LECTURE 010 – ECE 4430 REVIEW I

(READING: GHLM - Chap. 1)

Objective
The objective of this presentation is:
1.) Identify the prerequisite material as taught in ECE 4430
2.) Insure that the students of ECE 6412 are adequately prepared

Outline
• Models for Integrated-Circuit Active Devices
• Bipolar, MOS, and BiCMOS IC Technology
• Single-Transistor and Multiple-Transistor Amplifiers
• Transistor Current Sources and Active Loads

MODELS FOR INTEGRATED-CIRCUIT ACTIVE DEVICES

PN Junctions - Step Junction
Barrier potential-
\[ \psi_0 = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) = V_t \ln \left( \frac{N_A N_D}{n_i^2} \right) = U_T \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Depletion region widths-
\[
\begin{align*}
W_1 &= \sqrt{\frac{2\varepsilon_s (\psi_0 - V_D) N_D}{q N_A (N_A + N_D)}} \\
W_2 &= \sqrt{\frac{2\varepsilon_s (\psi_0 - V_D) N_A}{q N_D (N_A + N_D)}}
\end{align*}
\]
\[ W \propto \sqrt{\frac{1}{N}} \]

Depletion capacitance-
\[
C_j = A \sqrt{\frac{\varepsilon_s q N_A N_D}{2 (N_A + N_D)}} \frac{1}{\sqrt{\psi_0 - V_D}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\psi_0}}}
\]

Fig. 010-01
**PN-Junctions - Graded Junction**

Graded junction:

![Graded Junction Diagram](image)

Above expressions become:

Depletion region widths-

\[
W_1 = \frac{2\varepsilon_s (\psi_0 - V_D) N_D}{qN_D(N_A + N_D)}
\]

\[
W_2 = \frac{2\varepsilon_s (\psi_0 - V_D) N_A}{qN_D(N_A + N_D)}
\]

Depletion capacitance-

\[
C_j = A \left( \frac{\varepsilon_s q N_A N_D}{2(N_A + N_D)} \right)^m \frac{1}{(\psi_0 - V_D)^m} = \frac{C_{j0}}{\left( 1 - \frac{V_D}{\psi_0} \right)^m}
\]

where \(0.33 \leq m \leq 0.5\).

**Large Signal Model for the BJT in the Forward Active Region**

Large-signal model for an *npn* transistor:

\[
i_B = \frac{I_s}{\beta_F} \exp \left( \frac{V_{BE}}{V_t} \right)
\]

Assumes \(V_{BE}\) is a constant and \(i_B\) is determined externally

Large-signal model for an *pnp* transistor:

\[
i_B = -\frac{I_s}{\beta_F} \exp \left( -\frac{V_{BE}}{V_t} \right)
\]

Assumes \(V_{BE}\) is a constant and \(i_B\) is determined externally

Early Voltage:

Modified large signal model becomes

\[
i_C = I_s \left( 1 + \frac{V_{CE}}{V_A} \right) \exp \left( \frac{V_{BE}}{V_t} \right)
\]
**The Ebers-Moll Equations**

The reciprocity condition allows us to write,

\[ \alpha_{FE} I_E = \alpha_{R} I_C = I_S \]

Substituting into a previous form of the Ebers-Moll equations gives,

\[ i_C = I_S \left( \frac{v_{BE}}{V_t} + 1 \right) \frac{I_S}{\alpha_R} \left( \frac{v_{BC}}{V_t} + 1 \right) \]

and

\[ i_E = -\frac{I_S}{\alpha_F} \left( \frac{v_{BE}}{V_t} + 1 \right) + I_S \left( \frac{v_{BC}}{V_t} + 1 \right) \]

These equations are valid for all four regions of operation of the BJT.

Also:

- Dependence of \( \beta_F \) as a function of collector current
- The temperature coefficient of \( \beta_F \) is,

\[ TCF = \frac{1}{\beta_F} \frac{\partial \beta_F}{\partial T} = +7000 \text{ppm/}^\circ\text{C} \]

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**Simple Small Signal BJT Model**

Implementing the above relationships, \( i_C = g_m v_i + g_o v_{ce} \), and \( v_i = r\pi i_b \), into a schematic model gives,

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**Example:**

Find the small signal input resistance, \( R_{in} \), the output resistance, \( R_{out} \), and the voltage gain of the common emitter BJT if the BJT is unloaded \((R_L = \infty)\), \( v_{out}/v_{in} \), the dc collector current is 1mA, the Early voltage is 100V, and \( \beta_o = 100 \) at room temperature.

\[ g_m = \frac{I_C}{V_t} = \frac{1 \text{mA}}{26 \text{mV}} = \frac{1}{26} \text{mhos or Siemens} \]

\[ R_{in} = r\pi = \frac{\beta_o}{g_m} = 100 \cdot 26 = 2.6 \text{k}\Omega \]

\[ R_{out} = r_o = \frac{V_A}{I_C} = \frac{100 \text{V}}{1 \text{mA}} = 100 \text{k}\Omega \]

\[ \frac{v_{out}}{v_{in}} = -g_m r_o = -26 \text{mS} \cdot 100 \text{k}\Omega = -2600 \text{V/V} \]
Complete Small Signal BJT Model

The capacitance, $C_\pi$, consists of the sum of $C_{je}$ and $C_b$.

$$C_\pi = C_{je} + C_b$$

Example 1

Derive the complete small signal equivalent circuit for a BJT at $I_C = 1\text{mA}$, $V_{CB} = 3\text{V}$, and $V_{CS} = 5\text{V}$. The device parameters are $C_{je0} = 10\text{fF}$, $n_e = 0.5$, $\psi_{0e} = 0.9\text{V}$, $C_{\mu 0} = 10\text{fF}$, $n_c = 0.3$, $\psi_{0c} = 0.5\text{V}$, $C_{cs0} = 20\text{fF}$, $n_s = 0.3$, $\psi_{0s} = 0.65\text{V}$, $\beta_0 = 100$, $\tau_F = 10\text{ps}$, $V_A = 20\text{V}$, $r_b = 300\Omega$, $r_c = 50\Omega$, $r_{ex} = 5\Omega$, and $r_\mu = 10\beta_0 r_o$.

Solution

Because $C_{je}$ is difficult to determine and usually an insignificant part of $C_\pi$, let us approximate it as $2C_{je0}$.

$$C_\mu = \frac{C_{\mu 0}}{1 + \frac{V_{CB}}{\psi_{0c}}} = \frac{10\text{fF}}{1 + \frac{3}{0.5}0.3} = 5.6\text{fF} \quad \text{and} \quad C_{cs} = \frac{C_{cs0}}{1 + \frac{V_{CS}}{\psi_{0s}}} = \frac{20\text{fF}}{1 + \frac{5}{0.65}0.3} = 10.5\text{fF}$$

$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 38\text{mA/V} \quad C_b = \tau_F g_m = (10\text{ps})(38\text{mA/V}) = 0.38\text{pF}$$

$$r_\pi = \frac{\beta_0}{g_m} = 100 \cdot 26\Omega = 2.6\text{k}\Omega, \quad r_o = \frac{V_A}{I_C} = \frac{20\text{V}}{1\text{mA}} = 20\text{k}\Omega \quad \text{and} \quad r_\mu = 10\beta_0 r_o = 20\text{M}\Omega$$
**Transition Frequency, $f_T$**

$f_T$ is the frequency where the magnitude of the short-circuit, common-emitter current = 1.

Circuit and model:

Assume that $r_c \approx 0$. As a result, $r_o$ and $C_{cs}$ have no effect.

$$V_1 \approx \frac{r_\pi}{1 + r_\pi (C_\pi + C_\mu)} I_i \quad \text{and} \quad I_o \approx g_m V_1 \quad \Rightarrow \quad \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{g_m r_\pi}{(C_\pi + C_\mu)s} = \frac{\beta_o}{(C_\pi + C_\mu)s}$$

Now,

$$\beta(j\omega) = \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{\beta_o}{1 + \beta_o} \frac{(C_\pi + C_\mu)j\omega}{g_m}$$

At high frequencies,

$$\beta(j\omega) \approx \frac{g_m}{j\omega (C_\pi + C_\mu)} \quad \Rightarrow \quad \text{When } |\beta(j\omega)| = 1 \text{ then } \omega_T = \frac{g_m}{C_\pi + C_\mu} \quad \text{or} \quad f_T = \frac{1}{2\pi} \frac{g_m}{C_\pi + C_\mu}$$

**JFET Large Signal Model**

Large signal model:

Incorporating the channel modulation effect:

$$i_D = I_{DSS} \left(1 - \frac{v_{GS}}{V_p}\right)^2 (1 + \lambda v_{DS}) \quad , \quad v_{DS} \geq v_{GS} - V_p$$

Signs for the JFET variables:

<table>
<thead>
<tr>
<th>Type of JFET</th>
<th>$V_p$</th>
<th>$I_{DSS}$</th>
<th>$v_{GS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$-channel</td>
<td>Positive</td>
<td>Negative</td>
<td>Normally positive</td>
</tr>
<tr>
<td>$n$-channel</td>
<td>Negative</td>
<td>Positive</td>
<td>Normally negative</td>
</tr>
</tbody>
</table>
**Frequency Independent JFET Small Signal Model**

Schematic:

```
 vgs  g_m \cdot v_{gs}  r_o  v_{ds}
```

Parameters:

\[
g_m = \frac{dI_D}{dv_{GS}} \frac{1}{Q} = - \frac{2I_{DSS}}{V_p} \left( \frac{V_{GS}}{V_p} \right) = g_m(1 - \frac{V_{GS}}{V_p})
\]

where

\[
g_{m0} = \frac{2I_{DSS}}{V_p}
\]

\[
r_o = \frac{dI_D}{dv_{DS}} \frac{1}{Q} = \lambda I_{DSS} \left( 1 - \frac{V_{GS}}{V_p} \right)^2 = \frac{1}{\lambda I_D}
\]

Typical values of \(I_{DSS}\) and \(V_p\) for a \(p\)-channel JFET are -1mA and 2V, respectively. With \(\lambda = 0.02\text{V}^{-1}\) and \(I_D = 1\text{mA}\) we get \(g_m = 1\text{mA/V}\) or 1mS and \(r_o = 50\text{k}\Omega\).

**Frequency Dependent JFET Small Signal Model**

Complete small signal model:

```
 C_{gs}  C_{gd}  C_{gss}
```

All capacitors are reverse biased depletion capacitors given as,

\[
C_{gs} = \frac{C_{gs0}}{1 + \frac{V_{GS}}{\psi^o}}^{1/3} \quad \text{(capacitance from source to top and bottom gates)}
\]

\[
C_{gd} = \frac{C_{gd0}}{1 + \frac{V_{GD}}{\psi^o}}^{1/3} \quad \text{(capacitance from drain to top and bottom gates)}
\]

\[
C_{gss} = \frac{C_{gss0}}{1 + \frac{V_{GSS}}{\psi^o}}^{1/2} \quad \text{(capacitance from the gate (p-base) to substrate)}
\]

\[
\therefore f_T = \frac{1}{2\pi C_{gs} + C_{gd} + C_{gss}} = 30\text{MHz} \quad \text{if} \quad g_m = 1\text{mA/V} \text{ and } C_{gs} + C_{gd} + C_{gss} = 5\text{pF}
\]
Simple Large Signal MOSFET Model

N-channel reference convention:

Non-saturation:
\[ i_D = \frac{W \mu_C}{L} \left( (v_{GS} - V_T) v_{DS} - \frac{v_{DS}^2}{2} \right) \left( 1 + \lambda v_{DS} \right), \quad 0 < v_{DS} < v_{GS} - V_T \]

Saturation:
\[ i_D = \frac{W \mu_C}{2L} \left( (v_{GS} - V_T)^2 \left( 1 + \lambda v_{DS} \right) \right), \quad 0 < v_{GS} - V_T < v_{DS} \]

where:
- \( \mu_C \) = zero field mobility (cm²/volt·sec)
- \( C_o \) = gate oxide capacitance per unit area (F/cm²)
- \( \lambda \) = channel-length modulation parameter (volts⁻¹)
- \( V_T = V_{T0} + \gamma \sqrt{|2|\phi_f| + |V_{BS}| - \sqrt{2|\phi_f|}} \)
  - \( V_{T0} \) = zero bias threshold voltage
  - \( \gamma \) = bulk threshold parameter (volts⁻0.5)
  - \( 2|\phi_f| \) = strong inversion surface potential (volts)

For p-channel MOSFETs, use n-channel equations with p-channel parameters and invert current.

MOSFET Small-Signal Model

Complete schematic model:

where
\[ g_m = \frac{\partial i_D}{\partial v_{GS}} = \beta (V_{GS} - V_T) = \sqrt{2B I_D} \]
\[ g_{ds} = \frac{\partial i_D}{\partial v_{DS}} = \frac{\lambda i_D}{1 + \lambda v_{DS}} \approx \lambda i_D \]

and
\[ g_{mbs} = \frac{\partial i_D}{\partial V_{BS}} = \left( \frac{\partial i_D}{\partial v_{GS}} \right) \left( \frac{\partial v_{GS}}{\partial V_{BS}} \right) = \left( -\frac{\partial i_D}{\partial v_{T}} \right) \left( \frac{\partial v_{T}}{\partial V_{BS}} \right) = \frac{g_m}{2\sqrt{2|\phi_f| - V_{BS}}} = \eta g_m \]

Simplified schematic model:

Extremely important assumption:
\[ g_m \approx 10 g_{mbs} \approx 100 g_{ds} \]
MOSFET Depletion Capacitors - \( C_{BS} \) and \( C_{BD} \)

Model:

\[
C_{BS} = \frac{CJ \cdot AS}{1 - \frac{v_{BS}}{PB}} + \frac{CJSW \cdot PS}{1 - \frac{v_{BS}}{PB}}, \quad v_{BS} \leq FC \cdot PB
\]

and

\[
C_{BS} = \frac{CJ \cdot AS}{1 - (1+M)FC} \left(1 - (1+M)FC + MJ \frac{v_{BS}}{PB}\right)
\]

\[
+ \frac{CJSW \cdot PS}{1 - FC} \left(1 - (1+MJSW)FC + MJSW \frac{v_{BS}}{PB}\right), \quad v_{BS} > FC \cdot PB
\]

where

- \( AS \) = area of the source
- \( PS \) = perimeter of the source
- \( CJSW \) = zero bias, bulk source sidewall capacitance
- \( MJ \) = bulk-source sidewall grading coefficient

For the bulk-drain depletion capacitance replace "\( S \)" by "\( D \)" in the above equations.

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MOSFET Intrinsic Capacitors - \( C_{GD}, C_{GS} \) and \( C_{GB} \)

Cutoff Region:

\[
C_{GB} = C_2 + 2C_5 = C_{ox}(W_{eff})(L_{eff}) + 2CGBO(L_{eff})
\]

\[
C_{GS} = C_1 \approx C_{ox}(LD)W_{eff} = CGSO(W_{eff})
\]

\[
C_{GD} = C_3 \approx C_{ox}(LD)W_{eff} = CGDO(W_{eff})
\]

Saturation Region:

\[
C_{GB} = 2C_5 = CGBO(L_{eff})
\]

\[
C_{GS} = C_1 + (2/3)C_2 = C_{ox}(LD+0.67L_{eff})(W_{eff}) = CGSO(W_{eff}) + 0.67C_{ox}(W_{eff})(L_{eff})
\]

\[
C_{GD} = C_3 \approx C_{ox}(LD)W_{eff} = CGDO(W_{eff})
\]

Active Region:

\[
C_{GB} = 2C_5 = 2CGBO(L_{eff})
\]

\[
C_{GS} = C_1 + 0.5C_2 = C_{ox}(LD+0.5L_{eff})(W_{eff}) = (CGSO + 0.5C_{ox}L_{eff})W_{eff}
\]

\[
C_{GD} = C_3 + 0.5C_2 = C_{ox}(LD+0.5L_{eff})(W_{eff}) = (CGDO + 0.5C_{ox}L_{eff})W_{eff}
\]
**Small-Signal Frequency Dependent Model**

The depletion capacitors are found by evaluating the large signal capacitors at the DC operating point.

The charge storage capacitors are constant for a specific region of operation.

Gainbandwidth of the MOSFET:

Assume $V_{SB} = 0$ and the MOSFET is in saturation,

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{1}{2\pi} \frac{g_m}{C_{gs}}$$

Recalling that

$$C_{gs} \approx \frac{2}{3} C_{ox}WL \quad \text{and} \quad g_m = \mu_o C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

gives

$$f_T = \frac{3}{4\pi L^2} (V_{GS} - V_T)$$

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**Subthreshold MOSFET Model**

Weak inversion operation occurs when the applied gate voltage is below $V_T$ and pertains to when the surface of the substrate beneath the gate is weakly inverted.

Regions of operation according to the surface potential, $\phi_S$.

- $\phi_S < \phi_F$: Substrate not inverted
- $\phi_F < \phi_S < 2\phi_F$: Channel is weakly inverted (diffusion current)
- $2\phi_F < \phi_S$: Strong inversion (drift current)

Drift current versus diffusion current in a MOSFET:
**Large-Signal Model for Subthreshold**

Model:

\[ i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t}(1 - e^{-v_{DS}/V_t})(1 + \lambda v_{DS}) \]

where

- \( K_x \) is dependent on process parameters and the bulk-source voltage
- \( n \approx 1.5 - 3 \)

and

\[ V_t = \frac{kT}{q} \]

If \( v_{DS} > 0 \), then

\[ i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t} (1 + \lambda v_{DS}) \]

Small-signal model:

- \( g_m = \frac{\partial i_D}{\partial v_{GS}} \bigg|_Q = \frac{qI_D}{nkT} \)
- \( g_{ds} = \frac{\partial i_D}{\partial v_{DS}} \bigg|_Q \approx \frac{I_D}{V_A} \)

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**SUMMARY**

- **Models**
  - Large-signal
  - Small-signal
- **Components**
  - pn Junction
  - BJT
  - MOSFET
    - Strong inversion
    - Weak inversion
  - JFET
- **Capacitors**
  - Depletion
  - Parallel plate