**EXAMINATION NO. 1 - SOLUTIONS**

(Average Score = 66.2/100)

**Problem 1 - (25 points)**

A push-pull follower is shown with a 500Ω load. Assume that the MOSFETs have the following model parameters: \( K_N' = 100\mu A/V^2, V_TN = 0.5V, \) and \( K_P' = 50\mu A/V^2, V_{TP} = -0.5V. \) Ignore the bulk effects and assume \( \lambda = 0. \)

a.) Find the small signal voltage gain and the output resistance (not including \( R_L \)) for the conditions of part a.) if the dc current in M1 and M2 is 100µA.

b.) What is the output voltage when \( v_{IN} = 0.5V? \)

**Solution**

a.) The small-signal model is given as shown

where \( g_{m1} = \sqrt{2K_N'(W_1/L_1)I_{DL}} = \sqrt{2\cdot100\cdot50\cdot100} \)

\( g_{m1} = 1\)mS, \( g_{m2} = \sqrt{2\cdot50\cdot100\cdot100} = 1\)mS

Summing currents at the output node gives,

\[ g_{m1}(v_{in} - v_{out}) + g_{m2}(v_{in} - v_{out}) = G_Lv_{out} \]

\[ \therefore \frac{v_{out}}{v_{in}} = \frac{g_{m1} + g_{m2}}{g_{m1} + g_{m2} + G_L} = \frac{1+1}{1+1+2} = \frac{0.5V}{V} \]

and \( R_{out} = \frac{1}{g_{m1}+g_{m2}} = \frac{1}{2mS} = 500\Omega \)

b.) Under the condition of \( v_{IN} = 0.5V, \) the gate voltages are

\( V_{G1} = 0.5V+0.7V = 1.2V \) and \( V_{G1} = 0.5V-0.7V = -0.2V \)

We know that the output voltage can be expressed as \( V_{OUT} = (I_1-I_2)0.5k\Omega \) where \( I_1 \) and \( I_2 \) are the dc currents in M1 and M2.

Next we need to make an assumption about the operating region of the two transistors. Let us assume that M1 is saturated and M2 is cutoff. Therefore, \( I_2 = 0 \) and

\[ I_1 = 0.5(100)(50)[1.2-V_{OUT} - 0.5]^2 (\mu A) = 2.5(0.7-V_{OUT})^2 (mA) \]

\[ \therefore V_{OUT} = (I_1)0.5k\Omega = 1.25(0.7-V_{OUT})^2 \rightarrow 0.8V_{OUT} = 0.49 - 1.4V_{OUT} + V_{OUT}^2 \]

The resulting quadratic, \( V_{OUT}^2 - 2.2V_{OUT} + 0.49 = 0 \) gives

\[ V_{OUT} = 1.19\pm0.5\sqrt{2.2^2-4(0.49)} = 1.1 \pm 0.5(2.880) = 1.1\pm0.845 = 0.252V \]

Check the regions of M1 and M2.

M1: \( V_{DS1} = 1.5-0.252 = 1.25V > V_{GS1} - V_{TN} = 1.2-0.5 = 0.7 \) \( \therefore \) M1 is saturated.

M2: \( V_{SG2} = 0.252 - (-0.2) = 0.452 < |V_{TP}| = 0.5 \) \( \therefore \) M2 is cutoff

(Note: Many of you multiplied the gain of (a.) by 0.5 to get the answer which happened to be close. If you did this you lost 3 points because the current in M1 is no longer 100µA but 506µA and the current in M2 is 0µA. Your answer may be right but the approach is wrong. PA)
Problem 2 - (25 points)

An all-npn Darlington output stage is shown. For all devices assume that $V_{BE(on)} = 0.7V$, $V_{CE(sat)} = 0.2V$, and $\beta_F = 100$. The magnitude of the collector current in Q3 is 2mA.

a.) If $R_L = 8\Omega$, calculate the maximum positive and negative limits of $v_{OUT}$.

b.) Calculate the power dissipated in the circuit for $v_{OUT} = 0V$.

c.) Calculate the maximum average power that can be delivered to $R_L = 8\Omega$ before clipping occurs and the corresponding efficiency of the complete circuit. Assume that feedback is used around the circuit so that $v_{OUT}$ is approximately a sinusoidal.

Solution

a.) Assume that current to $R_L$ is not limited. Then,

$$V_o^+ = V_{CC} - V_{CE3(sat)} - V_{BE5} - V_{BE4} = 12 - 0.2 - 0.7 - 0.7 = 10.4V$$

$$V_o^- = -V_{CC} + V_{CE2(sat)} + V_{BE1} + V_{D1} = -12 + 0.2 + 0.7 + 0.7 = -10.4V$$

b.) For $v_{OUT} = 0V$, the 2mA flowing through Q3 flows through D3, D2 and Q1-Q2. The voltage drop across D2 and D3 is not enough to turn on D1, Q4, or Q5. Thus, the power dissipation is

$$P_Q = [+12-(-12)](2mA) = 48mW$$

c.) From part b.) we observe that the amplifier is operating in class B. The maximum load power is

$$P_{L(max)} = \frac{(V_{o}(peak))^2}{\sqrt{2} R_L} = \frac{(10.4)^2}{2 \cdot 8} = 6.76W$$

$$I_{Supply} = \frac{1}{\pi} I_{(peak)} = \frac{1}{\pi} \frac{10.4V}{8\Omega} = 0.4138A$$

$$\therefore P_{Supply} = [12-(-12)](0.4138) = 9.93W$$

$$\therefore \text{Efficiency} = \eta = \frac{P_{L(max)}}{P_Q + P_{Supply}} = \frac{6.76W}{0.048W + 9.93W} = 67.7\%$$
Problem 3 - (25 points)

Find the voltage transfer function of the common-gate amplifier shown. Identify the numerical values of the small-signal voltage gain, $v_{out}/v_{in}$, and the poles and zeros. Assume that $I_D$ = 500μA, $K_N’$ = 100μA/V², $V_{TN}$ = 0.5V, and $K_P’$ = 50μA/V², $V_{TP}$ = -0.5V, $\lambda$ = 0V⁻¹, $C_{gs}$ = 0.5pF and $C_{gd}$ = 0.1pF.

Solution

The small signal transconductance is,

$$g_m = \sqrt{2 \cdot K_N’(W/L)} I_D = \sqrt{2 \cdot 100 \cdot 10 \cdot 500} = 1\text{mS}$$

$r_{ds} = \infty$

The small signal model is,

![Small Signal Model Diagram]

The voltage gain can be expressed as follows,

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{gs}} \left( \frac{V_{gs}}{V_{in}} \right), \quad V_{out} = g_m \left( \frac{R_L (1/sC_{gd})}{R_L + (1/sC_{gd})} \right)$$

Sum currents at the source to get,

$$\frac{V_{in} + V_{gs}}{R_s} + g_m V_{gs} + sC_{gs} V_{gs} = 0 \quad \rightarrow \quad \frac{V_{gs}}{V_{in}} = \frac{-G_s}{G_s + g_m + sC_{gs}}$$

$$\therefore \quad \frac{V_{out}}{V_{in}} = \left( \frac{g_m R_L}{1 + g_m R_L} \right) \left( \frac{1}{sC_{gd} R_L + 1} \right) \left( \frac{sC_{gs}}{g_m + G_s + 1} \right)$$

The various values are,

Voltage gain $= \frac{g_m R_L}{1 + g_m R_L} = \frac{1 \cdot 10}{1 + 1} = 5\text{V/V}$

$$p_1 = \frac{-1}{C_{gd} R_L} = \frac{-1}{10^{-13} \cdot 10^4} = -10^9 \text{radians/sec.}$$

$$p_2 = \frac{-(g_m + G_s)}{C_{gs}} = \frac{-10^{-3} + 10^{-3}}{0.5 \times 10^{-12}} = 4 \times 10^9 \text{radians/sec.}$$
Problem 4 - (25 points)

For the circuit shown, assume all transistors are saturated and that their parasitic capacitors are negligible when compared with the ones shown. Assume that their respective $g_m$'s and $r_{ds}$'s are equal.

a.) Using either the open-circuit or short-circuit time-constant test, determine the 3-dB bandwidth ($f_{3\text{-}dB}$) of the circuit.

b.) Which capacitor is predominant in determining this frequency?

c.) Using either the open-circuit or short-circuit time-constant test, determine the location of the highest frequency pole, assuming all the poles are more than an order of magnitude away from each other.

d.) Which capacitor determines this high-frequency pole?

Solution:

a.) O.C.T.C. Tst:

\[ R_{oc2} = \frac{r_{ds, mp12}}{r_{ds, mnb12}} + R_2 + \frac{g_m(r_{ds, mp12}/r_{ds, mnb12})R_2}{1} \]

\[ = 250k + 100k + 2.5M = 2.8M \]

\[ R_{oc3} = \frac{r_{ds, mp12}}{r_{ds, mnb12}} = 250k \]

\[ W_{3\text{-}dB} = (\Sigma RC)^{-1} = \frac{1}{(2.8M)(2pF)} \]

\[ = 180k \text{ rad/s} \]

b.) $C_2$

c.) S.C.T.C. Tst:

\[ R_{sc2} = R_2 = 100k \]

\[ R_{sc3} = \frac{r_{ds, mp12}}{r_{ds, mnb12}}/(R_2/(1/g_m) = (250k)/(100k)/(10k) = 9k \]

\[ W_{hf} = \Sigma (1/RC) = 1/(100k)(2p) + 1/(9k)(2p) = 60M \text{ rad/s} \]

d.) $C_3$