LECTURE 180 – POWER SUPPLY REJECTION RATIO
(READING: GHLM – 434-439, AH – 286-293)

Objective
The objective of this presentation is:
1.) Illustrate the calculation of PSRR
2.) Examine the PSRR of the two-stage, Miller compensated op amp

Outline
• Definition of PSRR
• Calculation of PSRR for the two-stage op amp
• Conceptual reason for PSRR
• Summary

What is PSRR?

\[
PSRR = \frac{A_v(V_{dd}=0)}{A_{dd}(V_{in}=0)}
\]

How do you calculate PSRR?
You could calculate \(A_v\) and \(A_{dd}\) and divide, however

\[
V_{out} = A_{dd}V_{dd} + A_v(V_1-V_2) = A_{dd}V_{dd} - A_vV_{out} \quad \rightarrow \quad V_{out}(1+A_v) = A_{dd}V_{dd}
\]

\[
\therefore \frac{V_{out}}{V_{dd}} = 1 + \frac{A_{dd}}{A_v} = \frac{1}{PSRR^+} \quad \text{(Good for frequencies up to } GB)\]
The nodal equations are:
\[(g_{ds1} + g_{ds4})V_{dd} = (g_{ds2} + g_{ds4} + sC_{c} + sC_{l})V_{1} - (g_{m1} + sC_{c})V_{out}\]
\[(g_{m6} + g_{ds6})V_{dd} = (g_{m6} - sC_{c})V_{1} + (g_{ds6} + g_{ds7} + sC_{c} + sC_{ll})V_{out}\]

Using the generic notation the nodal equations are:
\[G_{I}V_{dd} = (G_{I} + sC_{c} + sC_{l})V_{1} - (g_{mI} + sC_{c})V_{out}\]
\[(g_{mII} + g_{ds6})V_{dd} = (g_{mII} - sC_{c})V_{1} + (G_{II} + sC_{c} + sC_{ll})V_{out}\]

where \(G_{I} = g_{ds1} + g_{ds4} = g_{ds2} + g_{ds4}, G_{II} = g_{ds6} + g_{ds7}, \) \(g_{mI} = g_{m1} = g_{m2} \) and \(g_{mII} = g_{m6}\)

We may solve for the approximate roots of numerator as
\[PSRR^{+} = \frac{V_{dd}}{V_{out}} = \left(\frac{g_{mII}g_{ds6}}{G_{I}g_{ds6}}\right)\left[\frac{(sC_{c})}{(g_{mI} + 1)(s(C_{c}C_{I} + C_{l}C_{ll} + C_{ll}C_{c}))} + 1\right]\]

where \(g_{mII} > g_{mI}\) and that all transconductances are larger than the channel conductances.

\[\therefore PSRR^{+} = \frac{V_{dd}}{V_{out}} = \left(\frac{g_{mII}g_{ds6}}{G_{I}g_{ds6}}\right)\left[\frac{(sC_{c})}{(g_{mI} + 1)(sC_{ll})} + 1\right] = \left(\frac{G_{I}A_{vo}}{g_{ds6}}\right)\left[\frac{s}{GB + 1}\right] + 1\]
Positive PSRR of the Two-Stage Op Amp - Continued

At approximately the dominant pole, the $PSRR$ falls off with a -20dB/decade slope and degrades the higher frequency $PSRR^+$ of the two-stage op amp.

Using the values of Example 6.3-1 we get:

$$PSRR^+(0) = 68.8\text{dB}, \quad z_1 = -5\text{MHz}, \quad z_2 = -15\text{MHz} \quad \text{and} \quad p_1 = -906\text{Hz}$$

Concept of the $PSRR^+$ for the Two-Stage Op Amp

1.) The M7 current sink causes $V_{SG6}$ to act like a battery.
2.) Therefore, $V_{dd}$ couples from the source to gate of M6.
3.) The path to the output is through any capacitance from gate to drain of M6.

Conclusion:
The Miller capacitor $C_c$ couples the positive power supply ripple directly to the output. Must reduce or eliminate $C_c$. 
Negative PSRR of the Two-Stage Op Amp with $V_{\text{Bias}}$ Grounded

Nodal equations for $V_{\text{Bias}}$ grounded:

\[
0 = (G_I + sC_c + sC_I)V_1 - (g_{mI} + sC_c)V_o
\]

\[
g_{mI}V_{ss} = (G_{II} + sC_c + sC_{II})V_1 + (G_I + sC_c + sC_{II})V_o
\]

Solving for $V_{out}/V_{ss}$ and inverting gives

\[
\frac{V_{ss}}{V_{out}} = s^{2}[C_cC_I + C_I C_{II} + C_{II}C_c] + s[G_I(C_c + C_{II}) + G_{II}(C_c + C_I) + C_c(g_{mII} - g_{mI})] + G_{II} + g_{mII}g_{mI}
\]

\[\frac{V_{ss}}{V_{out}} = \frac{sC_c + C_I + C_{II} + C_{II}C_c + s[G_I(C_c + C_{II}) + G_{II}(C_c + C_I) + C_c(g_{mII} - g_{mI})] + G_{II} + g_{mII}g_{mI}}{s(C_c + C_I) + G_I g_{mI}}\]

Comments:

\[PSRR^- = PSRR^+\text{ zeros}\]

DC gain $\approx$ Second-stage gain,

\[PSRR^-\text{ pole} = (\text{Second-stage gain}) \times (PSRR^+\text{ pole})\]

Assuming the values of Ex. 6.3-1 gives a gain of 23.7 dB and a pole -147 kHz. The dc value of $PSRR^-$ is very poor for this case, however, this case can be avoided by correctly implementing $V_{\text{Bias}}$ which we consider next.
Negative PSRR of the Two-Stage Op Amp with $V_{Bias}$ Connected to $V_{SS}$

If the value of $V_{Bias}$ is independent of $V_{ss}$, then the model shown results. The nodal equations for this model are

$$
0 = (G_I + sC_c + sC_I)V_1 - (g_{ml} + sC_c)V_{out}
$$

and

$$
(g_{ds7} + sC_{gd7})V_{ss} = (g_{ml} - sC_c)V_1 + (G_{II} + sC_c + sC_{II} + sC_{gd7})V_{out}
$$

Again, solving for $V_{out}/V_{ss}$ and inverting gives

$$
\frac{V_{ss}}{V_{out}} = \frac{s^2[C_cC_I + C_cC_{II} + C_cC_{II} + C_cC_{gd7} + C_cC_{gd7}]}{(sC_{gd7} + g_{ds7})[s(C_I + C_c) + sC_{gd7}]} + \frac{G_{II}A_{v0}}{g_{ds7}}
$$

for the approximate poles of both the numerator and denominator.

This equation can be rewritten as

$$
PSRR = \frac{V_{ss}}{V_{out}} \approx \frac{G_{II}A_{v0}}{g_{ds7}} \left[ \frac{s}{GB + 1} \left( \frac{s}{IP_2 + 1} \right) \right]
$$

Comments:

- DC gain has been increased by the ratio of $G_{II}$ to $g_{ds7}$
- Two poles instead of one, however the pole at $-g_{ds7}/C_{gd7}$ is large and can be ignored.

Using the values of Ex. 6.3-1 and assume that $C_{ds7} = 10fF$, gives,

$$
PSRR(0) = 76.7\text{dB} \quad \text{and} \quad \text{Poles at } -71.2\text{kHz and } -149\text{MHz}
$$
**Frequency Response of the Negative PSRR of the Two-Stage Op Amp with $V_{Bias}$ Connected to $V_{SS}$**

![Graph showing frequency response of the negative PSRR](image)

$$|PSRR(j\omega)| \text{ dB}$$

**Approximate Model for Negative PSRR with $V_{Bias}$ Connected to Ground**

![Circuit diagram](image)

Path through the input stage is not important as long as the CMRR is high.

Path through the output stage:

$$v_{out} \approx i_{ss} Z_{out} = g_m Z_{out} V_{SS}$$

$$\therefore \frac{V_{out}}{V_{SS}} = g_m Z_{out} = g_m R_{out} \left( \frac{1}{sR_{out}C_{out} + 1} \right)$$
Approximate Model for Negative PSRR with $V_{Bias}$ Connected to $V_{SS}$

What is $Z_{out}$?

$Z_{out} = \frac{V_I}{I_t} \Rightarrow$

$I_t = g_{mI}V_1 = g_{mII}\left(\frac{g_{mI}V_t}{G_I+sC_I+sC_C}\right)$

Thus, $Z_{out} = \frac{G_I+s(C_I+C_C)}{g_{mII}g_{mII}}$

$\therefore$

$\frac{V_{SS}}{V_{out}} = \frac{1+r_{ds7}}{Z_{out}1} = \frac{s(C_C+C_I) + G_I+8mgmIr_{ds7}}{s(C_C+C_I) + G_I}$

$\Rightarrow$ Pole at $\frac{-G_I}{C_C+C_I}$

The two-stage op amp will never have good PSRR because of the Miller compensation.
SUMMARY

• PSRR is a measure of the influence of power supply ripple on the op amp output voltage
• PSRR can be calculated by putting the op amp in the unity-gain configuration with the input shorted.
• The Miller compensation capacitor allows the power supply ripple at the output to be large
• The two-stage op amp will never have good PSRR unless some modifications are made.