INTRODUCTION

The objective of this presentation is to continue the ideas of the last lecture on compensation of op amps.

**Outline**

- Compensation of Op Amps
  - General principles
  - Miller, Nulling Miller
  - Self-compensation
  - Feedforward
- Summary

**Conditions for Stability of the Two-Stage Op Amp (Assuming \( p_2 \geq GB \))**

- Unity-gainbandwith is given as:
  \[
  GB = A_v(0) |p_1| = \left( \frac{g_{m1}g_{m1}R_1R_1}{g_{m2}R_2C} \right) = \frac{g_{m1}}{C_c} = \frac{g_{m1}g_{m2}R_1R_2}{g_{m2}R_1R_2C} = \frac{g_{m1}}{C_c}
  \]

- The requirement for 45° phase margin is:
  \[
  \pm 180° - \text{Arg}[AF] = \pm 180° - \tan^{-1}\left( \frac{\omega}{|p_1|} \right) - \tan^{-1}\left( \frac{\omega}{|p_2|} \right) - \tan^{-1}\left( \frac{\omega}{z} \right) = 45°
  \]

Let \( \omega = GB \) and assume that \( z \geq 10GB \), therefore we get,

\[
\pm 180° - \tan^{-1}\left( \frac{GB}{|p_1|} \right) - \tan^{-1}\left( \frac{GB}{|p_2|} \right) - \tan^{-1}\left( \frac{GB}{z} \right) = 45°
\]

\[
135° \approx \tan^{-1}(A_v(0)) + \tan^{-1}\left( \frac{GB}{|p_2|} \right) + \tan^{-1}(0.1) = 90° + \tan^{-1}\left( \frac{GB}{|p_2|} \right) + 5.7°
\]

\[
39.3° \approx \tan^{-1}\left( \frac{GB}{|p_2|} \right) \Rightarrow \frac{GB}{|p_2|} = 0.818 \Rightarrow |p_2| \geq 1.22GB
\]

- The requirement for 60° phase margin:
  \[
  |p_2| \geq 2.2GB \text{ if } z \geq 10GB
  \]

- If 60° phase margin is required, then the following relationships apply:
  \[
  \frac{g_{m6}}{C_c} > \frac{10g_{m1}}{C_c} \Rightarrow g_{m6} > 10g_{m1} \quad \text{and} \quad \frac{g_{m6}}{C_2} > \frac{2.2g_{m1}}{C_c} \Rightarrow C_c > 0.22C_2
  \]
Controlling the Right-Half Plane Zero

Why is the RHP zero a problem?

Because it boosts the magnitude but lags the phase - the worst possible combination for stability.

\[ j\omega \]

\[ j\omega_1 \]

\[ j\omega_2 \]

\[ j\omega_3 \]

\[ 180^\circ > \theta_1 > \theta_2 > \theta_3 \]

Solution of the problem:

If zeros are caused by two paths to the output, then eliminate one of the paths.

Use of Buffer to Eliminate the Feedforward Path through the Miller Capacitor

Model:

The transfer function is given by the following equation,

\[ \frac{V_o(s)}{V_{in}(s)} = \frac{(g_{ml})(g_{mII})(R_I)(R_{II})}{1 + s[R_I C_I + R_{II} C_{II} + R_I C_c + g_{mII} R_I R_{II} C_c] + s^2[R_I R_{II} C_{II}(C_I + C_c)]} \]

Using the technique as before to approximate \( p_1 \) and \( p_2 \) results in the following

\[ p_1 \equiv \frac{-1}{R_I C_I + R_{II} C_{II} + R_I C_c + g_{mII} R_I R_{II} C_c} = \frac{-1}{g_{mII} R_I R_{II} C_c} \]

and

\[ p_2 \equiv \frac{-g_{mII} C_c}{C_{II}(C_I + C_c)} \]

Comments:

Poles are approximately what they were before with the zero removed.

For 45° phase margin, \( |p_2| \) must be greater than \( GB \)

For 60° phase margin, \( |p_2| \) must be greater than \( 1.73 GB \)
Use of Buffer with Finite Output Resistance to Eliminate the RHP Zero

Assume that the unity-gain buffer has an output resistance of $R_O$.

Model:

It can be shown that if the output resistance of the buffer amplifier, $R_O$, is not neglected that another pole occurs at,

$$p_4 \cong -\frac{1}{R_O[C_IC_c/(C_I + C_c)]}$$

and a LHP zero at

$$z_2 \cong -\frac{1}{R_OC_c}$$

Closer examination shows that if a resistor, called a *nulling resistor*, is placed in series with $C_c$ that the RHP zero can be eliminated or moved to the LHP.

Use of Nulling Resistor to Eliminate the RHP Zero (or turn it into a LHP zero)$^\dagger$

Nodal equations:

\[
\begin{align*}
g_{ml}V_{in} + \frac{V_I}{R_I} + sC_I V_I + \left(\frac{sC_c}{1 + sC_cR_z}\right)(V_I - V_{out}) &= 0 \\
g_{mll}V_I + \frac{V_o}{R_{II}} + sC_{II} V_{out} + \left(\frac{sC_c}{1 + sC_cR_z}\right)(V_{out} - V_I) &= 0
\end{align*}
\]

Solution:

\[
\frac{V_{out}(s)}{V_{in}(s)} = \frac{a \{1 - s[(C_I/g_{mll}) - R_zC_c]\}}{1 + bs + cs^2 + ds^3}
\]

where

\[
\begin{align*}
a &= g_{ml}g_{mll}R_I R_{II} \\
b &= (C_{II} + C_c)R_{II} + (C_I + C_c)R_I + g_{mll}R_I R_{II}C_c + R_z C_c \\
c &= [R_I R_{II}(C_I C_{II} + C_c C_I + C_c C_{II}) + R_z C_c (R_I C_I + R_{II} C_{II})] \\
d &= R_I R_{II} R_z C_c C_{II} C_c
\end{align*}
\]

**Use of Nulling Resistor to Eliminate the RHP - Continued**

If $R_z$ is assumed to be less than $R_I$ or $R_{II}$ and the poles widely spaced, then the roots of the above transfer function can be approximated as

$$p_1 \approx \frac{-1}{(1 + g_{mII}R_{II})R_I C_c} \approx \frac{-1}{g_{mII}R_{II}R_IC_c}$$

$$p_2 \approx \frac{-g_{mII}C_c}{C_I C_{II} + C_c C_I + C_c C_{II}} \approx -\frac{g_{mII}}{C_{II}}$$

$$p_4 = \frac{-1}{R_z C_I}$$

and

$$z_1 = \frac{1}{C_c(1/g_{mII} - R_z)}$$

Note that the zero can be placed anywhere on the real axis.

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**Conceptual Illustration of the Nulling Resistor Approach**

![Diagram of the nulling resistor approach](Fig. 430-05)

The output voltage, $V_{out}$, can be written as

$$V_{out} = \frac{-g_{m6}R_{II} \left( R_z + \frac{1}{sC_c} \right)}{R_{II} + R_z + \frac{1}{sC_c}} V' + \frac{R_{II}}{R_{II} + R_z + \frac{1}{sC_c}} V'' = \frac{-R_{II} \left[ g_{m6} R_z + \frac{g_{m6}}{sC_c} - 1 \right]}{R_{II} + R_z + \frac{1}{sC_c}} V$$

when $V = V' = V''$.

Setting the numerator equal to zero and assuming $g_{m6} = g_{mII}$ gives,

$$z_1 = \frac{1}{C_c(1/g_{mII} - R_z)}$$
A Design Procedure that Allows the RHP Zero to Cancel the Output Pole, \( p_2 \)

We desire that \( z_1 = p_2 \) in terms of the previous notation. Therefore,

\[
\frac{1}{C_c(1/g_{mlI} - R_z)} = \frac{-g_{mlI}}{C_{II}}
\]

The value of \( R_z \) can be found as

\[
R_z = \left( \frac{C_c + C_{II}}{C_c} \right) \left( \frac{1}{g_{mlI}} \right)
\]

With \( p_2 \) canceled, the remaining roots are \( p_1 \) and \( p_4 \)(the pole due to \( R_z \)). For unity-gain stability, all that is required is that

\[
|p_4| > A_v(0) |p_1| = \frac{A_v(0)}{g_{mlI} R_{II} R_f C_c} = \frac{g_{mlI}}{C_{II}}
\]

and

\[
(1/R_z C_I) > (g_{mlI}/C_c) = GB
\]

Substituting \( R_z \) into the above inequality and assuming \( C_{II} >> C_c \) results in

\[
C_c > \sqrt{\frac{g_{mlI}}{g_{mlI} C_I C_{II}}}
\]

This procedure gives excellent stability for a fixed value of \( C_{II} \approx C_L \).

Unfortunately, as \( C_L \) changes, \( p_2 \) changes and the zero must be readjusted to cancel \( p_2 \).

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Increasing the Magnitude of the Output Pole

The magnitude of the output pole, \( p_2 \), can be increased by introducing gain in the Miller capacitor feedback path. For example,

The resistors \( R_1 \) and \( R_2 \) are defined as

\[
R_1 = \frac{1}{g_{ds2} + g_{ds4} + g_{ds9}} \quad \text{and} \quad R_2 = \frac{1}{g_{ds6} + g_{ds7}}
\]

where transistors M2 and M4 are the output transistors of the first stage.

Nodal equations:

\[
I_{in} = G_1 V_1 - g_{m8} V_{s8} = G_1 V_1 + \left( \frac{g_{m8} s C_c}{g_{m8} + s C_c} \right) V_{out} \quad \text{and} \quad 0 = g_m V_1 + \left[ G_2 + s C_2 + \frac{g_{m8} C_c}{g_{m8} + s C_c} \right] V_{out}
\]

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### Increasing the Magnitude of the Output Pole - Continued

Solving for the transfer function $\frac{V_{out}}{I_{in}}$ gives,

$$
\frac{V_{out}}{I_{in}} = \left[ \frac{-g_{m6}}{G_1G_2} \right] \left[ \frac{1 + \frac{sC_c}{g_{m8}}}{1 + \frac{C_c}{g_{m8}} + \frac{C_2}{G_2} + \frac{g_{m6}C_c}{G_1G_2} + s\frac{C_cC_2}{g_{m8}G_2}} \right]
$$

Using the approximate method of solving for the roots of the denominator gives

$$
p_1 = -1 - \frac{1}{g_{m6}r_{ds}\sqrt{2}C_c}
$$

and

$$
p_2 \approx \frac{-\frac{g_{m6}r_{ds}\sqrt{2}C_c}{C_cC_2}}{g_{m8}G_2} = \frac{g_{m8}r_{ds}2G_2}{6}\left(\frac{g_{m6}}{C_2}\right) = \frac{g_{m8}r_{ds}}{3}\left|p_2'\right|
$$

where all the various channel resistance have been assumed to equal $r_{ds}$ and $p_2'$ is the output pole for normal Miller compensation.

Result:

Dominant pole is approximately the same and the output pole is increased by $\approx g_{m}r_{ds}$.

### Concept Behind the Increasing of the Magnitude of the Output Pole

![Figure 430-08](image_url)

$$
R_{out} = r_{ds7}\|\frac{3}{g_{m6}g_{m8}r_{ds8}} = \frac{3}{g_{m6}g_{m8}r_{ds8}}
$$

Therefore, the output pole is approximately,

$$
|p_2| \approx \frac{g_{m6}g_{m8}r_{ds8}}{3C_II}
$$
FEEDFORWARD COMPENSATION

Use two parallel paths to achieve a LHP zero for lead compensation purposes.

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{AC_c}{C_c + C_{II}} \left( \frac{s + g_{mII}/AC_c}{s + 1/[R_{II}(C_c + C_{II})]} \right) \]

To use the LHP zero for compensation, a compromise must be observed.
- Placing the zero below GB will lead to boosting of the loop gain that could deteriorate the phase margin.
- Placing the zero above GB will have less influence on the leading phase caused by the zero.

Note that a source follower is a good candidate for the use of feedforward compensation.

SELF-COMPENSATED OP AMPS

Self compensation occurs when the load capacitor is the compensation capacitor (can never be unstable for resistive feedback)

Voltage gain:
\[ \frac{v_{out}}{v_{in}} = A_v(0) = G_mR_{out} \]

Dominant pole:
\[ p_1 = -\frac{1}{R_{out}C_L} \]

Unity-gainbandwidth:
\[ GB = A_v(0)\cdot|p_1| = \frac{G_m}{C_L} \]

Stability:
Large load capacitors simply reduce GB but the phase is still 90° at GB.
**SUMMARY**

**Compensation**
- Designed so that the op amp with unity gain feedback (buffer) is stable
- **Types**
  - Miller
  - Miller with nulling resistors
  - Self Compensating
  - Feedforward