LECTURE 100 – MULTIPLE-STAGE FREQUENCY RESPONSE - II
(READING: GHLM – 527-537)

Objective

The objective of this presentation is:

1.) Develop methods for the frequency analysis of multiple stage amplifiers

2.) Illustrate by examples

Outline

- Dominant Pole Approximation
- Zero-Value (Open-circuit) Time Constant Analysis
- Examples
- Short-Circuit Time Constants
- Examples
- Summary

Example 1 – Cascode Differential Amplifier

Calculate the low-frequency, small-signal voltage gain and the –3dB frequency of the circuit shown using the following data:

$R_I = 1\text{k}\Omega$	$R_E = 75\Omega$	$R_3 = 4\text{k}\Omega$	$R_L = 1\text{k}\Omega$
$R_1 = 4\text{k}\Omega$	$R_2 = 10\text{k}\Omega$	$V_{CC} = -V_{EE} = 10\text{V}$	$\beta = 200$
$V_{BE(\text{on})} = 0.7\text{V}$	$\tau_F = 0.25\text{ns}$	$r_b = 200\Omega$	$r_c(\text{active}) = 150\Omega$
$C_{je0} = 1.3\text{pF}$	$C_{\mu 0} = 0.6\text{pF}$	$\psi_{0c} = 0.6\text{V}$	$C_{cs0} = 2\text{pF}$
$\psi_{0s} = 0.58\text{V}$	$n_s = 0.5$		

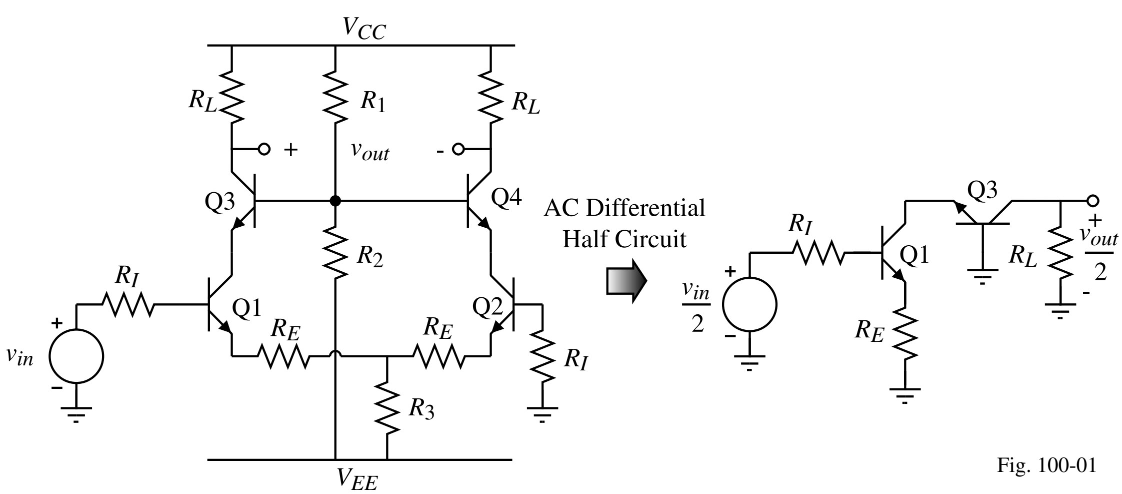

Fig. 100-01

Example 1 – Continued

DC Calculations:

$$V_{B3} = V_{CC} - \frac{R_1}{R_1+R_2}(V_{CC} - V_{EE}) = 10 - \frac{4}{14}(20) = 4.3\text{V}$$

$$V_{C1} = V_{B3} - V_{BE3(on)} = 3.6\text{V}$$

Assume the bases of Q1 and Q2 are at 0V,

$$I_{C1} = \frac{V_{EE} - V_{BE1(on)}}{R_E+2R_3} = \frac{10-0.7}{8.075}\text{ mA} = 1.15\text{mA}$$

$$V_{C3} = V_{CC} - I_{C1}R_L = 10\text{V}-1.15\text{V} = 8.85\text{V}$$

Small-signal model:

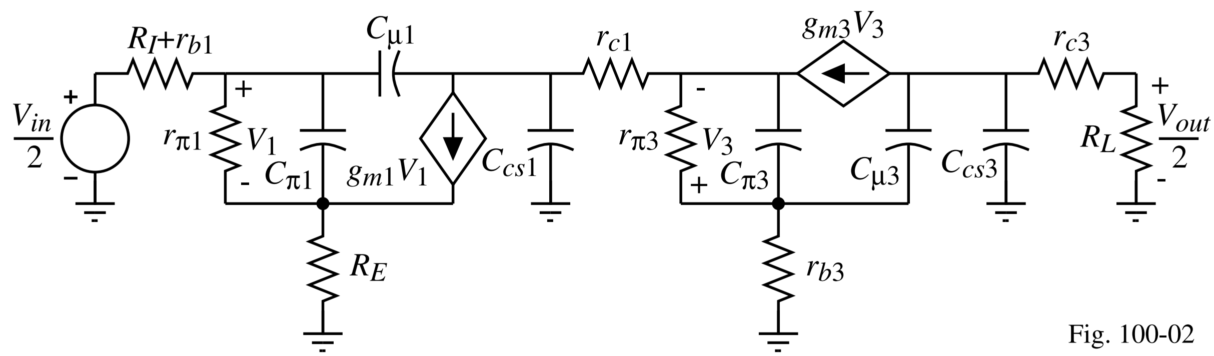

Fig. 100-02

Example 1 – Continued

The low-frequency gain is found as,

$$\frac{V_{out}}{V_{in}} \approx -\frac{R_{i1}}{R_{i1}+R_I}\frac{g_{m1}R_L}{1+g_{m1}R_E} \quad \text{where } R_{i1} = r_{\pi1} + (1+\beta)R_E$$

$$g_{m1} = g_{m3} = \frac{1.15\text{mA}}{26\text{mV}} = 44.2\text{mA/V} \qquad \text{and} \qquad r_{\pi1} = r_{\pi3} = \frac{\beta}{g_m} = \frac{200}{44.2}\text{ k}\Omega = 4525\Omega$$

Thus, $R_{i1} = 4.525\text{k}\Omega + 201(75\Omega) = 19.6\text{k}\Omega$

$$\therefore \quad \frac{V_{out}}{V_{in}} \approx -\frac{19.6}{19.6+1.2}\frac{0.0442\cdot1000}{1+0.0442\cdot75} = -0.942\cdot10.24 = -9.65\text{V/V}$$

-3dB frequency:

First calculate the capacitances-

$C_{je} \approx 2C_{je0} = 2.6\text{pF}$, $C_{b1} = \tau_F g_m = 0.25\text{x}10^{-9}\cdot44.2\text{x}10^{-3} = 11.1\text{pF}$, $\therefore C_{\pi1} = C_{b1}+C_{je1} = 13.7\text{pF}$

Also, $C_{\pi3} = C_{\pi1} = 13.7\text{pF}$ because the collector currents are equal.

$$C_{\mu1}(V_{CB1}=3.6\text{V}) = \frac{C_{\mu0}}{\sqrt{1+(V_{CB1}/\psi_{0c})}} = \frac{0.6}{\sqrt{1+(3.6/0.6)}}\text{ pF} = 0.23\text{pF}$$

$$C_{cs1}(V_{CS1}=13.6\text{V}) = \frac{C_{cs0}}{\sqrt{1+(V_{CS1}/\psi_{0s})}} = \frac{2}{\sqrt{1+(13.6/0.58)}}\text{ pF} = 0.40\text{pF}$$

Example 1 – Continued

$$C_{\mu3}(V_{CB3}{=}8.85\text{-}4.3{=}4.55\text{V}) = \frac{C_{\mu0}}{\sqrt{1+(V_{CB3}/\psi_{0c})}} = \frac{0.6}{\sqrt{1+(4.55/0.6)}}\text{pF} = 0.20\text{pF}$$

$$C_{cs3}(V_{CS3}{=}18.85\text{V}) = \frac{C_{cs0}}{\sqrt{1+(V_{CS3}/\psi_{0s})}} = \frac{2}{\sqrt{1+(18.85/0.58)}}\text{pF} = 0.35\text{pF}$$

Next, the O.C. time constants-

$R_{\pi01}$: Using the model shown, we get

$$I_t = \frac{V_t}{r_{\pi1}} + \frac{V_t+V_E}{R_I+r_{b1}}$$

and

$$I_t + \frac{V_E}{R_E} = \frac{V_t}{r_{\pi1}} + g_{m1}V_t \rightarrow V_E = R_E\left(\frac{V_t}{r_{\pi1}} + g_{m1}V_t - I_t\right)$$

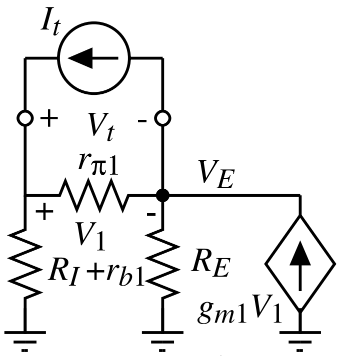

Fig. 100-03

Combining equations gives,

$$I_t\left(1+\frac{R_E}{R_I+r_{b1}}\right) = V_t\left(\frac{1}{r_{\pi1}} + \frac{1}{R_I+r_{b1}} + \frac{R_E}{r_{\pi1}(R_I+r_{b1})} + \frac{g_{m1}R_E}{R_I+r_{b1}}\right)$$

Multiplying through by $r_{\pi1}(R_I+r_{b1})$ gives,

$$I_t[r_{\pi1}(R_I+r_{b1}) + r_{\pi1}R_E] = V_t[R_I + r_{b1} + r_{\pi1} + R_E + g_{m1}r_{\pi1}R_E]$$

or

$$R_{\pi01} = \frac{V_t}{I_t} = \frac{r_{\pi1}(R_I+r_{b1}+R_E)}{R_I + r_{b1} + r_{\pi1} + R_E(1+\beta_1)} = \frac{4525(1000+200+75)}{1000+200+4525+75(1+200)} = 277\Omega$$

$\therefore \quad R_{\pi01}C_{\pi1} = 13.7\cdot0.277 \text{ ns} = 3.79\text{ns}$

Example 1 – Continued

R_{cs01}: This resistance is simply r_{c1} plus the input resistance to the CB stage, R_{i3}.

$$R_{i3} = \frac{1}{g_{m3}} + \frac{r_{b3}}{1+\beta} = \frac{1}{0.0442} + \frac{200}{1+200} = 23.6\Omega \rightarrow R_{cs01} = 23.6+150 = 174\Omega$$

$\therefore \quad R_{cs01}C_{cs1} = 0.4\cdot0.174\text{ns} = 0.07\text{ns}$

$R_{\mu01}$: Use the model shown.

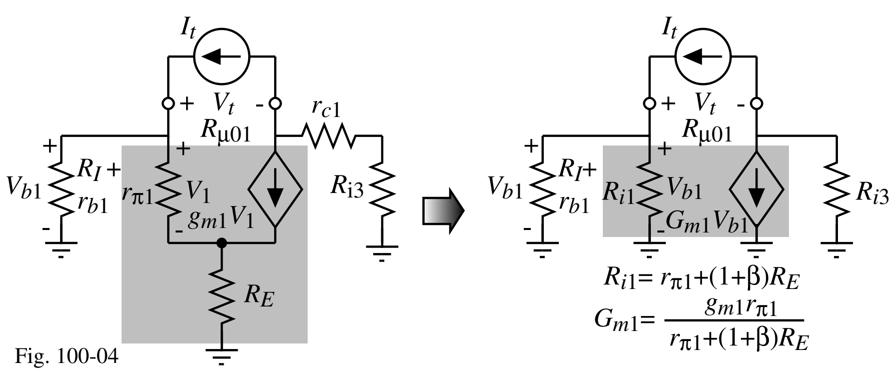

Fig. 100-04

From previous work, we can write,

$$V_t = I_tR_{i1}||(r_{b1}+R_I) + (I_t+G_{m1}V_{b1})R_{cs01} \quad \text{and} \quad V_{b1} = I_tR_{i1}||(r_{b1}+R_I)$$

$$\therefore \quad V_t = I_tR_{i1}||(r_{b1}+R_I) + I_tR_{cs01} + I_tG_{m1}[R_{i1}||(r_{b1}+R_I)]R_{cs01}$$

$$R_{\mu01} = R_{i1}||(r_{b1}+R_I) + R_{cs01} + G_{m1}R_{cs01}[R_{i1}||(r_{b1}+R_I)]$$

$$= 1131\Omega+174\Omega+0.0102\cdot174\cdot1131\Omega = 3.31\text{k}\Omega$$

$\therefore \quad R_{\mu01}C_{\mu1} = 0.23\cdot3.31\text{ns} = 0.76\text{ns}$

Example 1 – Continued

$R_{\pi 03}$: Use the model shown.

$$V_t = (I_t + g_{m3}V_3)r_{\pi 3} = I_t r_{\pi 3} + g_{m3}(-V_t)r_{\pi 3}$$

$$V_t(1+g_{m3}r_{\pi 3}) = I_t r_{\pi 3}$$

$$R_{\pi 03} = \frac{V_t}{I_t} = \frac{r_{\pi 3}}{1+\beta} = 22.5\Omega$$

$$\therefore \quad R_{\pi 03}C_{\pi 3} = 13.7\cdot 0.0023\text{ns} = 0.32\text{ns}$$

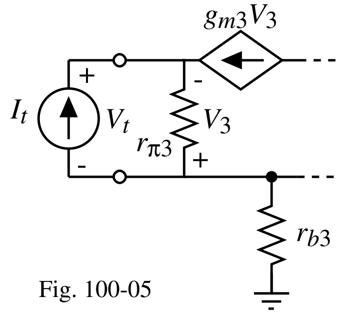

Fig. 100-05

$R_{\mu 03}$:

$$R_{\mu 03} = r_{b3} + r_{c3} + R_L = 200+150+1000 = 1350\Omega \rightarrow \quad R_{\mu 03}C_{\mu 3} = 0.2\cdot 1.35\text{ns} = 0.27\text{ns}$$

R_{cs03}:

$$R_{cs03} = r_{c3} + R_L = 150+1000 = 1150\Omega \rightarrow \quad R_{cs03}C_{cs3} = 0.35\cdot 1.15\text{ns} = 0.4\text{ns}$$

Finally,

$$\Sigma T_0 = (3.79 + 0.07 + 0.76 + 0.32 + 0.27 + 0.4)\text{ns} = 5.61\text{ns} \quad (C_{\pi 1} \text{ is the limitation})$$

$$\omega_{-3\text{dB}} \approx \frac{1}{\Sigma T_0} = \frac{1000\text{x}10^6}{5.61} = 178.25\text{x}10^6 \rightarrow \quad f_{-3\text{dB}} = 28.4\text{MHz}$$

(Computer simulation shows 6 poles with the lowest two of –35.8MHz and –253MHz resulting in a $f_{-3\text{dB}}$ = 34.7MHz)

Frequency Response of Current-Mirror Load Differential Amplifier

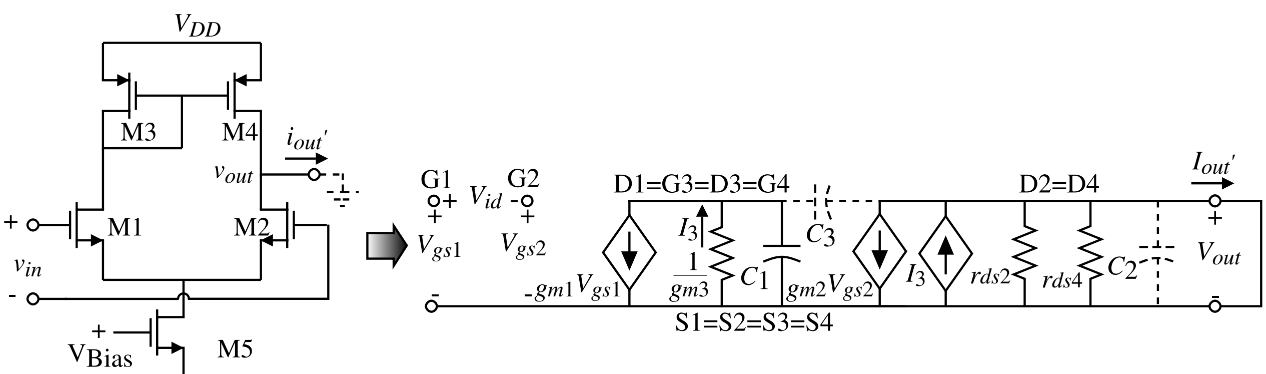

Fig. 100-06

$$I_{out}' = I_3 - g_{m2}V_{gs2} = -\left(\frac{\frac{1}{sC_1}}{\frac{1}{g_{m3}} + \frac{1}{sC_1}}\right) g_{m1}V_{gs1} - g_{m2}V_{gs2} = -\left(\frac{1}{s\frac{C_1}{g_{m3}} + 1}\right) g_{m1}\frac{V_{in}}{2} - g_{m2}\frac{V_{in}}{2}$$

$$I_{out}' = -g_{md}\left(\frac{s\frac{C_1}{g_{m3}} + 2}{s\frac{C_1}{g_{m3}} + 1}\right)\frac{V_{in}}{2} = -\, g_{md}\left(\frac{s\frac{C_1}{2g_{m3}} + 2}{s\frac{C_1}{g_{m3}} + 1}\right)V_{in} \qquad \text{where } g_{m1} = g_{m2} = g_{md}$$

The circuit has a zero at $-\,2g_{m3}/C_1$ and a pole at $-\,g_{m3}/C_1$.

(These roots are not dominant, C_2 will cause the –3dB frequency.)

Short-Circuit Time Constants

- Short-circuit time constants are used to identify the *largest* pole of a multipole system
- The short-circuit time constant approach is not very useful for IC circuits because they are dc-coupled (lowest root is most important).
- The short-circuit time constant approach is useful for larger of a two-pole system.

Derivation:

Based on,

$$I_1 = (g_{11}+sC_1)V_1 + g_{12}V_2 + g_{13}V_3 + \cdots$$
$$I_2 = g_{21}V_1 + (g_{22}+sC_2)V_2 + g_{23}V_3 + \cdots$$
$$I_3 = g_{31}V_1 + g_{32}V_2 + (g_{33}+sC_3)V_3 + \cdots$$
$$\vdots$$

The determinant can be expressed as,

$$\Delta(s) = K_0 + K_1 s + K_2 s^2 + K_3 s^3 = K_3\left(s^3 + \frac{K_2}{K_3}s^2 + \frac{K_1}{K_3}s + \frac{K_0}{K_3}\right)$$

$$\Delta(s) = K_3(s-p_1)(s-p_2)(s-p_3) = K_3[s^3 - s^2(p_1+p_2+p_3) + s(p_1p_3 + p_2p_3) - p_1p_2p_3]$$

$$\therefore \quad \frac{K_2}{K_3} = -(p_1+p_2+p_3) = -\sum_{i=1}^{3} p_i$$

Short-Circuit Time Constants – Continued

We know that, $K_3 = C_1C_2C_3$ and $K_2 = g_{11}C_2C_3 + g_{22}C_1C_3 + g_{33}C_1C_2$

$$\therefore \quad \frac{K_2}{K_3} = \frac{g_{11}}{C_1} + \frac{g_{22}}{C_2} + \frac{g_{33}}{C_3} = \frac{1}{r_{11}C_1} + \frac{1}{r_{22}C_2} + \frac{1}{r_{33}C_3}$$

where $r_{ii}C_i = \tau_{si}$ $(i = 1,2,3)$ are called the *short-circuit time constants* and r_{ii} is given as,

$$r_{11} = \frac{1}{g_{11}} = \left.\frac{V_1}{I_1}\right|_{V_2=V_3=0, C_1=0}, \quad r_{22} = \frac{1}{g_{22}} = \left.\frac{V_2}{I_2}\right|_{V_1=V_3=0, C_2=0}, \text{ and } \quad r_{33} = \frac{1}{g_{33}} = \left.\frac{V_3}{I_3}\right|_{V_1=V_2=0, C_3=0}$$

Therefore, if a circuit has n poles and $|p_n| > |p_1|$, $|p_n| > |p_2|$, $|p_n| > |p_3|$, $\cdots$ then

$$p_n \approx \sum_{i=1}^{n} p_i = -\sum_{i=1}^{n} \frac{1}{\tau_{si}} = -\sum_{i=1}^{n} \frac{1}{r_{ii}C_i}$$

The most useful application of this idea is to find p_2 where $|p_2| > |p_1|$.

Example 2

Consider the circuit shown below where R_I = 10kΩ, R_L = 10kΩ, g_m = 3mS, C_i = 1pF, and C_f = 20pF.

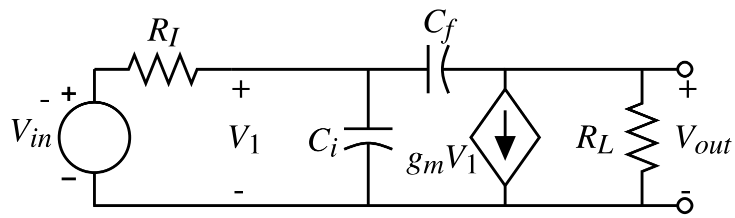

Fig. 100-07

The open-circuit time constants are:

$$\tau_{of} = C_fR_{fo} = C_f(R_I + R_L + g_mR_L R_I) = 20\text{pf}(10+10+30\cdot10)\text{k}\Omega = 6.4\mu\text{s}$$

$$\tau_{oi} = C_iR_{io} = C_iR_I = 1\text{pF}(10\text{k}\Omega) = 10\text{ns}$$

$$\therefore \quad p_1 \approx \frac{-1}{6.4\mu\text{s} + 0.01\mu\text{s}} = -156\text{krad/s}$$

The short-circuit time constants are:

$$\tau_{sf} = C_fR_L = 20\text{pf}(10\text{k}\Omega) = 200\text{ns}$$

$$\tau_{si} = C_i[R_I||(1/g_m)||R_L] = 1\text{pf}(10||0.33||10) = 0.312\text{ns}$$

$$\therefore \quad p_2 \approx -\left(\frac{1}{200\text{ns}} + \frac{1}{0.312\text{ns}}\right) = -3.21\text{Grad/s}$$

Exact analysis gives p_1= -156krad/s and p_2 = -3.20Grad/s

SUMMARY

- Exact methods of finding the poles of a circuit include:
  - Matrix methods
  - Simulation
- Approximate methods of finding the poles of a circuit include:
  - Miller approximation
  - Dominant pole approximation, good for –3dB frequency
  - Open-circuit time constants, good for finding the smallest pole of a multi-pole circuit
  - Short-circuit time constants, good for finding the largest pole of a multi-pole circuit

  For these methods to work well, the poles should not be closely spaced
- Probably the most important result is the information the above methods provide for the design of the circuit to achieve a given frequency performance
- We will come back to the frequency analysis of the 741 later