LECTURE 130 – COMPENSATION OF OP AMPS-II
(READING: GHLM – 638-652, AH – 260-269)

INTRODUCTION
The objective of this presentation is to continue the ideas of the last lecture on compensation of op amps.

Outline
• Compensation of Op Amps
  General principles
  Miller, Nulling Miller
  Self-compensation
  Feedforward
• Summary

Conditions for Stability of the Two-Stage Op Amp (Assuming \( p_3 \geq GB \))
• Unity-gainbandwith is given as:
  \[ GB = A_v(0) |p_1| = \left( \frac{g_m l g_m R_1 R_2}{g_m l R_1 R_2 C_c} \right) = \frac{g_m l}{C_c} = \frac{g_m 1 g_m 2 R_1 R_2}{g_m 2 R_1 R_2 C_c} = \frac{g_m 1}{C_c} \]
• The requirement for 45° phase margin is:
  \[ \pm 180° - \tan^{-1} \left( \frac{GB}{|p_1|} \right) - \tan^{-1} \left( \frac{GB}{|p_2|} \right) - \tan^{-1} \left( \frac{GB}{z} \right) = 45° \]
  Let \( \omega = GB \) and assume that \( z \geq 10GB \), therefore we get,
  \[ \pm 180° - \tan^{-1} \left( \frac{GB}{|p_1|} \right) - \tan^{-1} \left( \frac{GB}{|p_2|} \right) - \tan^{-1} \left( \frac{GB}{z} \right) = 45° \]
  \[ 135° = \tan^{-1}(A_v(0)) + \tan^{-1} \left( \frac{GB}{|p_2|} \right) + \tan^{-1}(0.1) = 90° + \tan^{-1} \left( \frac{GB}{|p_2|} \right) + 5.7° \]
  \[ 39.3° = \tan^{-1} \left( \frac{GB}{|p_2|} \right) \Rightarrow \frac{GB}{|p_2|} = 0.818 \Rightarrow |p_2| \geq 1.22GB \]
• The requirement for 60° phase margin:
  \[ |p_2| \geq 2.2GB \text{ if } z \geq 10GB \]
• If 60° phase margin is required, then the following relationships apply:
  \[ \frac{g_m 6}{C_c} > \frac{10g_m 1}{C_c} \Rightarrow g_m 6 > 10g_m 1 \text{ and } \frac{g_m 6}{C_2} > \frac{2.2g_m 1}{C_c} \Rightarrow C_c > 0.22C_2 \]
**Controlling the Right-Half Plane Zero**

Why is the RHP zero a problem?
Because it boosts the magnitude but lags the phase - the worst possible combination for stability.

![Diagram of RHP zero phase angles](image)

180° > θ₁ > θ₂ > θ₃

Solution of the problem:
If zeros are caused by two paths to the output, then eliminate one of the paths.

**Use of Buffer to Eliminate the Feedforward Path through the Miller Capacitor**

Model:

The transfer function is given by the following equation,

\[ V_o(s) = \frac{(g_{mI})(g_{mII})(R_I)(R_{II})}{1 + s[R_I C_I + R_{II} C_{II} + R_I C_c + g_{mII} R_I R_{II} C_c] + s^2[R_I R_{II} C_{II}(C_I + C_c)]} \]

Using the technique as before to approximate \( p_1 \) and \( p_2 \) results in the following

\[ p_1 \approx \frac{-1}{R_I C_I + R_{II} C_{II} + R_I C_c + g_{mII} R_I R_{II} C_c} \approx \frac{-1}{g_{mII} R_I R_{II} C_c} \]

and

\[ p_2 \approx \frac{-g_{mII} C_c}{R_{II}(C_I + C_c)} \]

Comments:
Poles are approximately what they were before with the zero removed.
For 45° phase margin, \( |p_2| \) must be greater than \( GB \)
For 60° phase margin, \( |p_2| \) must be greater than 1.73\( GB \)
Use of Buffer with Finite Output Resistance to Eliminate the RHP Zero

Assume that the unity-gain buffer has an output resistance of $R_o$.

Model:

It can be shown that if the output resistance of the buffer amplifier, $R_o$, is not neglected that another pole occurs at,

$$p_4 \approx \frac{-1}{R_o \left( C_f C_c \left( C_f + C_c \right) \right)}$$

and a LHP zero at

$$z_2 \approx \frac{-1}{R_o C_c}$$

Closer examination shows that if a resistor, called a nulling resistor, is placed in series with $C_c$ that the RHP zero can be eliminated or moved to the LHP.

Use of Nulling Resistor to Eliminate the RHP Zero (or turn it into a LHP zero)

Nodal equations:

$$g_{mI} V_{in} + \frac{V_I}{R_I} + sC_I V_I + \left( \frac{sC_c}{1 + sC_c R_z} \right) (V_I - V_{out}) = 0$$

$$g_{mII} V_I + \frac{V_I}{R_{II}} + sC_{II} V_{out} + \left( \frac{sC_c}{1 + sC_c R_z} \right) (V_{out} - V_I) = 0$$

Solution:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{a \left\{ 1 - s \left[ (C_c g_{mII}) - R_z C_c \right] \right\}}{1 + b s + c s^2 + d s^3}$$

where

$$a = g_{mII} g_{mII} R_I R_{II}$$

$$b = (C_{II} + C_c) R_{II} + (C_I + C_c) R_I + g_{mII} R_I R_{II} C_c + R_z C_c$$

$$c = [R_I R_{II} (C_I C_c + C_c C_I + C_c C_{II}) + R_z C_c (R_I C_I + R_{II} C_{II})]$$

$$d = R_I R_{II} R_z C_c C_{II} C_c$$

Use of Nulling Resistor to Eliminate the RHP - Continued

If \( R_z \) is assumed to be less than \( R_I \) or \( R_{II} \) and the poles widely spaced, then the roots of the above transfer function can be approximated as

\[
\begin{align*}
\text{p}_1 & \cong \frac{-1}{(1 + g_{mII}R_{II})R_I R_C} \cong \frac{-1}{g_{mII}R_{II}R_C} \\
\text{p}_2 & \cong \frac{-g_{mII}C_c}{C_I C_{II} + C_c C_I + C_c C_{II}} \cong \frac{-g_{mII}}{C_{II}} \\
\text{p}_4 & = \frac{-1}{R_z C_I} \quad (p_3 \text{ has been used previously for the mirror pole so we choose } p_4 \text{ for the nulling resistor pole})
\end{align*}
\]

and

\[
\begin{align*}
z_1 & = \frac{1}{C_c(1/g_{mII} - R_z)}
\end{align*}
\]

Note that the zero can be placed anywhere on the real axis.

Conceptual Illustration of the Nulling Resistor Approach

The output voltage, \( V_{out} \), can be written as

\[
V_{out} = \frac{-g_{m6}R_{II}}{R_{II} + R_z + \frac{1}{sC_c}} V' + \frac{R_{II}}{R_{II} + R_c + \frac{1}{sC_c}} V'' = \frac{-R_{II} \left[ g_{m6}R_z + \frac{g_{m6}}{sC_c} - 1 \right]}{R_{II} + R_z + \frac{1}{sC_c}} V
\]

when \( V = V' = V'' \).

Setting the numerator equal to zero and assuming \( g_{m6} = g_{mII} \) gives,

\[
z_1 = \frac{1}{C_c(1/g_{mII} - R_z)}
\]
A Design Procedure that Allows the RHP Zero to Cancel the Output Pole, \( p_2 \)

We desire that \( z_1 = p_2 \) in terms of the previous notation. Therefore,

\[
\frac{1}{C_c(1/g_m II - R_z)} = \frac{-g_m II}{C_c}
\]

The value of \( R_z \) can be found as

\[
R_z = \left( \frac{C_c + C_{II}}{C_c} \right) (1/g_m II)
\]

With \( p_2 \) canceled, the remaining roots are \( p_1 \) and \( p_4 \)(the pole due to \( R_z \)). For unity-gain stability, all that is required is that

\[
|p_4| > A_v(0)|p_1| = \frac{A_v(0)}{g_m II R_II R_I C_c} = \frac{g_m I}{C_c}
\]

and

\[
(1/R_z C_I) > (g_m II C_c) = GB
\]

Substituting \( R_z \) into the above inequality and assuming \( C_{II} >> C_c \) results in

\[
C_c > \sqrt{\frac{g_m I}{g_m II C_I C_{II}}}
\]

This procedure gives excellent stability for a fixed value of \( C_{II} (\approx C_L) \). Unfortunately, as \( C_L \) changes, \( p_2 \) changes and the zero must be readjusted to cancel \( p_2 \).

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Increasing the Magnitude of the Output Pole

The magnitude of the output pole, \( p_2 \), can be increased by introducing gain in the Miller capacitor feedback path. For example,

\[
I_{in} = G_1 V_1 - g_{m8} V_{s8} = G_1 V_1 \left( \frac{g_{m8} C_c}{s g_{m8} + s C_c} \right) V_{out} \quad \text{and} \quad 0 = g_{m6} V_1 + \left[ G_2 + s C_2 + \frac{g_{m8} C_c}{s g_{m8} + s C_c} \right] V_{out}
\]

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**Increasing the Magnitude of the Output Pole - Continued**

Solving for the transfer function $\frac{V_{out}}{I_{in}}$ gives,

\[
\frac{V_{out}}{I_{in}} = \left(\frac{-g_{m6}}{G_1G_2}\right) \left[1 + s \left(\frac{C_c}{g_{m8}} + \frac{C_c}{G_2} + \frac{g_{m6}C_c}{G_1G_2} + s^2 \left(\frac{C_cC_2}{g_{m8}G_2}\right)\right)\right]
\]

Using the approximate method of solving for the roots of the denominator gives

\[
p_1 = \frac{-1}{g_{m8} + \frac{C_c}{G_2} + \frac{g_{m6}C_c}{G_1G_2}} \approx \frac{-6}{g_{m6}r_{ds}^2C_c}
\]

and

\[
p_2 \approx \frac{-\frac{6}{C_cC_2}}{g_{m8}G_2} = \frac{g_{m8}r_{ds}^2G_2}{6} = \frac{g_{m8}r_{ds}}{3} |p_2'|
\]

where all the various channel resistance have been assumed to equal $r_{ds}$ and $p_2'$ is the output pole for normal Miller compensation.

**Result:**

Dominant pole is approximately the same and the output pole is increased by $\approx g_{m}r_{ds}$.

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**Concept Behind the Increasing of the Magnitude of the Output Pole**

\[
R_{out} = r_{ds7} \left|\frac{1}{g_{m6}g_{m8}r_{ds}}\right| \approx \frac{3}{g_{m6}g_{m8}r_{ds}}
\]

Therefore, the output pole is approximately,

\[
|p_2| \approx \frac{g_{m6}g_{m8}r_{ds}}{3C_II}
\]

Besides, the common gate amplifier stops the feedforward path preventing the RHP zero.
**FEEDFORWARD COMPENSATION**

Use two parallel paths to achieve a LHP zero for lead compensation purposes.

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{AC_c}{C_c + C_{II}} \left( s + \frac{g_{mII}AC_c}{s + 1/[R_{II}(C_c + C_{II})]} \right)
\]

To use the LHP zero for compensation, a compromise must be observed.

- Placing the zero below $GB$ will lead to boosting of the loop gain that could deteriorate the phase margin.
- Placing the zero above $GB$ will have less influence on the leading phase caused by the zero.

Note that a source follower is a good candidate for the use of feedforward compensation.

**SELF-COMPENSATED OP AMPS**

*Self compensation* occurs when the load capacitor is the compensation capacitor (can never be unstable for resistive feedback)

\[
\frac{v_{\text{out}}}{v_{\text{in}}} = A_{V}(0) = G_mR_{out}
\]

Dominant pole:

\[
p_1 = -\frac{1}{R_{out}C_L}
\]

Unity-gainbandwidth:

\[
GB = A_{V}(0)\cdot|p_1| = \frac{G_m}{C_L}
\]

Stability:

Large load capacitors simply reduce $GB$ but the phase is still $90^\circ$ at $GB$. 
SUMMARY

Compensation

- Designed so that the op amp with unity gain feedback (buffer) is stable
- Types
  - Miller
  - Miller with nulling resistors
  - Self Compensating
  - Feedforward