Homework Assignment No. 12

Problem 1 - (10 points)
Find the GB of a two-stage op amp using Miller compensation using a nulling resistor that has 60° phase margin where the second pole is \(-10 \times 10^6\) rads/sec and two higher poles both at \(-100 \times 10^6\) rads/sec. Assume that the RHP zero is used to cancel the second pole and that the load capacitance stays constant. If the input transconductance is 500\(\mu\)A/V, what is the value of \(C_c\)?

Solution
The resulting higher-order poles are two at \(-100 \times 10^6\) radians/sec. The resulting phase margin expression is,
\[
PM = 180° - \tan^{-1}(A_v(0)) - 2\tan^{-1}\left(\frac{GB}{10^7}\right) = 90° - 2\tan^{-1}\left(\frac{GB}{10^7}\right) = 60°
\]
\[
\Rightarrow 30° = 2\tan^{-1}\left(\frac{GB}{10^7}\right) \rightarrow \left(\frac{GB}{10^7}\right) = \tan(15°) = 0.2679
\]
\[
GB = 2.679 \times 10^7 = \frac{g_{m1}}{C_c} \rightarrow C_c = \frac{500 \times 10^{-6}}{26.79 \times 10^7} = 18.66\text{pF}
\]

Problem 2 – P7.2-4
Use the technique of Ex. 7.2-2 to extend the GB of the cascode op amp of Ex. 6.5-2 as much as possible that will maintain 60° phase margin. What is the minimum value of \(C_L\) for the maximum GB?

Solution
Assuming all channel lengths to be 1 \(\mu\)m, the total capacitance at the source of M7 is
\[
C_7 = C_{gs7} + C_{bd7} + C_{gd6} + C_{bd6}
\]
or,
\[
C_7 = 75 + 51 + 9 + 51 = 186\text{ fF}
\]
g_{m7} = 707 \(\mu\)S
Thus, the pole at the source of M7 is
\[
p_{S7} = -\frac{g_{m7}}{C_7} = -605\text{ MHz}.
\]
The total capacitance at the source of M12 is
\[
C_{12} = C_{gs12} + C_{bd12} + C_{gd11} + C_{bd11}
\]
or,
\[
C_{12} = 34 + 29 + 4 + 29 = 96\text{ fF}
\]
g_{m12} = 707 \(\mu\)S
Thus, the pole at the source of M12 is
\[
p_{S12} = -\frac{g_{m12}}{C_{12}} = -1170\text{ MHz}.
\]
The total capacitance at the drain of M4 is
\[
C_4 = C_{gs4} + C_{gs6} + C_{bd4} + C_{gd2} + C_{bd2}
\]
or,
\[
C_4 = 43 + 75 + 21 + 3 + 19 = 161\text{ fF}
\]
g_{m4} = 283 \(\mu\)S
Problem 2 - Continued

Thus, the pole at the drain of M4 is
\[ p_{D4} = \frac{g_{m4}}{C_4} = -280 \text{ MHz}. \]

The total capacitance at the drain of M8 is
\[ C_8 = C_{gd8} + C_{bd8} + C_{gs10} + C_{gs12} \]
or,
\[ C_8 = 9 + 51 + 34 + 34 = 128 \text{ fF} \]
\[ R_2 + \frac{1}{g_{m10}} = 3.4 \text{ KΩ} \]

Thus, the pole at the drain of M8 is
\[ p_{D8} = -\left(\frac{1}{R_2 + \frac{1}{g_{m10}}C_8}\right) = -366 \text{ MHz}. \]

For a phase margin of 60°, we have
\[ PM = 180° - \left[90° - \left\{\tan^{-1}\left(\frac{GB}{p_{s17}}\right) + \tan^{-1}\left(\frac{GB}{p_{s12}}\right) + \tan^{-1}\left(\frac{GB}{p_{D4}}\right) + \tan^{-1}\left(\frac{GB}{p_{D8}}\right)\right\}\right] \]

Solving the above equation
\[ GB \approx 65 \text{ MHz}. \]

And, \( A_v = 6925 \text{ V/V} \)

Thus, \( p_1 = 9.39 \text{ KHz}, \) and \( C_L \geq 1.54 \text{ pF} \)

Problem 3 - Problem 7.3-1

Compare the differential output op amps of Fig. 7.3-3, 7.3-5, 7.3-6, 7.3-7, 7.3-8 and 7.3-10 from the viewpoint of (a.) noise, (b.) PSRR, (c.) ICMR \([V_{ic}(max)\) and \(V_{ic}(min)\)], (d.) OCMR \([V_o)(max)\) and \(V_o)(min)\)], (e.) SR assuming all input differential currents are identical, and (f.) power dissipation if all current of the input differential amplifiers are identical and power supplies are equal.

Solution

<table>
<thead>
<tr>
<th>Noise</th>
<th>Fig. 7.3-3</th>
<th>Fig. 7.3-5</th>
<th>Fig. 7.3-6</th>
<th>Fig. 7.3-7</th>
<th>Fig. 7.3-8</th>
<th>Fig. 7.3-10</th>
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</thead>
<tbody>
<tr>
<td>Good</td>
<td>Poor</td>
<td>Good</td>
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<td>Okay</td>
<td>Poor</td>
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<td>PSRR</td>
<td>Poor</td>
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<td>( I_{ic}(min) )</td>
<td>( V_{SS}V_{ON} )</td>
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<td>( OCMR )</td>
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<td>( V_{o}(max) )</td>
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<tr>
<td>( V_{o}(min) )</td>
<td>( I_{SS}/C_c )</td>
<td>( I_{SS}/C_L )</td>
<td>( I_{SS}/C_c )</td>
<td>( I_{SS}/C_L )</td>
<td>( I_{SS}/C_L )</td>
<td>( I_{SS}/C_L )</td>
</tr>
</tbody>
</table>

| SR | \( I_{SS}/C_c \) | \( I_{SS}/C_L \) | \( I_{SS}/C_c \) | \( I_{SS}/C_L \) | \( I_{SS}/C_L \) | \( I_{SS}/C_L \) |
Problem 4 - Problem 7.3-7

(a.) If all transistors in Fig. 7.3-12 have a dc current of 50 µA and a W/L of 10 µm/1 µm, find the gain of the common mode feedback loop. (b.) If the output of this amplifier is cascoded, then repeat part (a.).

Solution

![Figure 7.3-12 Two-stage, Miller, differential-in, differential-out op amp with common-mode stabilization.](image)

The loop gain of the common-mode feedback loop is,

\[
\text{CMFB Loop gain} = -\frac{g_{m10}}{g_{ds9}} = -g_{m10}r_{ds9} \quad \text{or} \quad -\frac{g_{m11}}{g_{ds8}} = -g_{m11}r_{ds8}
\]

With \( I_D = 50 \mu A \) and \( W/L = 10 \mu m/1 \mu m, \) \( g_{m10} = \sqrt{\frac{2k'w_I_D}{L}} = \sqrt{2\cdot50\cdot10\cdot50} = 223.6 \mu S, \)
\( r_{dsN} = \frac{1}{\lambda N I_D} = \frac{25}{50 \mu A} = 0.5 \Omega \) and \( r_{dsP} = \frac{1}{\lambda p I_D} = \frac{20}{50 \mu A} = 0.4 \Omega \)

\[ \therefore \quad \text{CMFB Loop gain} = -g_{m10}r_{ds9} = -223.6(0.5) = -111.8 \text{V/V} \]

If the output is cascoded, the gain becomes,

\[
\text{CMFB Loop gain with cascoding} = -\frac{g_{m10}}{g_{ds9}} g_m(\text{cascode})r_{ds}(\text{cascode})
\]

\[ = -g_{m10}[(r_{ds9} g_m(\text{cascode})r_{ds}(\text{cascode}))||[g_{m7}r_{ds7} (r_{ds10})||r_{ds10}]]
\]

\[ g_{mP} = \sqrt{\frac{2k'w_I_D}{L}} = \sqrt{2\cdot110\cdot10\cdot50} = 331.67 \mu S
\]

\[ = -(223.6)[(0.5\cdot331.67\cdot0.5)][(223.6)(0.4)(0.2)] = 223.6(14.7) = -3,290 \text{ V/V}
\]

\[ \therefore \quad \text{CMFB Loop gain with cascoding} = -3.290 \text{V/V} \]
Problem 5 – Problem 7.4-1
Calculate the gain, GB, SR and \( P_{\text{diss}} \) for the folded cascode op amp of Fig. 6.5-7b if \( V_{DD} = -V_{SS} = 1.5 \text{V} \), the current in the differential amplifier pair is 50nA each and the current in the sources, M4 and M5, is 150nA. Assume the transistors are all 10\( \mu \text{m}/1\mu \text{m} \), the load capacitor is 2pF and that \( n_1 \) is 2.5 for NMOS and 1.5 for PMOS.

![Figure 6.5-7](image)

(a) Simplified version of an N-channel input, folded cascode op amp.  
(b) Practical version (a).

Solution

\[
g_m = \frac{I_D}{n_1(kT/q)} = \frac{50\text{nA}}{2.5 \times 25.9\text{mV}} = 0.772\mu\text{S}
\]

and \( r_{ds1} = r_{ds2} = \frac{1}{I_D\lambda_N} = 500\text{M}\Omega \)

\[
g_m = \frac{I_D}{n_1(kT/q)} = \frac{150\text{nA}}{1.5 \times 25.9\text{mV}} = 3.861\mu\text{S}
\]

and \( r_{ds4} = r_{ds5} = \frac{1}{I_D\lambda_N} = 133\text{M}\Omega \)

\[
g_m = \frac{I_D}{n_1(kT/q)} = \frac{100\text{nA}}{1.5 \times 25.9\text{mV}} = 2.574\mu\text{S}
\]

and \( r_{ds6} = r_{ds7} = \frac{1}{I_D\lambda_N} = 200\text{M}\Omega \)

\[
g_m = g_m = g_m = \frac{I_D}{n_1(kT/q)} = \frac{100\text{nA}}{2.5 \times 25.9\text{mV}} = 1.544\mu\text{S}
\]

and \( r_{ds8} = r_{ds9} = r_{ds10} = r_{ds11} = \frac{1}{I_D\lambda_N} = 250\text{M}\Omega \)

Gain: \( A_v(0) = g_m R_{out} \)

\[
R_{out} = r_{ds11} g_m r_{ds9} r_{ds10} r_{ds11} = 96.5\text{G}\Omega \| 34.23\text{G}\Omega = 25.269\text{G}\Omega
\]

\[
\therefore A_v(0) = 0.772\mu\text{S} \times 25.269\text{G}\Omega = 19.508 \text{V/V}
\]

\[
GB = g_m/C_L = 386\text{krads/sec} = 61.43\text{kHz}
\]

(this assumes all other poles are greater than \( GB \) which is the case if \( C_L \) makes \( R_B \) approximately the same as \( R_A \) at \( \omega = GB \).)

\[
SR = 100\text{nA} / 2\text{pF} = 0.05\text{V/\mu s}
\]

\[
P_{\text{diss}} = 3V\cdot(3\cdot150\text{nA}) = 1.35\mu\text{W}
\]