Objective
The objective of this presentation is:
1.) Illustrate the method of using return ratio to analyze feedback circuits
2.) Demonstrate using examples

Outline
• Concept of return ratio
• Closed-loop gain using return ratio
• Closed-loop impedance using return ratio
• Summary

Concept of Return Ratio
Instead of using two-port analysis, return ratio takes advantage of signal flow graph theory.
The return ratio for a dependent source in a feedback loop is found as follows:
1.) Set all independent sources to zero.
2.) Change the dependent source to an independent source and define the controlling variable as, \( s_r \), and the source variable as \( s_t \).
3.) Calculate the return ratio designated as \( RR = -\frac{s_r}{s_t} \).
**Example 1 – Calculation of Return Ratio**

Find the return ratio of the op amp with feedback shown if the input resistance of the op amp is $r_i$, the output resistance is $r_o$, and the voltage gain is $a_v$.

\[ v_r = \frac{(-a_v v_t) R_s || r_i}{r_o + R_F + R_S || r_i} \quad \rightarrow \quad RR = - \frac{v_r}{v_t} = \frac{(a_v) R_s || r_i}{r_o + R_F + R_S || r_i} \]

**Closed-Loop Gain Using Return Ratio**

Consider the following general feedback amplifier:

Note that $s_{oc} = ks_{ic}$.

Assume the amplifier is linear and express $s_{ic}$ and $s_{out}$ as linear functions of the two sources, $s_{in}$ and $s_{oc}$.

\[ s_{ic} = B_1 s_{in} - H s_{oc} \]
\[ s_{out} = d s_{in} + B_2 s_{oc} \]

where $B_1$, $B_2$, and $H$ are defined as

\[ B_1 = \frac{s_{ic}}{s_{in} s_{oc}=0} = \frac{s_{ic}}{s_{in} k=0} \quad \text{and} \quad B_2 = \frac{s_{out}}{s_{oc} s_{in}=0} \quad \text{and} \quad H = -\frac{s_{ic}}{s_{oc} s_{in}=0} \]
Closed-Loop Gain Using Return Ratio – Continued

Interpretation:

- $B_1$ is the transfer function from the input to the controlling signal with $k = 0$.
- $B_2$ is the transfer function from the controlling signal to the output with $s_{in} = 0$.
- $H$ is the transfer function from the output of the dependent source to the controlling signal with $s_{in} = 0$ and multiplied times a $-1$.

$d$ is defined as,

$$d = \frac{s_{out}}{s_{in}} \bigg|_{s_{oc}=0} = \frac{s_{out}}{s_{in}} \bigg|_{k=0}$$

$d$ is the direct signal feedthrough when the controlled source in $A$ is set to zero ($k=0$).

Closed-loop gain ($s_{out}/s_{in}$) can be found as,

$$s_{ic} = B_1 s_{in} - H s_{oc} = B_1 s_{in} - kH s_{ic} \quad \rightarrow \quad \frac{s_{ic}}{s_{in}} = \frac{B_1}{1 + kH}$$

$$s_{out} = d s_{in} + B_2 s_{oc} = d s_{in} + kB_2 s_{ic} = d s_{in} + \frac{B_1 kB_2}{1 + kH} s_{in}$$

2.) $A = \frac{s_{out}}{s_{in}} = \frac{B_1 kB_2}{1 + kH} + d = \frac{B_1 kB_2}{1 + RR} + d = \frac{g}{1 + RR} + d$

where $RR = kH$ and $g = B_1 kB_2$ (gain from $s_{in}$ to $s_{out}$ if $H = 0$ and $d = 0$)

Closed-Loop Gain Using Return Ratio – Continued

Further simplification:

$$A = \frac{g}{1 + RR} + d = \frac{g + d(1+RR)}{1 + RR} = \frac{g + dRR}{1 + RR} + \frac{d}{1 + RR} = \frac{gRR + dRR}{1 + RR} + \frac{d}{1 + RR}$$

Define

$$A_{\infty} = \frac{g}{RR} + d$$

3.) $A = A_{\infty} \frac{RR}{1 + RR} + \frac{d}{1 + RR}$

Note that as $RR \rightarrow \infty$, that $A = A_{\infty}$.

$A_{\infty}$ is the closed-loop gain when the feedback circuit is ideal (i.e., $RR \rightarrow \infty$ or $k \rightarrow \infty$).

Block diagram of the new formulation:

![Block diagram of the new formulation](image)

Note that $b = RR A_{\infty}$ is called the effective gain of the feedback amplifier.
Example 2 – Use of Return Ratio Approach to Calculate the Closed-Loop Gain

Find the closed-loop gain and the effective gain of the transistor feedback amplifier shown using the previous formulas. Assume that the BJT $g_m = 40\,\text{mS}$, $r_\pi = 5\,\text{k}\Omega$, and $r_o = 1\,\text{M}\Omega$.

Solution

The small-signal model suitable for calculating $A_\infty$ and $d$ is shown.

\[
A_\infty = \frac{s_{\text{out}}}{s_{\text{in}}} \quad k=\infty = \frac{v_o}{i_{\text{in}}} g_m=\infty = ?
\]

Remember that $A = \frac{a}{1+af} \to \frac{1}{f}$ as $a \to \infty$.

\[
f = \frac{v_o}{i_{\text{F}}} \bigg|_{v_{\text{in}}=0} = -\frac{1}{R_F}
\]

Therefore, $A_\infty = -R_F = -20\,\text{k}\Omega$

\[
d = \frac{s_{\text{out}}}{s_{\text{in}}} \quad k=0 = \frac{v_o}{i_{\text{in}}} g_m=0 = \frac{r_\pi}{r_\pi+R_F+(r_o||R_C)} (r_o||R_C) = \frac{5\,\text{k}\Omega}{5\,\text{k}\Omega+20\,\text{k}\Omega+1\,\text{M}\Omega||10\,\text{k}\Omega} = 1.42\,\text{k}\Omega
\]

\[\text{RR} = \frac{v_r}{i_{\text{F}}} = \frac{(g_m r_\pi) \frac{r_o||R_C}{r_\pi+R_F+r_o||R_C}}{1.42\,\text{k}\Omega} = 56.74\]

Now, the closed loop gain is found to be,

\[
A = A_\infty \frac{RR}{1+RR} + \frac{d}{1+RR} = (-20\,\text{k}\Omega) \left( \frac{56.74}{1+56.74} + \frac{1.4\,\text{k}\Omega}{1+56.74} \right) = -19.63\,\text{k}\Omega
\]

The effective gain is given as,

\[
b = \text{RR}A_\infty = 56.74(-20\,\text{k}\Omega) = -1135\,\text{k}\Omega
Closed-Loop Impedance Formula using the Return Ratio (Blackman’s Formula)

Consider the following linear feedback circuit where the impedance at port X is to be calculated.

Expressing the signals, \( v_x \) and \( s_{ic} \) as linear functions of the signals \( i_x \) and \( s_y \) gives,

\[
\begin{align*}
v_x &= a_1 i_x + a_2 s_y \\
s_{ic} &= a_3 i_x + a_4 s_y
\end{align*}
\]

The impedance looking into port X when \( k = 0 \) is,

\[
Z_{port}(k=0) = \left| \frac{v_x}{i_x} \right|_{k=0} = \left| \frac{v_x}{i_x} \right|_{s_y=0}
\]

Closed-Loop Impedance Formula using the Return Ratio – Continued

Next, compute the RR for the controlled source, \( k \), under two different conditions.

1.) The first condition is when port X is open (\( i_x = 0 \)).

\[
s_{ic} = a_4 s_y = a_4 s_t
\]

Also,

\[
s_r = k s_{ic} \quad \rightarrow \quad s_r = k a_4 s_t \quad \rightarrow \quad RR(\text{port open}) = -\frac{s_r}{s_t} = -k a_4
\]

2.) The second condition is when port X is shorted (\( v_x = 0 \)).

\[
i_x = -\frac{a_2}{a_1} s_y = -\frac{a_2}{a_1} s_t
\]

\[
s_{ic} = a_3 i_x + a_4 s_y = \left( a_4 - \frac{a_2 a_3}{a_1} \right) s_t
\]

The return signal is

\[
s_r = k s_{ic} = k \left( a_4 - \frac{a_2 a_3}{a_1} \right) s_t \quad \rightarrow \quad RR(\text{port shorted}) = -\frac{s_r}{s_t} = -k \left( a_4 - \frac{a_2 a_3}{a_1} \right)
\]

3.) The port impedance can be found as (Blackman’s formula),

\[
Z_{port} = Z_{port}(k=0) \left[ \frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})} \right]
\]
Example 3 – Application of Blackman’s Formula

Use Blackman’s formula to calculate the output resistance of Example 2.

**Solution**

We must calculate three quantities. They are $R_{out}(g_m=0)$, $RR$(output port shorted), and $RR$(output port open). Use the following model for calculations:

\[
R_{out}(g_m=0) = r_o || RC \parallel (r_\pi + R_F) = 7.09k\Omega
\]

$RR$(output port shorted) = 0 because $v_r = 0$.

$RR$(output port open) = $RR$ of Example 2 = 56.74

\[
\therefore R_{out} = R_{out}(g_m=0) \left[ \frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})} \right] = 7.09k\Omega \left( \frac{1}{1 + 56.74} \right) = 129\Omega
\]

Example 4 – Output Resistance of a Super-Source Follower

Find an expression for the small-signal output resistance of the circuit shown.

**Solution**

The appropriate small-signal model is shown where $g_{m2} = k$.

\[
R_{out}(g_{m2}=0) = r_{ds2} \quad \text{and} \quad RR(\text{output port shorted}) = 0 \text{ because } v_t = 0.
\]

\[
RR(\text{output port open}) = -\frac{s_r}{s_t} = -\frac{v_r}{v_t}
\]

\[
v_r = v_{out} - (g_{m1}v_2)r_{ds1} = v_{out} - g_{m1}r_{ds1}(-v_{out}) = v_{out}(1 + g_{m1}r_{ds1})
\]

\[
v_{out} = -g_{m2}r_{ds2}v_t \quad \implies \quad v_r = -(1 + g_{m1}r_{ds1})g_{m2}r_{ds2}v_t
\]

\[
RR(\text{output port open}) = -\frac{v_r}{v_t} = (1 + g_{m1}r_{ds1})g_{m2}r_{ds2}
\]

\[
\therefore R_{out} = R_{out}(g_{m2}=0) \left[ \frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})} \right] = r_{ds2} \left( \frac{1+0}{1+(1 + g_{m1}r_{ds1})g_{m2}r_{ds2}} \right) = \frac{1}{g_{m1}r_{ds1}g_{m2}}
\]
SUMMARY

• Return ratio is associated with a dependent source. If the dependent source is converted to an independent source, then the return ratio is the gain from the dependent source variable to the previously controlling variable.

• The closed-loop gain of a linear, negative feedback system can be expressed as

\[ A = A_\infty \frac{RR}{1 + RR} \frac{d}{1 + RR} \]

where

- \( A_\infty \) = the closed-loop gain when the loop gain is infinite
- \( RR \) = the return ratio
- \( d \) = the closed-loop gain when the amplifier gain is zero

• The resistance at a port can be found from Blackman’s formula which is

\[ Z_{\text{port}} = Z_{\text{port}(k=0)} \frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})} \]

where \( k \) is the gain of the dependent source chosen for the return ratio calculation.

• This stuff is all great but of little use as far as calculations are concerned. Small-signal analysis is generally quicker and easier than the two-port approach or the return ratio approach.

• Why study feedback? Because it is a great tool for understanding a circuit and for knowing how to modify the performance in design.