LECTURE 360 – CHARACTERIZATION OF COMPARATORS
(READING: AH – 439-444)

Objective
The objective of this presentation is:
1.) Introduction to the comparator
2.) Characterization of the comparator

Outline
• Static characterization
• Dynamic characterization
• Summary

What is a Comparator?
The comparator is essentially a 1-bit analog-digital converter.
  Input is analog
  Output is digital
Types of comparators:
1.) Open-loop (op amps without compensation)
2.) Regenerative (use of positive feedback - latches)
3.) Combination of open-loop and regenerative comparators
**Circuit Symbol for a Comparator**

![Circuit Symbol for a Comparator](image)

**Fig. 8.1-1**

**Static Characteristics**
- Gain
- Output high and low states
- Input resolution
- Offset
- Noise

**Dynamic Characteristics**
- Propagation delay
- Slew rate

**Noninverting and Inverting Comparators**

The comparator output is binary with the two-level outputs defined as,

- $V_{OH} = \text{the high output of the comparator}$
- $V_{OL} = \text{the low level output of the comparator}$

Voltage transfer function of an Noninverting and Inverting Comparator:

![Voltage Transfer Function](image)

**Noninverting Comparator**

- $V_{OH}$
- $V_{OL}$
- $v_o$ vs $v_P - v_N$

**Inverting Comparator**

- $V_{OH}$
- $V_{OL}$
- $v_o$ vs $v_P - v_N$

**Fig. 8.1-2A**
**Static Characteristics - Zero-order Model for a Comparator**

Voltage transfer function curve:

\[
\begin{align*}
\text{Vo} & = \text{VOH} \\
\text{vP-vN} & = \text{VOL} \\
\end{align*}
\]

Model:

\[
f_0(v_{P-v_N}) = \begin{cases} 
\text{VOH} & \text{for } (v_{P-v_N}) > 0 \\
\text{VOL} & \text{for } (v_{P-v_N}) < 0 
\end{cases}
\]

Gain = \( A_v = \lim_{\Delta V \to 0} \frac{V_{OH}V_{OL}}{\Delta V} \) where \( \Delta V \) is the input voltage change

**Static Characteristics - First-Order Model for a Comparator**

Voltage transfer curve:

\[
\begin{align*}
\text{vo} & = \text{VOH} \\
\text{vP-vN} & = \text{VOL} \\
\end{align*}
\]

where for a noninverting comparator,

\[
V_{IH} = \text{smallest input voltage at which the output voltage is } V_{OH} \\
V_{IL} = \text{largest input voltage at which the output voltage is } V_{OL}
\]

Model:

\[
f_1(v_{P-v_N}) = \begin{cases} 
\text{VOH for } (v_{P-v_N}) > V_{IH} \\
A_v(v_{P-v_N}) & \text{for } V_{IL} < (v_{P-v_N}) < V_{IH} \\
\text{VOL for } (v_{P-v_N}) < V_{IL} 
\end{cases}
\]

The voltage gain is \( A_v = \frac{V_{OH} - V_{OL}}{V_{IH} - V_{IL}} \)
Static Characteristics - First-Order Model including Input Offset Voltage

Voltage transfer curve:

\[ V_{OS} = \text{the input voltage necessary to make the output equal} \frac{V_{OH}+V_{OL}}{2} \text{ when } v_{P} = v_{N}. \]

Model:

Other aspects of the model:

- \( ICMR \) = input common mode voltage range (all transistors remain in saturation)
- \( R_{in} \) = input differential resistance
- \( R_{icm} \) = common mode input resistance

Static Characteristics - Comparator Noise

Noise of a comparator is modeled as if the comparator were biased in the transition region.

Noise leads to an uncertainty in the transition region which causes jitter or phase noise.
Dynamic Characteristics - Propagation Time Delay

Rising propagation delay time:

![Diagram of rising propagation delay time]

\[ v_o = \frac{V_{OH} + V_{OL}}{2} \]

\[ t_p = \frac{V_{IH} + V_{IL}}{2} \]

Propagation delay time = \( \frac{\text{Rising propagation delay time} + \text{Falling propagation delay time}}{2} \)

Dynamic Characteristics - Single-Pole Response

Model:

\[ A_v(s) = \frac{A_v(0)}{s + \frac{1}{\omega_c \tau_c + 1}} = \frac{A_v(0)}{s + 1/\omega_c} \]

where

\[ A_v(0) = \text{dc voltage gain of the comparator} \]

\[ \omega_c = \frac{1}{\tau_c} = -3\text{dB frequency of the comparator or the magnitude of the pole} \]

Step Response:

\[ v_o(t) = A_v(0) \left[ 1 - e^{-t/\tau_c} \right] V_{in} \]

where

\[ V_{in} = \text{the magnitude of the step input.} \]
Dynamic Characteristics - Propagation Time Delay

The rising propagation time delay for a single-pole comparator is:

\[
\frac{V_{OH} - V_{OL}}{2} = A_v(0) \left[ 1 - e^{-tp/\tau_c} \right] V_{in} \quad \rightarrow \quad tp = \tau_c \ln \left[ \frac{1}{1 - \frac{V_{OH} - V_{OL}}{2A_v(0)V_{in}}} \right]
\]

Define the minimum input voltage to the comparator as,

\[
V_{in}(\text{min}) = \frac{V_{OH} - V_{OL}}{A_v(0)} \quad \rightarrow \quad tp = \tau_c \ln \left[ \frac{1}{1 - \frac{V_{in}(\text{min})}{2V_{in}}} \right]
\]

Define \( k \) as the ratio of the input step voltage, \( V_{in} \), to the minimum input voltage, \( V_{in}(\text{min}) \),

\[
k = \frac{V_{in}}{V_{in}(\text{min})} \quad \rightarrow \quad tp = \tau_c \ln \left[ \frac{2k}{2k-1} \right]
\]

Thus, if \( k = 1 \), \( tp = 0.693 \tau_c \).

Illustration:

Obviously, the more overdrive applied to the input, the smaller the propagation delay time.

Dynamic Characteristics - Slew Rate of a Comparator

If the rate of rise or fall of a comparator becomes large, the dynamics may be limited by the slew rate.

Slew rate comes from the relationship,

\[
i = C \frac{dv}{dt}
\]

where \( i \) is the current through a capacitor and \( v \) is the voltage across it.

If the current becomes limited, then the voltage rate becomes limited.

Therefore for a comparator that is slew rate limited we have,

\[
 tp = \Delta T = \frac{AV}{SR} = \frac{V_{OH} - V_{OL}}{2 \cdot SR}
\]

where

\[
SR = \text{slew rate of the comparator.}
\]
Example 1 - Propagation Delay Time of a Comparator

Find the propagation delay time of an open loop comparator that has a dominant pole at $10^3$ radians/sec, a dc gain of $10^4$, a slew rate of $1V/\mu s$, and a binary output voltage swing of $1V$. Assume the applied input voltage is $10mV$.

Solution

The input resolution for this comparator is $1V/10^4$ or $0.1mV$. Therefore, the $10mV$ input is 100 times larger than $v_{in}(min)$ giving a $k$ of 100. Therefore, we get

$$t_p = \frac{1}{10^3} \ln\left(\frac{2 \cdot 100}{2 \cdot 100 - 1}\right) = 10^{-3} \ln\left(\frac{200}{199}\right) = 5.01\mu s$$

For slew rate considerations, we get

$$t_p = \frac{1}{2 \cdot 1 \times 10^6} = 0.5\mu s$$

Therefore, the propagation delay time for this case is the larger or $5.01\mu s$.

SUMMARY

- A comparator is a one-bit ADC
- Comparators can be noninverting or inverting
- Types of comparators include:
  - Open-loop
  - Regenerative
  - Open-loop and regenerative
- Static Characteristics
  - Gain
  - Output high and low states
  - Input resolution
  - Offset
  - Noise
- Dynamic Characteristics
  - Propagation delay
  - Slew rate