Objective
The objective of this presentation is:
1.) Illustrate the frequency analysis of single stage amplifiers
2.) Introduce the Miller technique and the approximate method of solving for two poles

Outline
• Differential and Common Frequency Response of the Differential Amplifier
• Emitter/Source Follower Frequency Response
• Common Base/Gate Frequency Response
• Summary

Emitter Follower Input Impedance

If we let $z_\pi = \frac{r_\pi}{1+sC_i r_\pi}$, then

$$V_i = V_b = I_{in}(r_b + z_\pi) + (I_{in} + g_m z_\pi I_{in}) R_L \rightarrow Z_i = \frac{V_i}{I_{in}} = r_b + z_\pi + (1 + g_m z_\pi) R_L$$

$$Z_i = r_b + \frac{r_\pi}{1+sC_i r_\pi} + \left( \frac{g_m r_\pi}{1+1+sC_i r_\pi} \right) R_L = r_b + \left( \frac{(1+g_m R_L) r_\pi}{1+sC_i r_\pi} \right) + R_L$$

$$Z_i = r_b + \frac{(1+g_m R_L) r_\pi}{1+g_m R_L} + R_L = r_b + \frac{R}{1+sC_R + R_L}$$

where $R = (1+g_m R_L) r_\pi$ and $C = C_i/(1+g_m R_L)$
**Emitter Follower Output Impedance**

From the previous model (or from the impedance transformation aspect of a BJT) we can write,

\[ Z_o = \frac{V_{out}}{I_o} = \frac{z_\pi + R_I + r_b}{1 + g_m z_\pi} = \frac{r_\pi + R_f'}{1 + sC_i r_\pi} + \frac{R_I'}{1 + g_m r_\pi} = \frac{r_\pi + R_f' + sC_i r_\pi R_I'}{1 + sC_i r_\pi} \]

Multiplying top and bottom by \( R_f'/\beta_0 \), gives

\[ Z_o \approx \left( \frac{1 + \frac{R_f'}{\beta_0} + sC_i r_\pi \frac{R_f'}{\beta_0}}{R_f'} \right) \left( \frac{1 + \frac{R_f'}{\beta_0} + sC_i r_\pi \frac{R_f'}{\beta_0}}{R_f'} \right) \frac{r_\pi + R_f' + sC_i r_\pi R_f'}{R_2 + sL} \]

assuming \( \beta_0 \gg 1 \).

**Equivalent output circuit:**

\[ V_{out} \]

\[ Z_o \]

\[ R_1 = \frac{1}{g_m} + \frac{R_f'}{\beta_0} \]

\[ L = C_\pi r_\pi \frac{R_f'}{\beta_0} \]

\[ R_2 = R_f' \]

\[ v_{out} \]

\[ v_{in} \]

\[ p_1 = -\frac{1}{R_1 C_{gs}} \]

\[ R_1 = \frac{1 + g_m R_f'}{1 + g_m R_f'} \approx \frac{1}{g_m} \]

**Source Follower Frequency Response**

From the previous lecture for the MOSFET \( r_\pi = \infty \), \( r_b = 0 \), and \( R'_L = \frac{1}{g_{mbs}} || R_L = \frac{R_L}{1 + g_{mbs} R_L} \)

\[ \frac{v_{out}}{v_{in}} = \frac{g_m R_L'}{1 + g_m R_L'} \left[ \frac{s}{1 - z_1} \right] \rightarrow \frac{v_{out}}{v_{in}} = \frac{g_m R_L'}{1 + g_m R_L'} \left[ \frac{s}{1 - z_1} \right] \]

where \( z_1 = \frac{g_m}{C_{gs}}, \quad p_1 = -\frac{1}{R_1 C_{gs}} \) and \( R_1 = \frac{R_f + R_L'}{1 + g_m R_L'} \approx \frac{1}{g_m} \)

**Example**

Calculate the transfer function for a source follower with \( C_{gs}=7.33pF, K'W/L=100mA/V^2, R_L=2k\Omega, R_f=190\Omega, \) and \( I_D=4mA \). Let \( g_{mbs} = 0, C_{gd} = 0, C_{gb} = 0, \) and \( C_{bs} = 0 \).

**Solution**

\[ g_m = \sqrt{2(100)4} \text{ mA/V} = 28.2 \text{ mA/V}. \]

\[ |z_1| = \omega R = \frac{g_m}{C_{gs}} = 3.85 \times 10^9 \text{ rad/s}, \quad R_1 = \frac{R_f + R_L'}{1 + g_m R_L'} = \frac{190 + 2000}{1 + 1 / 28.2 \times 2} = 38.2 \Omega, \]

\[ p_1 = -\frac{10^{12}}{38.2 \times 7.33} = -3.57 \times 10^9 \text{ rad/s} \]

\( (C_{gd}, C_{gb}, \) and \( C_{bs} \) cause two poles, 1 zero)
Source Follower Output Impedance

The output impedance of the source follower can be found from the previous general analysis or from the following model:

\[ V_{out} = \frac{V_{gs}}{1+sC_{gs}R_I} \]

Summing currents at the output,

\[ I_o + g_mV_{gs} + sC_{gs}V_{gs} = g_{mb}V_{out} \quad \text{and} \quad V_{gs} = -\frac{V_{out}}{sC_{gs}R_I+1} \]

\[ I_o = g_{mb}V_{out} - (g_m+sC_{gs})\left(\frac{-V_{out}}{sC_{gs}R_I+1}\right) \]

\[ Z_o = \frac{V_{out}}{I_o} = \frac{1+sC_{gs}R_I}{(g_m+g_{mb})+sC_{gs}(R_Ig_{mb}+1)} = \frac{1}{g_m+g_{mb}} \left[ \frac{1+sC_{gs}R_I}{1+sC_{gs}(g_m+g_{mb})} \right] \]

Identification of the Output Impedance

Find the value of \( R_1, R_2, \) and \( L \) in the equivalent output impedance model shown for the source follower.

Note that,

\[ Z_o = \frac{R_2(R_1+sL)}{R_1+R_2+sL} \approx \frac{R_2(R_1+sL)}{R_2+sL} \quad \text{if} \quad R_1 \ll R_2 \]

The best way to solve this problem is to use the limits of \( Z_o \).

\[ \lim_{s \to 0} Z_o = \frac{R_1R_2}{R_1+R_2} = R_1 = \frac{1}{g_m+g_{mb}} \quad \text{where} \quad R_1 \ll R_2 \]

\[ \lim_{s \to \infty} Z_o = R_2 = \frac{R_I}{R_Ig_{mb}+1} \]

\[ L = \frac{C_{gs}(1+R_Ig_{mb})}{g_m+g_{mb}} = \frac{C_{gs}R_I}{R_2} \]

If one includes \( C_{gd} \) in parallel with the equivalent circuit, the potential for resonance of the output impedance will occur roughly at \( \sqrt{\frac{1}{LC_{gd}}} \) if \( R_1 \) is small. Using the values of the previous example with \( g_{mb} = 0.1g_m \) and \( C_{gd} = 0.5 \text{ pf} \) gives \( R_1 = 32.2 \Omega, R_2 = 123.7 \Omega \) and \( L = (7.33\text{pF} \cdot 190\Omega \cdot 32.2\Omega/123.7\Omega) = 0.362\text{nH} \) \( \Rightarrow f_{osc} = 11.8\text{GHz} \)
**Frequency Response of Current Buffers**

Current buffers include the common base and common gate configurations.

![Image of common base and common gate configurations](image)

Summing the currents at the input node (neglecting $R_I$),

$$i_i + \frac{v_1}{z_\pi} + g_m v_1 \approx 0 \quad \text{where} \quad z_\pi = \frac{r_\pi}{1 + sC_i r_\pi} \rightarrow \quad i_i = -v_1 \left( g_m + \frac{1}{r_\pi} + sC_i \right)$$

The short-circuit output current gain can be found as,

$$i_o = -g_m v_1 = \frac{g_m}{g_m + \frac{1}{r_\pi} + sC_i} \quad i_i = \frac{g_m r_\pi}{1 + g_m r_\pi + sC_i r_\pi} i_i \quad \rightarrow \quad \frac{i_o}{i_i} = \frac{g_m r_\pi}{1 + g_m r_\pi + \frac{r_\pi}{1 + sC_i r_\pi}}$$

For MOS: $r_b = 0$ and $r_\pi \approx \infty$

**Common-Base Amplifier Frequency Response**

Replace $C_i$ with $C_\pi$ gives,

$$\frac{i_o}{i_i} = \frac{g_m r_\pi}{1 + g_m r_\pi} \frac{1}{1 + \frac{r_\pi}{1 + sC_\pi C_\mu}} \approx \frac{\beta_0}{1 + \beta_0} \frac{1}{1 + \frac{sC_\pi}{g_m}} \quad \text{if} \ \beta_0 \gg 1 \ \text{where} \ \beta_0 = g_m r_\pi$$

∴ The low frequency gain, $\frac{i_o}{i_i} = \alpha_0$ and a pole is at $p_1 = -\frac{g_m}{C_\pi} \approx -\omega_T$

If the output current flows through $R_L$, then the current gain has another pole due to $C_\mu$:

$$\frac{i_o}{i_i} = \frac{g_m r_\pi}{1 + g_m r_\pi} \frac{1}{1 + \frac{r_\pi}{1 + sR_L C_\mu}} \approx \frac{\beta_0}{1 + \beta_0} \left( \frac{1}{1 + sRC_\mu} \right) = A_i \left( \frac{1}{1 + \frac{1}{p_1}} \right) \left( \frac{1}{1 + \frac{1}{p_2}} \right)$$

Example:

If $I_C = 1\ mA$, $\beta_0 = 100$, $C_\pi = 10pF$, $C_\mu = 0.5pF$, $C_{cs} = 1pf$, and $R_L = 2k\Omega$, evaluate the CB amplifier.

$g_m = 1/26 \ mS$

$A_i = 0.99$, $p_1 = -2.6 \times 10^{12} \ \text{rad/s}$, and $p_2 = -\frac{R_L(C_\mu + C_{cs})}{1 + \frac{1}{p_1}} = -0.333 \times 10^9 \ \text{rad/s}$
Common Gate Amplifier Frequency Response

Short-circuit current gain:

\[
\frac{i_o}{i_i} = \frac{g_m r_\pi}{1 + g_m r_\pi} \frac{1}{r_\pi} \quad (r_\pi = \infty \text{ and } g_m \leftrightarrow g_m + g_{mbs})
\]

\[
\frac{i_o}{i_i} = \frac{1}{1 + s r_\pi C_i}
\]

Current gain \((R_L \neq 0)\):

\[
\frac{i_o}{i_i} = \frac{1}{C_{gs} + C_{bs}} \left( \frac{1}{1 + s R_L C_{gd}} \right) = A_i \left( \frac{1}{1 + p_1} \right) \left( \frac{1}{1 + p_2} \right)
\]

where

\[A_i = 1, \quad p_1 = \frac{g_m + g_{mbs}}{C_{gs} + C_{bs}} \quad \text{and} \quad p_2 = \frac{1}{R_L C_{gd}}\]

Example

Calculate the transfer function for a common-gate amplifier with \(C_{gs} = 7.33\,\mu\text{F}\), \(K'W/L = 100\,\text{mA/V}^2\), \(R_L = 2\,\text{k}\Omega\), and \(I_D = 4\,\text{mA}\). Let \(g_{mbs} \approx 0\), \(C_{gd} = 1\,\mu\text{F}\), and \(C_{bs} = 2\,\mu\text{F}\).

\[g_m = \sqrt{2 \cdot 4 \cdot 100} \, \text{mS} = 28.2\,\text{mS} \quad \therefore \quad p_1 = \frac{28.2 \times 10^{-3}}{9.33 \times 10^{-12}} = 3.03 \times 10^9 \, \text{rad/s}\]

and \[p_2 = \frac{1}{2000 \cdot 10^{-12}} = 0.5 \times 10^9 \, \text{rad/s}\]

SUMMARY

- The emitter follower and source follower have very high frequency responses
- The –3dB frequency will most likely be caused by the pole at the output of the follower
- The equivalent output of the emitter follower is inductive
- The common base and common gate amplifiers have a current gain of 1
- The CB and CG amplifiers have a high frequency response because of the low input resistance at the input
- If \(R_L \neq 0\), the pole at the output of the CB and CG amplifiers causes the –3dB frequency
- The common base amplifier has an input impedance that is inductive
- More detailed analysis of these amplifiers leads to complex poles which will influence the high frequency behavior