Homework Assignment No. 2 - Solutions

Problem 1 - (10 points)

a) For the emitter follower output stage shown below, find the value of $R_I$ for maximum efficiency and find the value of that efficiency. $V_{CC} = -V_{EE} = 2.5\,\text{V}$, $V_{CE(sat)} = 0.2\,\text{V}$, $R_L = 10\,\text{k}\Omega$, $V_{BE(on)} = 0.7\,\text{V}$.

b) The load resistance $R_L$ is replaced with a capacitor of 100pF. If the input voltage suddenly drops from 2.5V to -2.5V, explain what happens at the output and accurately sketch the output voltage as a function of time, specifying its initial and final values and time interval.

Solution

The $I_Q$ for maximum efficiency is found as,

$$I_Q = \left( \frac{V_{CC} - V_{CE(sat)}}{R_L} \right) = 230\,\text{µA}$$

$$R_I = \left( \frac{-V_{EE} - V_{BE}}{I_Q} \right) = 7.826k\Omega$$

$$P_{L(max)} = \left( \frac{V_{CC} - V_{CE(sat)}}{\sqrt{2}} \right) \left( \frac{I_Q}{\sqrt{2}} \right) = 0.5(2.3\,\text{V})(0.23\,\text{mA}) = 0.2645\,\text{mW}$$

$$P_{supply} = 2V_{CC}I_Q = 2(2.5)(0.23\,\text{mA}) = 1.15\,\text{mW}$$

$$\eta = \frac{P_{L(max)}}{P_{supply}} = \frac{1}{4} \left( 1 - \frac{V_{CE(sat)}}{V_{CC}} \right) = 23\%$$

b) The output would slew under such condition. The current will be limited by the bias current:

Slew rate $= 0.23\,\text{mA}/100\,\text{pF} = 2.3\,\text{V/µs}$
Problem 2 - (10 points)

Six versions of a source follower are shown below. Assume that \( K'_N = 2K'_P \), \( \lambda_P = 2\lambda_N \), all W/L ratios of all devices are equal, and that all bias currents in each device are equal. Neglect bulk effects in this problem and assume no external load resistor. Identify which circuit or circuits have the following characteristics: (a.) highest small-signal voltage gain, (b.) lowest small-signal voltage gain, (c.) the highest output resistance, (d.) the lowest output resistance, (e.) the highest \( v_{out(\text{max})} \) and (f.) the lowest \( v_{out(\text{max})} \).

Solution

(a.) and (b.) - Voltage gain.

Small signal model:

The voltage gain is found as:

\[
\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + G_L}
\]

where \( G_L \) is the load conductance. Therefore we get:

\[
\begin{array}{cccccc}
\text{Circuit} & 1 & 2 & 3 & 4 & 5 & 6 \\
\frac{v_{out}}{v_{in}} & \frac{g_mN}{g_mN+g_mN} & \frac{g_mP}{g_mP+g_mP} & \frac{g_mN}{g_mN+g_mN} & \frac{g_mP}{g_mP+g_mP} & \frac{g_mN}{g_mN+g_dsN+g_dsP} & \frac{g_mP}{g_mP+g_dsN+g_dsP} \\
\end{array}
\]

But \( g_mN = \sqrt{2} g_{mp} \) and \( g_dsN = 0.5 g_{dsp} \), therefore

\[
\begin{array}{cccccc}
\text{Circuit} & 1 & 2 & 3 & 4 & 5 & 6 \\
\frac{v_{out}}{v_{in}} & \frac{1}{2} & \frac{1}{2} & 0.5858 & 0.4142 & \frac{g_mP}{g_mP+(g_{dsP}+g_{dsN})/\sqrt{2}} & \frac{g_mP}{g_mP+g_{dsP}+g_{dsN}} \\
\end{array}
\]

Thus, circuit 5 has the highest voltage gain and circuit 4 the lowest gain.

(c.) and (d.) - Output resistance.

The denominators of the first table show the following:

Ckt. 6 has the highest output resistance and Ckt. 1 the lowest output resistance.

(e.) Assuming no current has to be provided by the output, circuits 2, 4, and 6 can pull the output to \( V_{DD} \). ∴ Circuits 2, 4 and 6 have the highest output swing.

(f.) Assuming no current has to be provided by the output, circuits 1, 3, and 5 can pull the output to ground. ∴ Circuits 1, 3 and 5 have lowest output swing.

Summary

(a.) Ckt. 5 has the highest voltage gain
(b.) Ckt. 4 has the lowest voltage gain
(c.) Ckt. 6 has the highest output resistance
(d.) Ckt. 1 has the lowest output resistance
(e.) Ckts. 2,4 and 6 have the highest output
(f.) Ckts. 1,3 and 5 have the lowest output
Problem 3 - (10 points)

A push-pull follower is shown which uses an NPN BJT and a p-channel MOSFET. In this problem, ignore the bulk effect, the channel length modulation, and the Early voltage. The parameters for the NPN BJT are $\beta_F = 100$, $I_s = 10fA$ and $V_t = 25.9mV$. The model parameters for the PMOS are $Kp' = 50\mu A/V^2$ and $V_T = -0.7V$. (a.) Find the value of the dc batteries, $V_1$ and $V_2$, which will cause $100\mu A$ to flow in Q1 and M2 when the dc value of $v_{IN} = 0V_{DC}$. (b.) Find the small-signal input resistance, output resistance and voltage gain when the dc value of $v_{IN} = 0V_{DC}$.

Solution

(a.) $V_1 = V_{BE1} = V_t \ln \left( \frac{i_C}{I_s} \right) = 0.0259 \ln \left( \frac{100\mu A}{10fA} \right) = 0.5964V$ \quad $V_1 = 0.5964V$

$b.) V_2 = V_{SG2} = \sqrt{\frac{2ID}{Kp'(W/L)}} + |V_{TP}| = \sqrt{\frac{2 \cdot 100}{50 \cdot 100} + 0.7} = 0.9V$ \quad $V_2 = 0.9V$

(b.) Small-signal model (simplified):

$g_m1 = \frac{I_{C1}}{V_t} = \frac{100\mu A}{25.9mV} = 3.86mS$

$r_{\pi1} = \frac{1 + \beta_F}{g_m1} = 26.159k\Omega$

$g_m2 = \sqrt{\frac{2Kp'W_2D_2}{L_2}} = \sqrt{2 \cdot 50 \cdot 100} = 1mS$

$R_in : v_{in} = r_{\pi1}i_{in} + (i_{in} + g_m1v_{\pi1} + g_m2v_{gs2})R_L = r_{\pi1}i_{in} + (i_{in} + g_m1r_{\pi1}i_{in} + g_m2r_{\pi1}i_{in})R_L$

$R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi1} + R_L + g_m1r_{\pi1}R_L + g_m2r_{\pi1}R_L = r_{\pi1} + R_L(1 + \beta_F) + g_m2r_{\pi1}R_L$

$\therefore \quad R_{in} = 26.159k\Omega + 101\cdot100\Omega + 1 \cdot 26.159k\Omega \cdot 0.1 = 38.875k\Omega$ \quad $R_{in} = 38.875k\Omega$

$R_{out} : R_{out} = \frac{1}{g_m1} \parallel \frac{1}{g_m2} = \frac{1}{3.86mS + 1mS} = 205.8k\Omega$ \quad $R_{out} = 205.8k\Omega$

$\frac{v_{out}}{v_{in}} : \frac{v_{out}}{i_{in}} = \frac{R_L(1 + \beta_F) + g_m2r_{\pi1}R_L}{r_{\pi1} + R_L(1 + \beta_F) + g_m2r_{\pi1}R_L} = \frac{R_L(1 + \beta_F) + g_m2r_{\pi1}R_L}{38.875} = 0.3271$

$\frac{v_{out}}{v_{in}} = 0.3271V/V$
Problem 4 - (10 points)

Find an algebraic expression for the voltage gain, $v_{out}/v_{in}$, and the output resistance, $R_{out}$, of the source follower shown in terms of the small-signal model parameters, $g_m$ and $R_L$ (ignore $r_{ds}$). If the bias current is 1mA find the numerical value of the voltage gain and the output resistance. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $K_P' = 50\mu A/V^2$, $V_{TP} = -0.7V$.

Solution

A small-signal model for this circuit is shown below neglecting $r_{ds}$ of the transistors.

![Small-signal model](image)

Summing currents at the output node gives,

$$g_m v_{gs1} = g_m v_{gs3} + G_L v_{out}$$

Also, $v_{gs3} = -g_m v_{gs1} (1/g_m)$

$$g_m v_{gs1} = g_m \left( \frac{g_m}{g_m} \right) v_{gs3} + G_L v_{out}$$

$$g_m v_{gs1} \left( 1 + \frac{g_m}{g_m} \right) = G_L v_{out} \rightarrow$$

$$v_{out} v_{in} = g_m \left( 1 + \frac{g_m}{g_m} \right) + G_L$$

Setting $v_{in} = 0$ and applying $i_t$ and solving for $v_{out}$ and ignoring $R_L$ gives,

$$i_t = g_m v_{gs3} + g_m v_{out} = g_m \left( \frac{g_m}{g_m} \right) v_{out} + g_m v_{out}$$

$$v_{out} \frac{i_t}{i_t} = g_m \left( 1 + \frac{g_m}{g_m} \right)$$

$$R_{out} = \frac{1}{g_m \left( 1 + \frac{g_m}{g_m} \right)}$$

Note that the 1mA splits between M1(M2) and M3 in a ratio of 1 to 100. Therefore, $I_{D1} = I_{D2} = 9.9\mu A$ and $I_{D3} = 990.1\mu A$.

$$g_m = \sqrt{2 \cdot 110 \cdot 100 \cdot 9.9} = 466.71\mu S, g_m = \sqrt{2 \cdot 50 \cdot 1.99} = 31.47\mu S$$

and $g_m = \sqrt{2 \cdot 110 \cdot 100 \cdot 990.1} = 3146.7\mu S$

$$\frac{v_{out}}{v_{in}} = \frac{466.71 \cdot 101}{466.71 \cdot 101 + 1/50} = 47.137$$

$$R_{out} = \frac{1000}{47.137} = 21.2\Omega$$
A three stage amplifier connected in unity gain configuration is used as the output buffer. The first two stages are class-A whereas the last stage is class-AB.

The inF load capacitance makes the pole present at node nout the dominant pole. Therefore, there is no need to apply additional compensation to the three stage amplifier.

DC gain of the amplifier in unity gain configuration = \( A_v/(1+A_v) \)

where \( A_v \propto g_m \beta r_{ds} \)

Output impedance of the unity gain buffer = \( r_{ds}(1+A_v) \).

Mr1 and Mr2 act as a linear resistor, allowing a voltage drop between n5a and n5b. The net result is reduced quiescent current at the output stage.
*output buffer
.option brief
.options
 + post
 + ingold=2
 + scale=1e-6
 + accurate
 + delmax=0.5n
 + method=gear lvltim=2
 + probe
*

vdd dd 0 dc 2v
vss ss 0 dc -2v
*vin in 0 dc 0v
vin in 0 sin (0 1.5 100k 0)
*vin in 0 pulse (-1 1 2u 2n 2n 1u 2u)

mb1 n1 n1 dd dd cmosp l=2u w=200u
mb2 n2 n1 dd dd cmosp l=2u w=200u
mb3 n6 n6 ss ss cmosn l=5u w=4u
mb4 n1 n1 n6 ss cmosn l=5u w=4u
mi1 n3 in n2 n2 cmosp l=2u w=200u
mi2 n4 nout n2 n2 cmosp l=2u w=200u
ml1 n3 n3 ss ss cmosn l=2u w=200u
ml2 n4 n3 ss ss cmosn l=2u w=200u
m21 n5a n4 ss ss cmosn l=2u w=200u
m22 n5b n1 dd dd cmosp l=2u w=200u
mr1 n5a ss n5a dd cmosp l=20u w=8u
mr2 n5b dd n5a ss cmosn l=20u w=4u
mout1 nout n5a ss ss cmosn l=2u w=100u
mout2 nout n5b dd dd cmosp l=2u w=200u
rl nout 0 100
cl nout ss 1nF
*.dc vin -2 2 1m
.meas tran current_vdd avg i(vdd) from=0 to=50u
.meas tran power_vdd param='4*current_vdd'
.tran 1u 50u
.model cmosn nmos kp=110u vto=0.7 lambda=0.01 gamma=0.4 phi=0.7
.model cmosp pmos kp=50u vto=0.7 lambda=0.01 gamma=0.7 phi=0.8
.probe v(in) v(n1) v(n2) v(n3) v(n4) v(n5) v(nout) v(n5a) v(n5b) i(vdd) i(vss)
.op
.end
***** operating point information

***** operating point status is all

node = voltage node = voltage node = voltage
+0:dd = 2.000e+00 0:in = 0. 0:n1 = 1.197e+00
+0:n2 = 7.723e-01 0:n3 = -1.250e+00 0:n4 = -1.230e+00
+0:n5a = -3.641e-01 0:n5b = 3.185e-01 0:n6 = -5.283e-01
+0:nout = -1.441e-05 0:ss = -2.000e+00

***** voltage sources

subckt

.element 0:vdd 0:vss 0:vin
.volt 2.000e+00 -2.000e+00 0.
.current -2.537e-03 2.537e-03 0.
.power 5.073e-03 5.074e-03 0.

.total voltage source power dissipation = 1.015e-02 watts

***** resistors

subckt

.element 0:r1
.r value 1.000e+02
.v drop -1.441e-05
.current -1.441e-07
.power 2.075e-12

***** mosfets

subckt

.element 0:mb1 0:mb2 0:mb3 0:mb4 0:mi1 0:mi2
.model 0:cmosp 0:cmosp 0:cmosn 0:cmosn 0:cmosp 0:cmosp
.id -2.659e-05 -2.670e-05 2.659e-05 2.659e-05 -1.335e-05 -1.335e-05
.ibs 0. 0. 0. -1.472e-14 0. 0.
.ibd 8.027e-15 1.228e-14 -1.472e-14 -3.197e-14 2.023e-14 2.003e-14
.vgs -8.027e-01 -8.027e-01 1.471e+00 1.725e+00 -7.723e-01 -7.724e-01
.vds -8.027e-01 -1.227e+00 1.471e+00 1.725e+00 -2.023e+00 -2.003e+00
.vbs 0. 0. 0. -1.471e+00 0. 0.
.vth -7.000e-01 -7.000e-01 7.000e-01 9.548e-01 -7.000e-01 -7.000e-01
.vdsat -1.027e-01 -1.027e-01 7.717e-01 7.708e-01 -7.235e-02 -7.236e-02
.beta 5.040e-03 5.061e-03 8.930e-05 8.952e-05 5.101e-03 5.100e-03
.gam eff 7.000e-01 7.000e-01 4.000e-01 4.000e-01 7.000e-01 7.000e-01
.gds 2.638e-07 2.638e-07 2.620e-07 2.614e-07 1.308e-07 1.309e-07
.cdtot 1.478e-28 2.261e-28 0. 0. 3.726e-28 3.689e-28
.cbdtot 6.331e-27 6.331e-27 0. 0. 8.414e-27 8.412e-27
.cgd 1.478e-28 2.261e-28 0. 0. 3.726e-28 3.689e-28

subckt

.element 0:ml1 0:ml2 0:m21 0:m22 0:mr1 0:mr2
.model 0:cmosn 0:cmosn 0:cmosn 0:cmosn 0:cmosn 0:cmosn
.id 1.335e-05 1.335e-05 2.682e-05 -2.682e-05 1.100e-05 1.582e-05
.ibs 0. 0. 0. 0. 0. 0. 2.364e-14 -1.636e-14
vgs  7.491e-01  7.491e-01  7.693e-01  -8.027e-01  -1.635e+00  2.364e+00
vds  7.491e-01  7.693e-01  1.635e+00  -1.681e+00  6.826e-01  6.826e-01
vbs  0.          0.          0.          0.          2.364e+00  -1.635e+00
vth  7.000e-01  7.000e-01  7.000e-01  -7.000e-01  -1.176e+00  9.767e-01
vdsat 4.908e-02  4.908e-02  6.927e-02  -1.027e-01  -6.826e-01  6.826e-01
beta  1.108e-02  1.108e-02  1.18e-02   5.084e-03   2.014e-05   2.215e-05
gam_eff 4.000e-01  4.000e-01  4.000e-01  7.000e-01  7.000e-01  4.000e-01
gm   5.440e-04  5.441e-04  7.744e-04  5.222e-04  1.375e-05  1.512e-05
gds  1.325e-07  1.325e-07  2.639e-07  2.638e-07  9.357e-06  1.577e-05
gmb  1.300e-04  1.301e-04  1.851e-04  2.044e-04  3.054e-06  1.979e-06

Opening plot unit= 15
file=/buffer.tr0

*****
*current source
****** transient analysis
******
current_vdd  =  -4.5829E-03  from=  .0000E+00  to=  5.0000E-05
power_vdd   =  -1.8332E-02  average power dissipation

***** job concluded
****** Star-HSPICE --  97.2.1 (970915) 14:15:30  98/06/01  pa
******
*current source
****** job statistics summary
******
total memory used  159 kbytes
# nodes = 12  # elements= 19
# diodes = 0  # bjts = 0  # jfets = 0  # mosfets = 14

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**total cpu time**: 216.72 seconds

job started at 14:15:30  98/06/01
job ended  at 14:19:15  98/06/01

lic: Release token(s)
HSPICE job buffer.sp completed.
Mon Jun 1 14:19:15 PDT 1998