pF}$	$C_{gd1} = C_{gd2} = 1\text{pF}$
$C_{bd1} = C_{bd2} = 2\text{pF}$	$g_{m1} = 3\text{mA/V}$	$g_{m2} = 6\text{mA/V}$

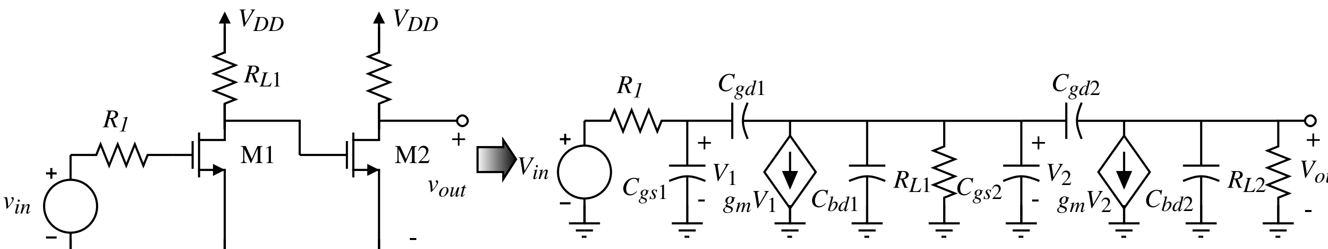


Fig. 090-06

Finding the open-circuit time constants:

C_{gs1}: $R_{gs01} = R_I = 10\text{k}\Omega$ C_{gs2}: $R_{gs02} = R_{L1} = 10\text{k}\Omega$

C_{gd1}: (Use the model to the right)

$$V_t = I_tR_I + (I_t + g_{m1}V_1)R_{L1} = I_tR_I + (I_t + g_{m1}I_tR_I)R_{L1}$$

$$\therefore \quad R_{gd01} = \frac{V_t}{I_t} = R_I + R_{L1} + g_{m1}R_IR_{L1} = 20\text{k}\Omega + 300\text{k}\Omega = 320\text{k}\Omega$$

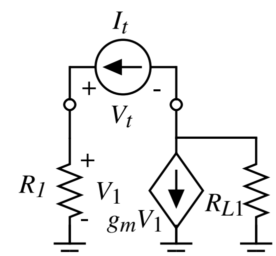


Fig. 090-07

Example 2 – Continued

C_{gd2}: (Can use the results of C_{gd1})

$$\therefore \quad R_{gd02} = R_{L1} + R_{L2} + g_{m2}R_{L1}R_{L2} = 10\text{k}\Omega + 5\text{k}\Omega + 300\text{k}\Omega = 315\text{k}\Omega$$

C_{bd1}: $R_{bd01} = R_{L1} = 10\text{k}\Omega$ C_{bd2}: $R_{bd02} = R_{L2} = 5\text{k}\Omega$

$$\omega_{\text{-3dB}} \approx \frac{1}{\Sigma T_0} = \frac{1}{R_{gs01}C_{gs1}+R_{gs02}C_{gs2}+R_{gd01}C_{gd1}+R_{gd02}C_{gd2}+R_{bd01}C_{bd1}+R_{bd02}C_{bd2}}$$

$$= \frac{10^9}{10\cdot5+10\cdot10+\cdot320\cdot1+315\cdot1+10\cdot2+5\cdot2} = \frac{10^9}{815} = 1.227\text{x}10^6 \text{ rad/s} \rightarrow f_{\text{-3dB}} = 195\text{kHz}$$

- Computer simulation gives poles at –205kHz, -4.02MHz, and –39.98MHz and two zeros at +477MHz and +955MHz.
- How important is it for the circuit to have a dominant pole for the open-circuit time constant approach?

  For a circuit with two identical poles, the –3dB frequency is

  $$\omega_{\text{-3dB}} = \omega_x\sqrt{\sqrt{2} - 1)} = 0.64\omega_x$$

  The open-circuit time constant approach gives

  $$\omega_{\text{-3dB}} = \omega_x/2 = 0.5\omega_x \quad \rightarrow \quad 22\% \text{ error and is pessimistic}$$

Example 3 - Cascode Voltage-Amplifier Frequency Response

Calculate the –3dB frequency of the cascade voltage amplifier shown which has the following parameters:

$R_I = 10\text{k}\Omega$	$R_{L2} = 10\text{k}\Omega$	$C_{gs1} = 5\text{pF}$
$C_{gs2} = 10\text{pF}$	$C_{gd1} = C_{gd2} = 1\text{pF}$	$C_{bd1} = C_{bd2} = 2\text{pF}$
$g_{m1} = 3\text{mA/V}$	$g_{m2} = 6\text{mA/V}$	$r_{ds2} = 50\text{k}\Omega$

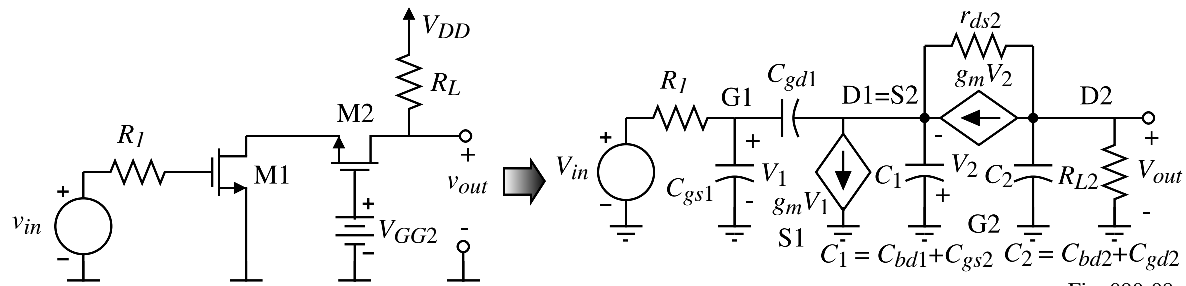


Fig. 090-08

Finding the open-circuit time constants:

C_{gs1}: $R_{gs01} = R_I = 10\text{k}\Omega$

C_{gd1}: $R_{gd01} = ?$

$$V_t = I_tR_I + (I_t + g_{m1}V_1)R_{S2} = I_tR_I + (I_t + g_{m1}I_tR_I)R_{S2}$$

$$\therefore \quad R_{gd01} = \frac{V_t}{I_t} = R_I + R_{S2} + g_{m1}R_IR_{S2} = R_I + R_{S2}(1+g_{m1}R_I)$$

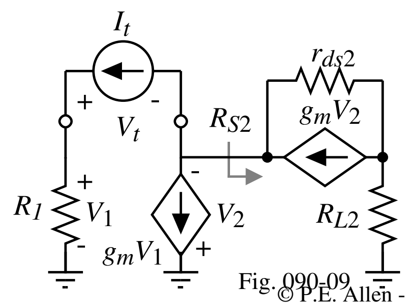


Fig. 090-09

Example 3 – Continued

But what is R_{S2}?

Using the model shown, we see that

$$I_t = g_{m2}V_t\left(\frac{r_{ds2}}{r_{ds2}+R_{L2}}\right) \quad \Rightarrow \quad R_{S2} = \frac{V_t}{I_t} = \frac{r_{ds2}+R_{L2}}{g_{m2}r_{ds2}}$$

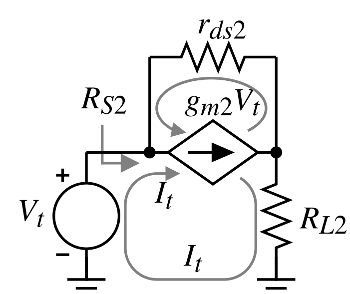


Fig. 090-10

Note that unless $r_{ds2} > R_{L2}$ that R_{S2} is greater than $1/g_{m1}$.

$$\therefore \quad R_{gd01} = R_I + (1+g_{m1}R_I)\left(\frac{r_{ds2}+R_{L2}}{g_{m2}r_{ds2}}\right) = 10\text{k}\Omega + 31\cdot 0.2\text{k}\Omega = 16.2\text{k}\Omega$$

C_1: $R_{01} = R_{S2} = \dfrac{r_{ds2}+R_{L2}}{g_{m2}r_{ds2}} = 0.2\text{k}\Omega$ $\qquad C_2$: $R_{01} = R_{L2} = 10\text{k}\Omega$

$$\omega_{\text{-3dB}} \approx \frac{1}{\Sigma T_0} = \frac{1}{R_{gs01}C_{gs1}+R_{gd01}C_{gd1}+R_{01}C_1+R_{02}C_2}$$

$$= \frac{1}{R_{gs01}C_{gs1}+R_{gd01}C_{gd1}+R_{01}(C_{bd1}+C_{gs2})+R_{02}(C_{bd2}+C_{gd2})}$$

$$= \frac{10^9}{10\cdot 5+16.2\cdot 1+0.2\cdot 12+10\cdot 3} = \frac{10^9}{64.8} = 15.43\text{x}10^6 \text{ rad/s} \rightarrow f_{\text{-3dB}} = 2.46\text{MHz}$$

Note that the Miller effect, R_{gd01}, is less in the cascode amplifier (16.2kΩ compared with 320kΩ).

SUMMARY

- Developed the background for the open-circuit time constant analysis
  - Good for amplifiers with multiple capacitors
  - Works well if one of the poles is dominant, okay if not (pessimistic approx.)
- Illustrated the open-circuit time constant analysis
  - Cascaded MOSFET amplifier
  - Cascode MOSFET amplifier
- The input impedance to the cascoding stage depends on what is connected to the output of the cascoding stage.

$$R_{S2} = \frac{V_t}{I_t} = \frac{r_{ds2}+R_{L2}}{g_{m2}r_{ds2}}$$

We will continue the multiple amplifier analysis techniques in the following lecture.