LECTURE 080 – SINGLE-STAGE FREQUENCY RESPONSE - II
(READING: GHLM – 504-516)

Objective

The objective of this presentation is:

1.) Illustrate the frequency analysis of single stage amplifiers

2.) Introduce the Miller technique and the approximate method of solving for two poles

Outline

- Differential and Common Frequency Response of the Differential Amplifier
- Emitter/Source Follower Frequency Response
- Common Base/Gate Frequency Response
- Summary

Emitter Follower Input Impedance

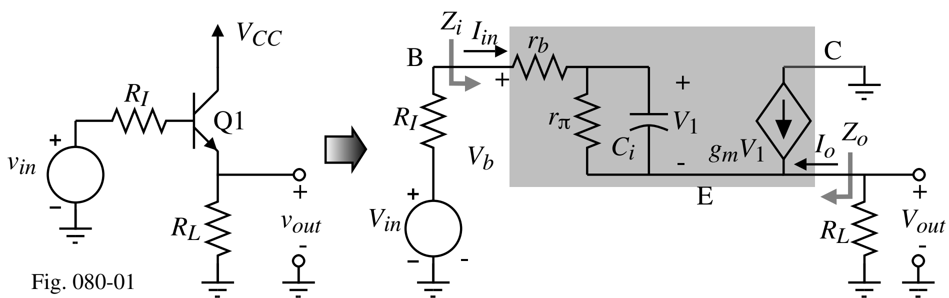

Fig. 080-01

If we let $z_\pi = \dfrac{r_\pi}{1+sC_ir_\pi}$, then

$$V_i = V_b = I_{in}(r_b + z_\pi) + (I_{in} + g_m z_\pi I_{in})R_L \rightarrow \quad Z_i = \frac{V_i}{I_{in}} = r_b + z_\pi + (1 + g_m z_\pi)R_L$$

$$\therefore \quad Z_i = r_b + \frac{r_\pi}{1+sC_ir_\pi} + \left(1+\frac{g_mr_\pi}{1+sC_ir_\pi}\right)R_L = r_b + \left(\frac{(1+g_mR_L)r_\pi}{1+sC_ir_\pi}\right) + R_L$$

$$Z_i = r_b + \frac{(1+g_mR_L)r_\pi}{1+\frac{sC_i}{1+g_mR_L}(1+g_mR_L)r_\pi} + R_L = r_b + \frac{R}{1+sCR} + R_L$$

where $R = (1+g_mR_L)r_\pi$ and $C = C_i/(1+g_mR_L)$

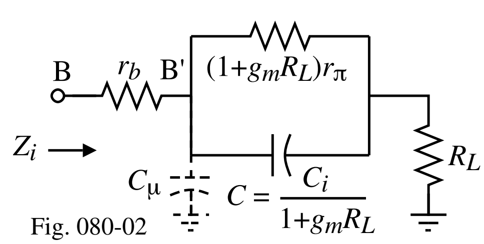

Fig. 080-02

Emitter Follower Output Impedance

From the previous model (or from the impedance transformation aspect of a BJT) we can write,

$$Z_o = \frac{V_{out}}{I_o} = \frac{z_\pi + R_I + r_b}{1 + g_m z_\pi} = \frac{z_\pi + R_I'}{1 + g_m z_\pi} = \frac{\dfrac{r_\pi}{1+sC_i r_\pi} + R_I'}{1 + \dfrac{g_m r_\pi}{1+sC_i r_\pi}} = \frac{r_\pi + R_I' + sC_i r_\pi R_I'}{\beta_0 + 1 + sC_i r_\pi}$$

Multiplying top and bottom by R_I'/β_0, gives

$$Z_o \approx \frac{\left(\dfrac{1}{g_m} + \dfrac{R_I'}{\beta_0} + sC_i r_\pi \dfrac{R_I'}{\beta_0}\right) R_I'}{R_I' + sC_i r_\pi \dfrac{R_I'}{\beta_0}} = \frac{(R_1 + sL)R_2}{R_2 + sL} \qquad \text{assuming } \beta_0 >> 1.$$

Equivalent output circuit:

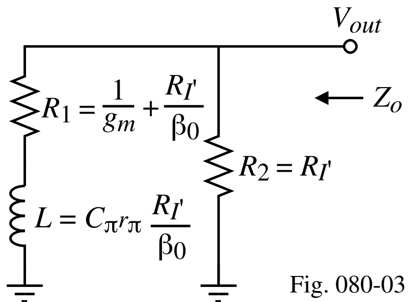

Fig. 080-03

Source Follower Frequency Response

From the previous lecture for the MOSFET $r_\pi = \infty$, $r_b = 0$, and $R'_L = \dfrac{1}{g_{mbs}} || R_L = \dfrac{R_L}{1+g_{mbs}R_L}$

$$\frac{v_{out}}{v_{in}} = \frac{g_m R'_L + \dfrac{R'_L}{r_\pi}}{1 + g_m R'_L + \dfrac{R'_I + R'_L}{r_\pi}} \left[\frac{1 - \dfrac{s}{z_1}}{1 - \dfrac{s}{p_1}}\right] \quad \rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{g_m R'_L}{1 + g_m R'_L} \left[\frac{1 - \dfrac{s}{z_1}}{1 - \dfrac{s}{p_1}}\right]$$

where $z_1 = -\dfrac{g_m}{C_{gs}}$, $\quad p_1 = -\dfrac{1}{R_1 C_{gs}}$ and $R_1 = \dfrac{R_I + R'_L}{1 + g_m R'_L} \approx \dfrac{1}{g_m}$

Example

Calculate the transfer function for a source follower with C_{gs}=7.33pF, $K'W/L$=100mA/V^2, R_L=2kΩ, R_I=190Ω, and I_D=4mA. Let $g_{mbs} \approx 0$, $C_{gd} = 0$, $C_{gb} = 0$, and $C_{bs} = 0$.

Solution

$$g_m = \sqrt{2(100)4}\ \text{mA/V} = 28.2\text{mA/V}. \qquad \frac{v_{out}}{v_{in}} = \frac{28.2\cdot 2}{1 + 28.2\cdot 2} = 0.983\text{V/V}$$

$$|z_1| = \omega_T = \frac{g_m}{C_{gs}} = 3.85\text{x}10^9 \text{ rads/s}, \qquad R_1 = \frac{R_I + R'_L}{1 + g_m R'_L} = \frac{190+2000}{1+1\ 28.2\cdot 2} = 38.2\Omega,$$

$$p_1 = -\frac{10^{12}}{38.2\cdot 7.33} = -3.57\text{x}10^9 \text{ rads/s}$$

(C_{gd}, C_{gb}, and C_{bs} cause two poles, 1 zero)

Source Follower Output Impedance

The output impedance of the source follower can be found from the previous general analysis or from the following model:

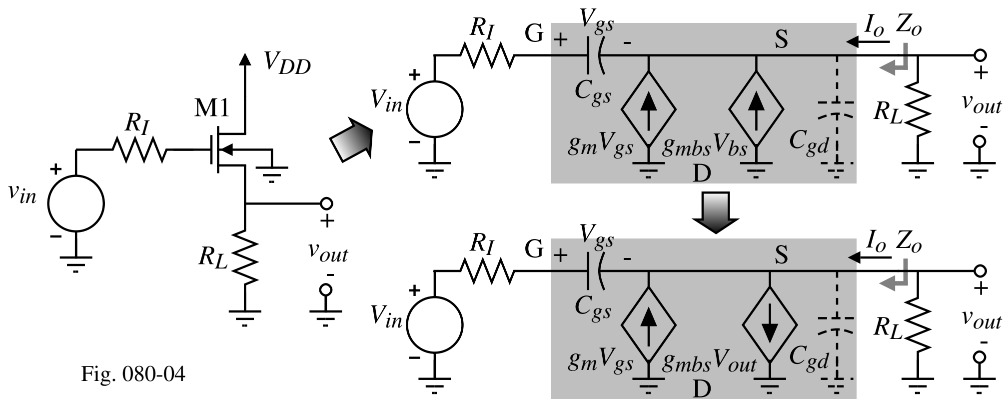

Fig. 080-04

Summing currents at the output,

$$I_o + g_m V_{gs} + sC_{gs}V_{gs} = g_{mbs}V_{out} \quad \text{and} \quad V_{gs} = -\frac{V_{out}}{sC_{gs}R_I+1}$$

$$\therefore \quad I_o = g_{mbs}V_{out} - (g_m+sC_{gs})\left(\frac{-V_{out}}{sC_{gs}R_I+1}\right) \quad \rightarrow \quad \frac{I_o}{V_{out}} = g_{mbs} + \frac{g_m + sC_{gs}}{1+sC_{gs}R_I}$$

$$Z_o = \frac{V_{out}}{I_o} = \frac{1+sC_{gs}R_I}{(g_m+g_{mbs}) + sC_{gs}(R_I g_{mbs}+1)} = \frac{1}{g_m+g_{mbs}}\left[\frac{1+sC_{gs}R_I}{1+sC_{gs}\left(\frac{R_I g_{mbs}+1}{g_m+g_{mbs}}\right)}\right]$$

Identification of the Output Impedance

Find the value of R_1, R_2, and L in the equivalent output impedance model shown for the source follower.

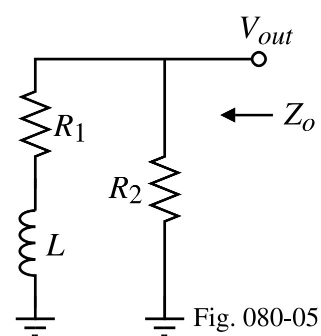

Fig. 080-05

Note that,

$$Z_o = \frac{R_2(R_1+sL)}{R_1+R_2+sL} \approx \frac{R_2(R_1+sL)}{R_2+sL} \quad \text{if} \quad R_1 << R_2$$

The best way to solve this problem is to use the limits of Z_o.

$$lim\ Z_o\ (s\rightarrow 0) = \frac{R_1R_2}{R_1+R_2} \approx R_1 = \frac{1}{g_m+g_{mbs}} \quad \text{where } R_1 << R_2$$

$$lim\ Z_o\ (s\rightarrow \infty) = R_2 = \frac{R_I}{R_I g_{mbs}+1}$$

$$\therefore \quad L = \frac{C_{gs}(1+R_I g_{mbs})}{g_m+g_{mbs}} = \frac{C_{gs}R_I R_1}{R_2}$$

If one includes C_{gd} in parallel with the equivalent circuit, the potential for resonance of the output impedance will occur roughly at $\sqrt{\frac{1}{LC_{gd}}}$ if R_1 is small. Using the values of the previous example with $g_{mbs} = 0.1g_m$ and $C_{gd} = 0.5$ pf gives $R_1 = 32.2\Omega$, $R_2 = 123.7\Omega$ and $L = (7.33\text{pF}\cdot 190\Omega\cdot 32.2\Omega/123.7\Omega) = 0.362\text{nH} \Rightarrow \quad f_{osc} = 11.8\text{GHz}$

Frequency Response of Current Buffers

Current buffers include the common base and common gate configurations.

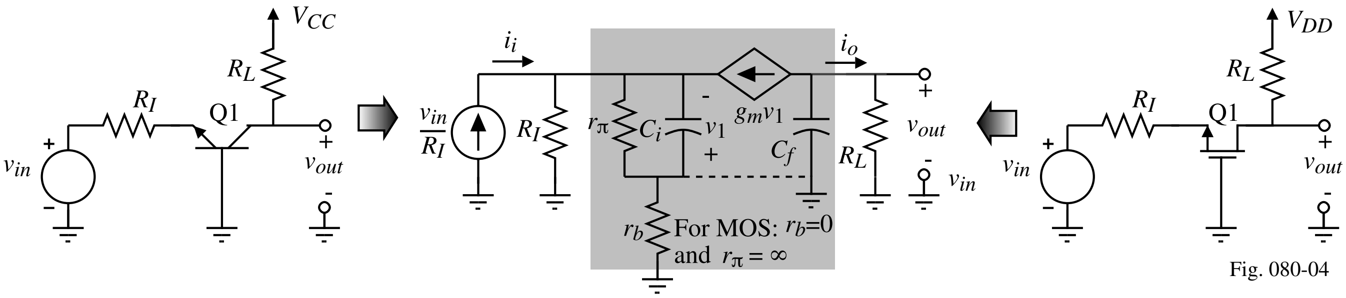

Fig. 080-04

Summing the currents at the input node (neglecting R_I),

$$i_i + \frac{v_1}{z_\pi} + g_m v_1 \approx 0 \qquad \text{where} \qquad z_\pi = \frac{r_\pi}{1+sC_i r_\pi} \rightarrow \quad i_i = -v_1\left(g_m + \frac{1}{r_\pi} + sC_i\right)$$

The short-circuit output current gain can be found as,

$$i_o = -g_m v_1 = \frac{g_m}{g_m + \frac{1}{r_\pi} + sC_i}\, i_i = \frac{g_m r_\pi}{1 + g_m r_\pi + sC_i r_\pi}\, i_i \quad \rightarrow \quad \frac{i_o}{i_i} = \frac{g_m r_\pi}{1+g_m r_\pi}\,\frac{1}{1 + s\frac{r_\pi}{1+g_m r_\pi}C_i}$$

Common-Base Amplifier Frequency Response

Replace C_i with C_π gives,

$$\frac{i_o}{i_i} = \frac{g_m r_\pi}{1+g_m r_\pi}\,\frac{1}{1 + s\frac{r_\pi}{1+g_m r_\pi}C_\pi} \approx \frac{\beta_0}{1+\beta_0}\,\frac{1}{1 + s\frac{C_\pi}{g_m}} \quad \text{if } \beta_0{>>}1 \text{ where } \beta_0 = g_m r_\pi$$

$\therefore$ The low frequency gain, $\frac{i_o}{i_i} = \alpha_0$ and a pole is at $p_1 = -\frac{g_m}{C_\pi} \approx -\omega_T$

If the output current flows through R_L, then the current gain has another pole due to C_μ:

$$\frac{i_o}{i_i} = \frac{g_m r_\pi}{1+g_m r_\pi}\,\frac{1}{1 + s\frac{r_\pi}{1+g_m r_\pi}C_\pi}\,\frac{1}{1+sR_L C_\mu} \approx \frac{\beta_0}{1+\beta_0}\left(\frac{1}{1 + s\frac{C_\pi}{g_m}}\right)\left(\frac{1}{1+sR_L C_\mu}\right) = A_i\left(\frac{1}{1+\frac{s}{p_1}}\right)\left(\frac{1}{1+\frac{s}{p_2}}\right)$$

Example:

If I_C = 1mA, β_0 = 100, C_π = 10pF, C_μ = 0.5pF, C_{cs} = 1pf, and R_L = 2kΩ, evaluate the CB amplifier.

g_m = 1/26 mS

$$A_i = 0.99,\ p_1 = -2.6\text{x}10^{12} \text{ rad/s, and } p_2 = -\frac{1}{R_L(C_\mu + C_{cs})} = -0.333\text{x}10^{9} \text{ rad/s}$$

Common Gate Amplifier Frequency Response

Short-circuit current gain:

$$\frac{i_o}{i_i} = \frac{g_m r_\pi}{1+g_m r_\pi}\frac{1}{1 + s\frac{r_\pi}{1+g_m r_\pi}C_i} \quad \xrightarrow{(r_\pi = \infty \text{ and } g_m \leftarrow g_m + g_{mbs})} \quad \frac{i_o}{i_i} = \frac{1}{1 + s\frac{C_{gs}+C_{bs}}{g_m+g_{mbs}}}$$

Current gain ($R_L \neq 0$):

$$\frac{i_o}{i_i} = \frac{1}{1 + s\frac{C_{gs}+C_{bs}}{g_m+g_{mbs}}}\left(\frac{1}{1+sR_LC_{gd}}\right) = A_i\left(\frac{1}{1+\frac{s}{p_1}}\right)\left(\frac{1}{1+\frac{s}{p_2}}\right)$$

where

$$A_i = 1, \quad p_1 = -\frac{g_m+g_{mbs}}{C_{gs}+C_{bs}} \quad \text{and} \quad p_2 = -\frac{1}{R_LC_{gd}}$$

Example

Calculate the transfer function for a common-gate amplifier with C_{gs}=7.33pF, $K'W/L$=100mA/V^2, R_L=2kΩ, and I_D=4mA. Let $g_{mbs} \approx 0$, C_{gd} = 1pF, and C_{bs} = 2pF.

$$g_m = \sqrt{2\cdot 4\cdot 100}\text{ mS} = 28.2\text{mS} \quad \therefore \quad p_1 = -\frac{28.2\text{x}10^{-3}}{9.33\text{x}10^{-12}} = -3.03\text{x}10^9 \text{ rad/s}$$

$$\text{and} \quad p_2 = -\frac{1}{2000\cdot 10^{-12}} = 0.5\text{ x}10^9 \text{ rad/s}$$

SUMMARY

- The emitter follower and source follower have very high frequency responses
- The –3dB frequency will most likely be caused by the pole at the output of the follower
- The equivalent output of the emitter follower is inductive
- The common base and common gate amplifiers have a current gain of 1
- The CB and CG amplifiers have a high frequency response because of the low input resistance at the input
- If $R_L \neq 0$, the pole at the output of the CB and CG amplifiers causes the –3dB frequency
- The common base amplifier has an input impedance that is inductive
- More detailed analysis of these amplifiers leads to complex poles which will influence the high frequency behavior