LECTURE 330 – LOW POWER OP AMPS
(READING: AH – 393-402)

Objective
The objective of this presentation is:
1.) Examine op amps that have minimum static power
   - Minimize power dissipation
   - Work at low values of power supply
   - Tradeoff speed for less power

Outline
• Weak inversion
• Methods of creating an overdrive
• Examples
• Summary

Subthreshold Operation
Most micropower op amps use transistors in the subthreshold region.
Subthreshold characteristics:

\[ i_D = \frac{W}{L} I_{DO} \exp \left( \frac{qV_{GS}}{nkT} \right) \left( 1 + \lambda v_{DS} \right) \]

\[ \Rightarrow g_m = \frac{qI_D}{nkT} \text{ and } g_{ds} \approx \lambda I_D \]

Operation with channel length = \( L_{min} \) also will normally be in weak inversion.
Two-Stage, Miller Op Amp Operating in Weak Inversion

![Circuit Diagram](Fig.7.4-1)

Low frequency response:

\[ A_{vo} = g_{m2}g_{m6} \left( \frac{r_{o2}r_{o4}}{r_{o2} + r_{o4}} \right) \left( \frac{r_{o6}r_{o7}}{r_{o6} + r_{o7}} \right) = \frac{1}{n_2n_6(kT/q)^2(\lambda_2 + \lambda_4)(\lambda_6 + \lambda_7)} \]

(No longer \( \propto \frac{1}{I_D} \))

**GB and SR:**

\[ GB = \frac{I_{D1}}{(n_1kT/q)C} \quad \text{and} \quad SR = \frac{I_{D5}}{C} = 2 \frac{I_{D1}}{C} = 2GB \left( \frac{n_1kT}{q} \right) = 2GBn_1V_T \]

---

**Example 7.4-1 Gain and GB Calculations for Subthreshold Op Amp.**

Calculate the gain, GB, and SR of the op amp shown above. The currents are \( I_{D5} = 200 \text{ nA} \) and \( I_{D7} = 500 \text{ nA} \). The device lengths are 1 \( \mu \text{m} \). Values for \( n \) are 1.5 and 2.5 for p-channel and n-channel transistors respectively. The compensation capacitor is 5 pF. Use Table 3.1-2 as required. Assume that the temperature is 27 \( ^\circ \text{C} \). If \( V_{DD} = 1.5\text{V} \) and \( V_{SS} = -1.5\text{V} \), what is the power dissipation of this op amp?

**Solution**

The low-frequency small-signal gain is,

\[ A_V = \frac{1}{(1.5)(2.5)(0.026)^2(0.04 + 0.05)(0.04 + 0.05)} = 43,701 \text{ V/V} \]

The gain bandwidth is

\[ GB = \frac{100 	imes 10^{-9}}{2.5(0.026)(5 	imes 10^{-12})} = 307,690 \text{ rps} \approx 49.0 \text{ kHz} \]

The slew rate is

\[ SR = (2)(307690)(2.5)(0.026) = 0.04 \text{ V/\mu s} \]

The power dissipation is,

\[ P_{diss} = 3(0.7\mu \text{A}) = 2.1\mu \text{W} \]
**Push-Pull Output Op Amp in Weak Inversion**

First stage gain is,

\[
A_{vo} = \frac{g_{m2}}{g_{m4}} = \frac{I_{D2}n_4V_i}{I_{D4}n_2V_i} = \frac{I_{D2}n_4}{I_{D4}n_2} \approx 1
\]

Total gain is,

\[
A_{vo} = \frac{g_{m1}(S_6/S_4)}{(g_{ds6} + g_{ds7})} = (\lambda_6 + \lambda_7)n_1V_i
\]

At room temperature \((V_t = 0.0259V)\) and for typical device lengths, gains of 60dB can be obtained.

The GB is,

\[
GB = \frac{g_{m1}b}{C} = \frac{g_{m1}b}{C}
\]

---

**Increasing the Gain of the Previous Op Amp**

1.) Can reduce the currents in M3 and M4 and introduce gain in the current mirrors.

2.) Use a cascode output stage (can’t use self-biased cascode, currents are too low).

\[
A_v = \left(\frac{g_{m1} + g_{m2}}{2}\right)R_{out}
\]

\[
= \frac{g_{m1}}{g_{ds68}g_{ds10} + g_{ds78}g_{ds11}} + \frac{g_{m10}}{g_{m11}}
\]

\[
= \frac{I_5}{2n_nV_t} + \frac{I_7^2\lambda_n^2}{I_7} + \frac{I_7^2\lambda_p^2}{I_7} = \left(\frac{I_5}{2V_t}\right)\left(\frac{1}{n_nV_t^2(n_n\lambda_n^2 + n_p\lambda_p^2)}\right)
\]

Can easily achieve gains greater than 80dB with power dissipation of less than 1µW.
**Increasing the Output Current for Weak Inversion Operation**

A significant disadvantage of the weak inversion is that very small currents are available to drive output capacitance so the slew rate becomes very small.

Dynamically biased differential amplifier input stage:

![Dynamically Biased Differential Amplifier](image)

Note that the sinking current for M1 and M2 is

\[ I_{sink} = I_5 + A(i_2-i_1) + A(i_1-i_2) \]

where \((i_2-i_1)\) and \((i_1-i_2)\) are only positive or zero.

If \(v_{i1}>v_{i2}\), then \(i_2>i_1\) and the sinking current is increased by \(A(i_2-i_1)\).

If \(v_{i2}>v_{i1}\), then \(i_1>i_2\) and the sinking current is increased by \(A(i_1-i_2)\).

**Dynamically Biased Differential Amplifier - Continued**

How much output current is available from this circuit if there is no current gain from the input to output stage?

Assume transistors M18 through M21 are equal to M3 and M4 and that transistors M22 through M27 are all equal.

Let

\[ \frac{W_{28}}{L_{28}} = A \left( \frac{W_{26}}{L_{26}} \right) \quad \text{and} \quad \frac{W_{29}}{L_{29}} = A \left( \frac{W_{27}}{L_{27}} \right) \]

The output current available can be found by assuming that \(v_{in} = v_{i1} - v_{i2} > 0\).

\[ i_1 + i_2 = I_5 + A(i_2-i_1) \]

The ratio of \(i_2\) to \(i_1\) can be expressed as

\[ \frac{i_2}{i_1} = \exp \left( \frac{v_{in}}{nV_t} \right) \]

Defining the output current as \(i_{OUT} = b(i_2-i_1)\) and combining the above two equations gives,

\[ i_{OUT} = \frac{bI_5 \left[ \exp \left( \frac{v_{in}}{nV_t} \right) - 1 \right]}{(1+A) - (A-1)\exp \left( \frac{v_{in}}{nV_t} \right)} \quad \Rightarrow \quad i_{OUT} = \infty \quad \text{when} \quad A = 2.16 \quad \text{and} \quad \frac{v_{in}}{nV_t} = 1 \]

where \(b\) corresponds to any current gain through current mirrors (M5-M4 and M8-M3).
Overdrive of the Dynamically Biased Differential Amplifier

The enhanced output current is accomplished by the use of positive feedback (M28-M2-M19-M28).

The loop gain is,

\[
LG = \frac{g_{m28}(g_{m19})}{g_{m4}(g_{m26})} = A \frac{g_{m19}}{g_{m4}} = A
\]

Note that as the output current increases, the transistors leave the weak inversion region and the above analysis is no longer valid.

Increasing the Output Current for Strong Inversion Operation

An interesting technique is to bias the output transistor of a current mirror in the active region and then during large overdrive cause the output transistor to become saturated causing a significant current gain.

Illustration:
Example 7.4-2  Current Mirror with M2 operating in the Active Region

Assume that M2 has a voltage across the drain-source of 0.1 \( V_{ds} \) (sat). Design the \( W_2/L_2 \) ratio so that \( I_1 = I_2 = 100 \mu A \) if \( W_1/L_1 = 10 \). Find the value of \( I_2 \) if M2 is saturated.

**Solution**

Using the parameters of Table 3.1-2, we find that the saturation voltage of M2 is

\[
V_{ds1(sat)} = \sqrt{\frac{2I_1}{K_N' (W_2/L_2)}} = \sqrt{\frac{200}{110 \cdot 10}} = 0.4264 V
\]

Now using the active equation of M2, we set \( I_2 = 100 \mu A \) and solve for \( W_2/L_2 \).

\[
100 \mu A = K_N' (W_2/L_2) [V_{ds1(sat)} \cdot V_{ds2} - 0.5 V_{ds2}^2]
\]

\[
= 110 \mu A / V^2 (W_2/L_2) [0.426 \cdot 0.0426 - 0.5 \cdot 0.0426^2] V^2 = 1.883 \times 10^6 (W_2/L_2)
\]

Thus,

\[
100 = 1.883(W_2/L_2) \rightarrow \frac{W_2}{L_2} = 53.12
\]

Now if M2 should become saturated, the value of the output current of the mirror with 100\( \mu A \) input would be 531\( \mu A \) or a boosting of 5.31 times \( I_1 \).

Implementation of the Current Mirror Boosting Concept

\[k = \text{overdrive factor of the current mirror}\]
**A Better Way to Achieve the Current Mirror Boosting**

It was found that when the current mirror boosting idea illustrated on the previous slide was used that when the current increased through the cascode device (M16) that $V_{GS16}$ increased limiting the increase of $V_{DS12}$. This can be overcome by the following circuit.

![Fig. 7.4-7A](image)

**SUMMARY**

- Operation of transistors is generally in weak inversion
- Boosting techniques are needed to get output sourcing and sinking currents that are larger than that available during quiescent operation
- Be careful about using circuits at weak inversion, i.e. the self-biased cascode will cause the resistor to be too large