

## LECTURE 380 – TWO-STAGE OPEN-LOOP COMPARATORS - II

### (READING: AH – 445-461)

#### Trip Point of an Inverter

In order to determine the propagation delay time, it is necessary to know when the second stage of the two-stage comparator begins to “turn on”.

Second stage:

Trip point:

Assume that M6 and M7 are saturated. (We know that the steepest slope occurs for this condition.)

Equate  $i_6$  to  $i_7$  and solve for  $v_{in}$  which becomes the trip point.

$$\therefore v_{in} = V_{TRP} = V_{DD} - |V_{TP}| - \sqrt{\frac{K_N(W_7/L_7)}{K_P(W_6/L_6)}} (V_{Bias} - V_{SS} - V_{TN})$$

Example:

If  $W_7/L_7 = W_6/L_6$ ,  $V_{DD} = 2.5V$ ,  $V_{SS} = -2.5V$ , and  $V_{Bias} = 0V$  the trip point for the circuit above is

$$V_{TRP} = 2.5 - 0.7 - \sqrt{110/50} (0 + 2.5 - 0.7) = -0.870V$$

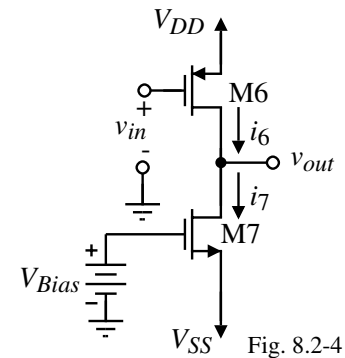


Fig. 8.2-4

#### Propagation Delay Time of a Slewing, Two-Stage, Open-Loop Comparator

Previously we calculated the propagation delay time for a nonslewing comparator.

If the comparator slews, then the propagation delay time is found from

$$i_i = C_i \frac{dv_i}{dt_i} = C_i \frac{\Delta v_i}{\Delta t_i}$$

where

$C_i$  is the capacitance to ground at the output of the  $i$ -th stage

The propagation delay time of the  $i$ -th stage is,

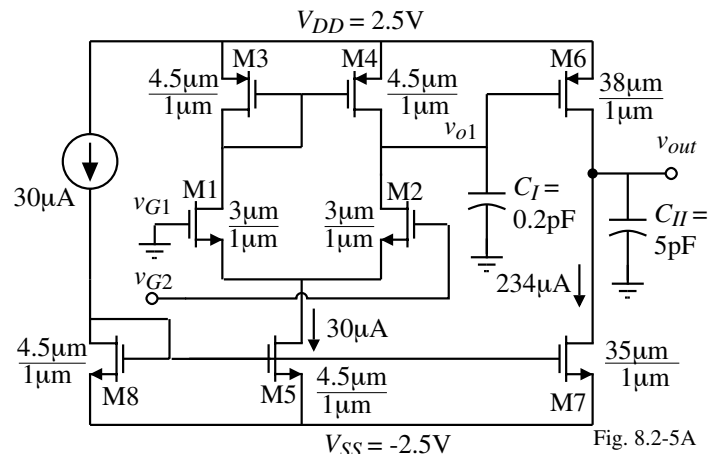
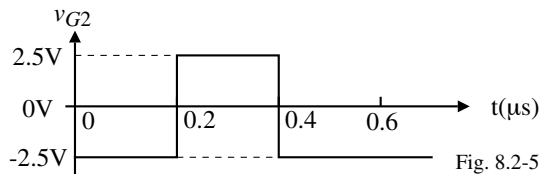
$$t_i = \Delta t_i = C_i \frac{\Delta V_i}{I_i}$$

The propagation delay time is found by summing the delays of each stage.

$$t_p = t_1 + t_2 + t_3 + \dots$$

### Example 2-5 - Propagation Time Delay of a Two-Stage, Open-Loop Comparator

For the two-stage comparator shown assume that  $C_I = 0.2\text{pF}$  and  $C_{II} = 5\text{pF}$ . Also, assume that  $v_{G1} = 0\text{V}$  and that  $v_{G2}$  has the waveform shown. If the input voltage is large enough to cause slew to dominate, find the propagation time delay of the rising and falling output of the comparator and give the propagation time delay of the comparator.



#### Solution

- Total delay = sum of the first and second stage delays,  $t_1$  and  $t_2$
- First, consider the change of  $v_{G2}$  from  $-2.5\text{V}$  to  $2.5\text{V}$  at  $0.2\mu\text{s}$ .  
The last row of Table 8.2-1 gives  $v_{o1} = +2.5\text{V}$  and  $v_{out} = -2.5\text{V}$
- $t_{f1}$ , requires  $C_I$ ,  $\Delta V_{o1}$ , and  $I_5$ .  $C_I = 0.2\text{pF}$ ,  $I_5 = 30\mu\text{A}$  and  $\Delta V_1$  can be calculated by finding the trip point of the output stage/

### Example 2-5 - Continued

- The trip point of the output stage by setting the current of M6 when saturated equal to  $234\mu\text{A}$ .

$$\frac{\beta_6}{2} (V_{SG6} - |V_{TP}|)^2 = 234\mu\text{A} \rightarrow V_{SG6} = 0.7 + \sqrt{\frac{234 \cdot 2}{110 \cdot 38}} = 1.035\text{V}$$

Therefore, the trip point of the second stage is  $V_{TRP2} = 2.5 - 1.035 = 1.465\text{V}$

Therefore,  $\Delta V_1 = 2.5\text{V} - 1.465\text{V} = V_{SG6} = 1.035\text{V}$ . Thus the falling propagation time delay of the first stage is

$$t_{fo1} = 0.2\text{pF} \left( \frac{1.035\text{V}}{30\mu\text{A}} \right) = 6.9\text{ns}$$

- The rising propagation time delay of the second stage requires  $C_{II}$ ,  $\Delta V_{out}$ , and  $I_6$ .  $C_{II}$  is given as  $5\text{pF}$ ,  $\Delta V_{out} = 2.5\text{V}$  (assuming the trip point of the circuit connected to the output of the comparator is  $0\text{V}$ ), and  $I_6$  can be found as follows:

$$V_{G6}(\text{guess}) \approx 0.5[V_{G6}(I_6=234\mu\text{A}) + V_{G6}(\text{min})]$$

$$V_{G6}(\text{min}) = V_{G1} - V_{GS1}(I_{SS}/2) + V_{DS2} \approx -V_{GS1}(I_{SS}/2) = -0.7 - \sqrt{\frac{2 \cdot 15}{110 \cdot 3}} = -1.00\text{V}$$

$$V_{G6}(\text{guess}) \approx 0.5(1.465\text{V} - 1.00\text{V}) = 0.232\text{V}$$

Therefore  $V_{SG6} = 2.27\text{V}$  and  $I_6 = \frac{\beta_6}{2} (V_{SG6} - |V_{TP}|)^2 = \frac{38 \cdot 50}{2} (2.27 - 0.7)^2 = 2,342\mu\text{A}$

**Example 2-5 - Continued**

6.) The rising propagation time delay for the output can be expressed as

$$t_{rout} = 5\text{pF} \left( \frac{2.5\text{V}}{2,342\mu\text{A} - 234\mu\text{A}} \right) = 5.93\text{ns}$$

Thus the total propagation time delay of the rising output of the comparator is approximately 12.8ns and most of this delay is attributable to the first stage.

7.) Next consider the change of  $v_{G2}$  from 2.5V to -2.5V which occurs at 0.4 $\mu$ s. We shall assume that  $v_{G2}$  has been at 2.5V long enough for the conditions of Table 8.2-1 to be valid. Therefore,  $v_{o1} \approx V_{SS} = -2.5\text{V}$  and  $v_{out} \approx V_{DD}$ . The propagation time delays for the first and second stages are calculated as

$$t_{ro1} = 0.2\text{pF} \left( \frac{1.465\text{V} - (-1.13\text{V})}{30\mu\text{A}} \right) = 17.3\text{ns}$$

$$t_{fout} = 5\text{pF} \left( \frac{2.5\text{V}}{234\mu\text{A}} \right) = 53.42\text{ns}$$

8.) The total propagation time delay of the falling output is 70.72ns. Taking the average of the rising and falling propagation time delays gives a propagation time delay for this two-stage, open-loop comparator of about 41.76ns.

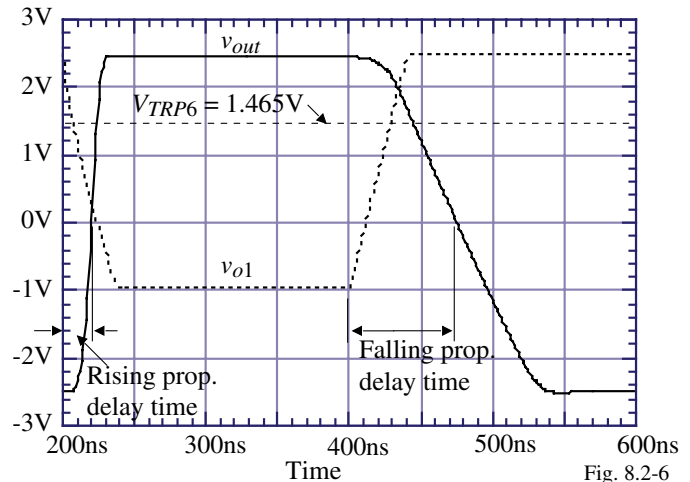


Fig. 8.2-6

**Design of a Two-Stage, Open-Loop Comparator**

Table 8.2-2 Design of the Two-Stage, Open-Loop Comparator of Fig. 8.2-3 for a Linear Response.

Step	Design Relationships	Comments
1	Specifications: $t_p, C_{II}, V_{in}(\min), V_{OH}, V_{OL}, V_{icm}^+, V_{icm}^-$ , and overdrive Constraints: Technology, $V_{DD}$ and $V_{SS}$ $ p_{I1}  =  p_{I2}  = \frac{1}{t_p \sqrt{m k}}$ and $I_7 = I_6 = \frac{ p_{I2}  C_{II}}{\lambda_N + \lambda_P}$	Choose $m = 1$
2	$\frac{W_6}{L_6} = \frac{2 \cdot I_6}{K_P (V_{SD6}(\text{sat}))^2}$ and $\frac{W_7}{L_7} = \frac{2 \cdot I_7}{K_N (V_{DS7}(\text{sat}))^2}$	$V_{SD6}(\text{sat}) = V_{DD} - V_{OH}$ $V_{DS7}(\text{sat}) = V_{OL} - V_{SS}$
3	Guess $C_I$ as 0.1pF to 0.5pF $\therefore I_5 = I_7 \frac{2C_I}{C_{II}}$	A result of choosing $m = 1$ . Will check $C_I$ later
4	$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{I_5}{K_P (V_{SG3} -  V_{TP} )^2}$	$V_{SG3} = V_{DD} - V_{icm}^+ + V_{TN}$
5	$g_{m1} = \frac{A_v(0)(g_{ds2} + g_{ds4})(g_{ds6} + g_{ds7})}{g_{m6}}$ $\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}^2}{K_N I_5}$	$g_{m6} = \sqrt{\frac{2K_P W_6 I_6}{L_6}}$ $A_v(0) = \frac{V_{OH} + V_{OL}}{V_{in}(\min)}$
6	Find $C_I$ and check assumption $C_I = C_{gd2} + C_{gd4} + C_{gs6} + C_{bd2} + C_{bd4}$	If $C_I$ is greater than the guess in step 3, then increase $C_I$ and repeat steps 4 through 6
7	$V_{DS5}(\text{sat}) = V_{icm}^- - V_{GS1} - V_{SS}$ $\frac{W_5}{L_5} = \frac{2 \cdot I_5}{K_N (V_{DS5}(\text{sat}))^2}$	If $V_{DS5}(\text{sat})$ is less than 100mV, increase $W_1/L_1$ .

**Example 2-6 - Two-Stage, Open-Loop Comparator Design for a Linear Response.**

Assume the specifications of the comparator shown are given below.

$$t_p = 50\text{ns} \quad V_{OH} = 2\text{V} \quad V_{OL} = -2\text{V}$$

$$V_{DD} = 2.5\text{V} \quad V_{SS} = -2.5\text{V} \quad C_{II} = 5\text{pF}$$

$$V_{in}(\text{min}) = 1\text{mV} \quad V_{icm}^+ = 2\text{V} \quad V_{icm}^- = -1.25\text{V}$$

Also assume that the overdrive will be a factor of 10. Use this architecture to achieve the above specifications and assume that all channel lengths are to be  $1\mu\text{m}$ .

**Solution**

Following the procedure outlined in Table 8.2-2, we choose  $m = 1$  to get

$$|p_I| = |p_{II}| = \frac{10^9}{50\sqrt{10}} = 6.32 \times 10^6 \text{ rads/sec}$$

This gives

$$I_6 = I_7 = \frac{6.32 \times 10^6 \cdot 5 \times 10^{-12}}{0.04 + 0.05} = 351 \mu\text{A} \rightarrow I_6 = I_7 = 400 \mu\text{A}$$

Therefore,

$$\frac{W_6}{L_6} = \frac{2 \cdot 400}{(0.5)^2 \cdot 50} = 64 \quad \text{and} \quad \frac{W_7}{L_7} = \frac{2 \cdot 400}{(0.5)^2 \cdot 110} = 29$$

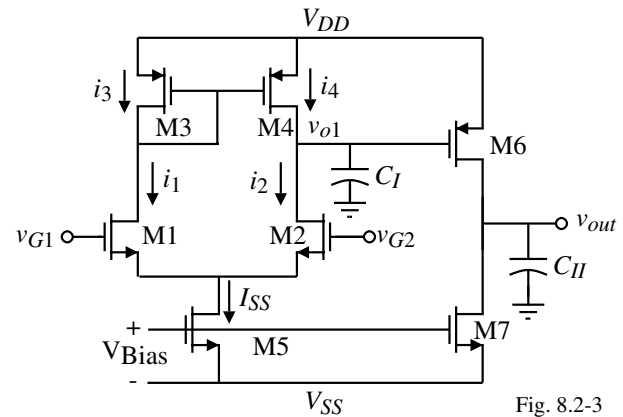


Fig. 8.2-3

**Example 2-6 - Continued**

Next, we guess  $C_I = 0.2\text{pF}$ . This gives  $I_5 = 32\mu\text{A}$  and we will increase it to  $40\mu\text{A}$  for a margin of safety. Step 4 gives  $V_{SG3}$  as  $1.2\text{V}$  which results in

$$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{40}{50(1.2-0.7)^2} = 3.2 \quad \rightarrow \quad \frac{W_3}{L_3} = \frac{W_4}{L_4} = 4$$

The desired gain is found to be 4000 which gives an input transconductance of

$$g_{m1} = \frac{4000 \cdot 0.09 \cdot 20}{44.44} = 162 \mu\text{S}$$

This gives the  $W/L$  ratios of M1 and M2 as

$$\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{(162)^2}{110 \cdot 40} = 5.96 \quad \rightarrow \quad \frac{W_1}{L_1} = \frac{W_2}{L_2} = 6$$

To check the guess for  $C_I$  we need to calculate it which is done as

$$C_I = C_{gd2} + C_{gd4} + C_{gs6} + C_{bd2} + C_{bd4} = 0.9\text{fF} + 1.3\text{fF} + 119.5\text{fF} + 20.4\text{fF} + 36.8\text{fF} = 178.9\text{fF}$$

which is less than what was guessed so we will make no changes.

**Example 2-6 - Continued**

Finally, the  $W/L$  value of M5 is found by finding  $V_{GS1}$  as 0.946V which gives  $V_{DS5}(\text{sat}) = 0.304\text{V}$ . This gives

$$\frac{W_5}{L_5} = \frac{2 \cdot 40}{(0.304)^2 \cdot 110} = 7.87 \approx 8$$

Obviously, M5 and M7 cannot be connected gate-gate and source-source. The value of  $I_5$  and  $I_7$  must be derived separately as illustrated below. The  $W$  values are summarized below assuming that all channel lengths are  $1\mu\text{m}$ .

$$W_1 = W_2 = 6\mu\text{m} \quad W_3 = W_4 = 4\mu\text{m} \quad W_5 = 8\mu\text{m} \quad W_6 = 64\mu\text{m} \quad W_7 = 29\mu\text{m}$$

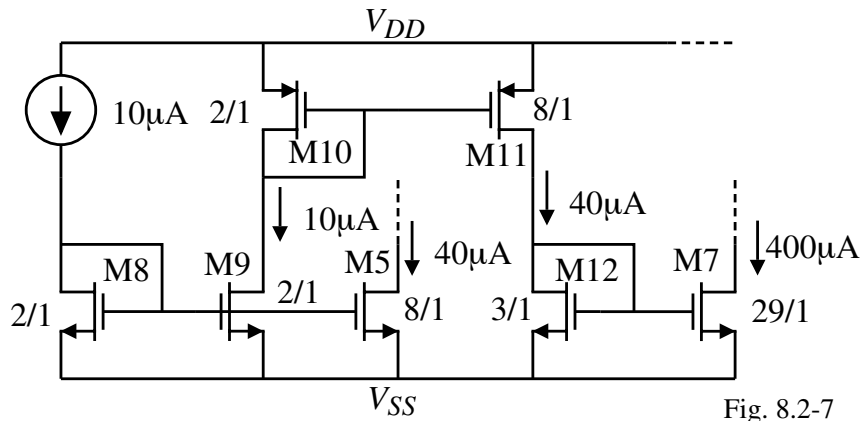


Fig. 8.2-7

**Design of a Two-Stage Comparator for a Slewing Response**

Table 8.2-3 Two-Stage, Open-Loop Comparator Design for a Slewing Response.

Step	Design Relationships	Comments
	Specifications: $t_p, C_{II}, V_{in}(\text{min}), V_{OH}, V_{OL}, V_{icm}^+, V_{icm}^-$	Constraints: Technology, $V_{DD}$ and $V_{SS}$
1	$I_7 = I_6 = C_{II} \frac{dv_{out}}{dt} = \frac{C_{II}(V_{OH}-V_{OL})}{t_p}$	Assume the trip point of the output is $(V_{OH}-V_{OL})/2$ .
2	$\frac{W_6}{L_6} = \frac{2 \cdot I_6}{K_P'(V_{SD6}(\text{sat}))^2}$ and $\frac{W_7}{L_7} = \frac{2 \cdot I_7}{K_N'(V_{DS7}(\text{sat}))^2}$	$V_{SD6}(\text{sat}) = V_{DD} - V_{OH}$ $V_{DS7}(\text{sat}) = V_{OL} - V_{SS}$
3	Guess $C_I$ as 0.1pF to 0.5pF $\therefore I_5 = I_7 \frac{2C_I}{C_{II}}$	Typically $0.1\text{pF} < C_I < 0.5\text{pF}$
4	$I_5 = C_I \frac{dv_{o1}}{dt} \approx \frac{C_I(V_{OH}-V_{OL})}{t_p}$	Assume that $v_{o1}$ swings between $V_{OH}$ and $V_{OL}$ .
5	$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{I_5}{K_P'(V_{SG3}-V_{TPl})^2}$	$V_{SG3} = V_{DD} - V_{icm}^+ + V_{TN}$
6	$g_{m1} = \frac{A_v(0)(g_{ds2}+g_{ds4})(g_{ds6}+g_{ds7})}{g_{m6}} \quad \frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}^2}{K_N'I_5}$	$g_{m6} = \sqrt{\frac{2K_P'W_6I_6}{L_6}} \quad A_v(0) = \frac{V_{OH}+V_{OL}}{V_{in}(\text{min})}$
7	Find $C_I$ and check assumption $C_I = C_{gd2} + C_{gd4} + C_{gs6} + C_{bd2} + C_{bd4}$	If $C_I$ is greater than the guess in step 3, increase the value of $C_I$ and repeat steps 4 through 6
8	$V_{DS5}(\text{sat}) = V_{icm}^- - V_{GS1} - V_{SS} \quad \frac{W_5}{L_5} = \frac{2 \cdot I_5}{K_N'(V_{DS5}(\text{sat}))^2}$	If $V_{DS5}(\text{sat})$ is less than 100mV, increase $W_1/L_1$ .

**Example 2-7 - Two-Stage, Open-Loop Comparator Design for a Slewing Response**

Assume the specifications of Fig. 8.2-3 are given below.

$$t_p = 50\text{ns} \quad V_{OH} = 2\text{V} \quad V_{OL} = -2\text{V} \quad V_{DD} = 2.5\text{V} \quad V_{SS} = -2.5\text{V}$$

$$C_{II} = 5\text{pF} \quad V_{in}(\text{min}) = 1\text{mV} \quad V_{icm}^+ = 2\text{V} \quad V_{icm}^- = -1.25\text{V}$$

Design a two-stage, open-loop comparator using the circuit of Fig. 8.2-3 to the above specifications and assume all channel lengths are to be  $1\mu\text{m}$ .

**Solution**

Following the procedure outlined in Table 8.2-3, we calculate  $I_6$  and  $I_7$  as

$$I_6 = I_7 = \frac{5 \times 10^{-12.4}}{50 \times 10^{-9}} = 400\mu\text{A}$$

Therefore,

$$\frac{W_6}{L_6} = \frac{2 \cdot 400}{(0.5)^2 \cdot 2.50} = 64 \quad \text{and} \quad \frac{W_7}{L_7} = \frac{2 \cdot 400}{(0.5)^2 \cdot 1.10} = 29$$

Next, we guess  $C_I = 0.2\text{pF}$ . This gives

$$I_5 = \frac{0.2\text{pF}(4\text{V})}{50\text{ns}} = 16\mu\text{A} \quad \rightarrow \quad I_5 = 20\mu\text{A}$$

Step 5 gives  $V_{SG3}$  as  $1.2\text{V}$  which results in

$$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{20}{50(1.2-0.7)^2} = 1.6 \quad \rightarrow \quad \frac{W_3}{L_3} = \frac{W_4}{L_4} = 2$$

**Example 2-7 - Continued**

The desired gain is found to be 4000 which gives an input transconductance of

$$g_{m1} = \frac{4000 \cdot 0.09 \cdot 10}{44.44} = 81\mu\text{S}$$

This gives the  $W/L$  ratios of M1 and M2 as

$$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{(81)^2}{110 \cdot 40} = 1.49 \quad \rightarrow \quad \frac{W_1}{L_1} = \frac{W_2}{L_2} = 2$$

To check the guess for  $C_I$  we need to calculate it which done as

$$C_I = C_{gd2} + C_{gd4} + C_{gs6} + C_{bd2} + C_{bd4} = 0.9\text{fF} + 0.4\text{fF} + 119.5\text{fF} + 20.4\text{fF} + 15.3\text{fF} = 156.5\text{fF}$$

which is less than what was guessed.

Finally, the  $W/L$  value of M5 is found by finding  $V_{GS1}$  as  $1.00\text{V}$  which gives  $V_{DS5}(\text{sat}) = 0.25\text{V}$ . This gives

$$\frac{W_5}{L_5} = \frac{2 \cdot 20}{(0.25)^2 \cdot 1.10} = 5.8 \approx 6$$

As in the previous example, M5 and M7 cannot be connected gate-gate and source-source and a scheme like that of Example 8.2-6 must be used. The  $W$  values are summarized below assuming that all channel lengths are  $1\mu\text{m}$ .

$$W_1 = W_2 = 2\mu\text{m} \quad W_3 = W_4 = 4\mu\text{m} \quad W_5 = 6\mu\text{m} \quad W_6 = 64\mu\text{m} \quad W_7 = 29\mu\text{m}$$

## **SUMMARY**

- The two-stage, open-loop comparator has two poles which should be as large as possible
- The transient response of a two-stage, open-loop comparator will be limited by either the bandwidth or the slew rate
- It is important to know the initial states of a two-stage, open-loop comparator when finding the propagation delay time
- If the comparator is gainbandwidth limited then the poles should be as large as possible for minimum propagation delay time
- If the comparator is slew rate limited, then the current sinking and sourcing ability should be as large as possible