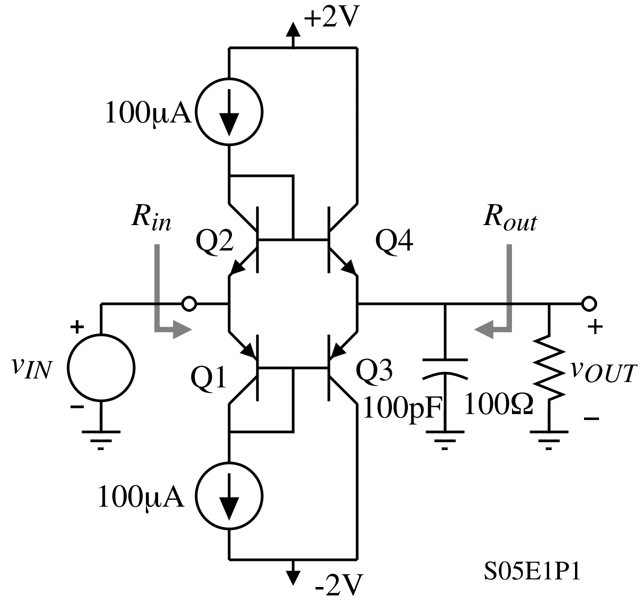
EXAMINATION NO. 1

(Average score = 56.2/100)

Problem 1 - (25 points)

An emitter follower, push-pull output stage is shown. Assume that $\beta_N = \beta_P = 100$, $V_t = 25\text{mV}$, and $I_s = 10\text{fA}$.



S05E1P1

a.) If the emitter areas of Q1 and Q2 are 10µm^2, find the emitter area of Q3 and Q4 so that the collector current in Q3 and Q4 is 1mA when $v_{IN} = v_{OUT} = 0$.

b.) What is the ±peak output voltage of this amplifier? Assume the 100µA sources can have a minimum voltage across them of 0.2V.

c.) What is the ±slew rate of this amplifier in V/µs?

d.) What is the small-signal input and output resistance of this amplifier when $v_{IN} = v_{OUT} = 0$? (Do not include the load resistance in the output resistance.)

Solution

a.) $V_{EB1}+V_{BE2} = V_{BE4}+V_{EB3} \;\rightarrow\; \dfrac{I_{C1}{}^2}{I_{s1}I_{s2}} = \dfrac{I_{C3}{}^2}{I_{s3}I_{s4}} \;\rightarrow\; I_{s3} = I_{s4} = \dfrac{I_{C3}I_{s1}}{I_{C1}} = 10I_{s1}$

$\therefore\; A_{E3} = A_{E4} = 10A_{E1} = \underline{\underline{100\mu\text{m}^2}}$

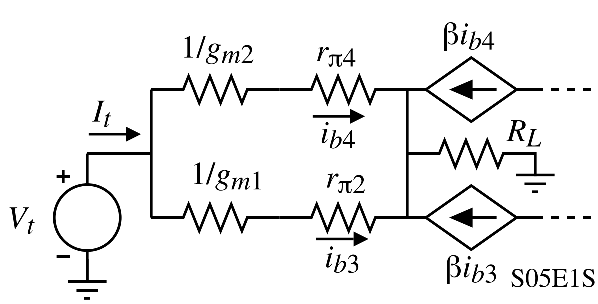
b.) $V_{peak} = \pm(100\mu\text{A})(1+\beta_o)R_L = \pm 100\mu\text{A}\cdot 101\cdot 100\Omega = \pm 1.01\text{V}$

Check to make sure this answer is okay. $V_{BE4} = V_t \ln\left(\dfrac{10.1\text{mA}}{10\text{fA}}\right) = 0.691\text{V}$

$\therefore$ Maximum swing is 2-0.691-0.2 = 1.109V so $V_{peak} = \underline{\underline{\pm 1.01\text{V}}}$

c.) $\pm SR = \left(\dfrac{10.1\text{mA}}{100\text{pF}}\right) = \underline{\underline{101\text{V}/\mu\text{s}}}$

d.) Small signal model:



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$$V_t = \frac{I_t}{2}\left(\frac{1}{g_{m2}} + r_{\pi 4}\right) + R_L I_t(1+\beta_o)$$

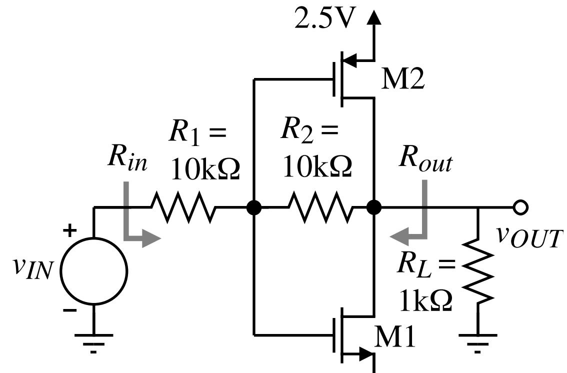
$$g_{m2} = \frac{100\mu\text{A}}{25\text{mV}} = \frac{1}{250\Omega}$$

$$g_{m4} = \frac{1\text{mA}}{25\text{mV}} = \frac{1}{25\Omega} \qquad R_{in} = 0.5[250+25(101)] + 101(100) = \underline{\underline{11.487\text{k}\Omega}}$$

$$R_{out} = 0.5\left[\frac{1}{g_{m4}} + \frac{1/g_{m2}}{1+\beta_o}\right] = 0.5[25 + (250/101)] = \underline{\underline{13.37\Omega}}$$

Problem 2 - (25 points)

Find the value for the small-signal output resistance R_{out} ignoring R_L and the value of the small-signal input resistance for the amplifier shown. Let the dc currents through M1 and M2 be 500µA, W_1/L_1 = 100µm/1µm and W_2/L_2 = 200µm/1µm. Assume the parameters of the NMOS transistors are K_N'=110V/µA^2, V_{TN} = 0.7V, and for the PMOS transistors are K_P'=50V/µA^2, V_{TP} = -0.7V. Ignore r_{ds1} and r_{ds2}.



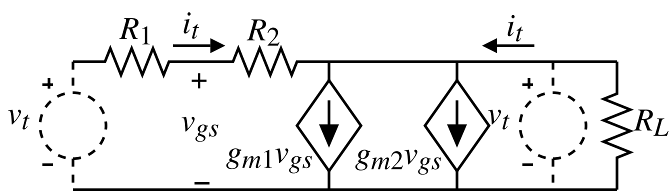
S05E1P2

Solution

Calculating the small-signal parameters gives,

$$g_{m1} = \sqrt{2\cdot 110\cdot 500\cdot 100} = 3.316\text{mS},\quad g_{m2} = \sqrt{2\cdot 50\cdot 500\cdot 200} = 3.162\text{mS}$$

The small-signal model is given as,



S05E1S2

For R_{out}, sum the currents at the output (with the LH v_t =0) to get,

$$i_t = v_t\left[\frac{1}{R_1+R_2} + \frac{g_{m1}+g_{m2}}{2}\right] \quad\rightarrow\quad R_{out} = \frac{v_t}{i_t} = \left[\frac{1}{R_1+R_2} + \frac{g_{m1}+g_{m2}}{2}\right]^{-1} = \underline{\underline{308\Omega}}$$

For R_{in}, remove the RH v_t and write a loop equation at the input to get,

$$v_t = i_t(R_1+R_2) + (i_t - g_{m1}v_{gs} - g_{m2}v_{gs})R_L = i_t(R_1+R_2+R_L) - (g_{m1}+g_{m2})v_{gs}$$

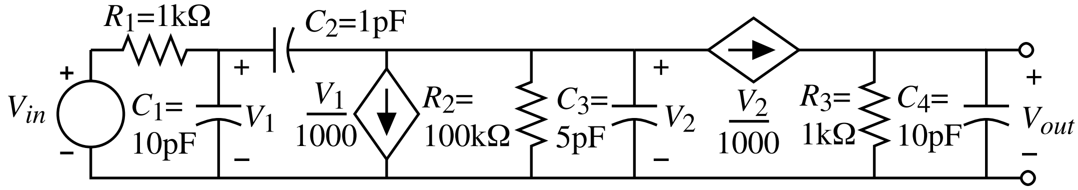
But $v_{gs} = v_t - i_t R_1$ which gives,

$$R_{in} = \frac{v_t}{i_t} = \frac{R_1+R_2+R_L+(g_{m1}+g_{m2})R_LR_1}{1+(g_{m1}+g_{m2})R_L} = \frac{201\text{k}\Omega+(3.316+3.162)(1)(100\text{k}\Omega)}{1+(3.316+3.162)(1)}$$

$$R_{in} = \underline{\underline{113.5\text{k}\Omega}}$$

Problem 3 - (25 points)

Find the midband voltage gain and the –3dB frequency in Hertz for the circuit shown.



S05E1P3

Solution

The midband voltage gain can be expressed as,

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_2}\frac{V_2}{V_1}\frac{V_1}{V_{in}} = (1)\left(\frac{-R_2}{R_2+1000}\right)(1) = \underline{\underline{-0.99\text{V/V}}}$$

Finding the open-circuit, time constants:

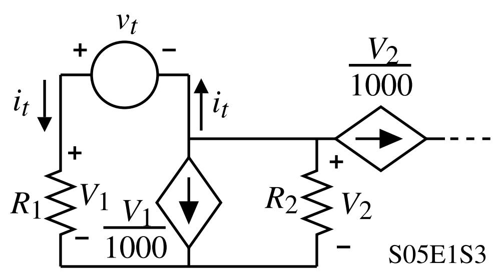
R_{C1O}: $\quad R_{C1O} = R_1 = 1\text{k}\Omega \quad \rightarrow \quad R_{C1O}C_1 = 10\text{ns}$

R_{C2O}:

$$v_t = R_1 i_t + R_2\left[i_t + \frac{V_1}{1000} + \frac{V_2}{1000}\right]$$

But $v_t = V_1 - V_2$ and $V_1 = R_1 i_t$,

$$\therefore \quad v_t = R_1 i_t + R_2 i_t + \frac{2R_1R_2 i_t}{1000} - \frac{R_2 v_t}{1000}$$



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$$R_{C2O} = \frac{v_t}{i_t} = \frac{R_1+R_2+0.002R_1R_2}{1+0.001R_2}$$

$$= \frac{1\text{k}\Omega+100\text{k}\Omega+200\text{k}\Omega}{1+100} = 2.98\text{k}\Omega \quad \rightarrow \quad R_{C2O}C_2 = 2.98\text{ns}$$

R_{C3O}: $\quad R_{C3O} = R_2 \| 1\text{k}\Omega = 0.99\text{k}\Omega \quad \rightarrow \quad R_{C3O}C_3 = 4.95\text{ns}$

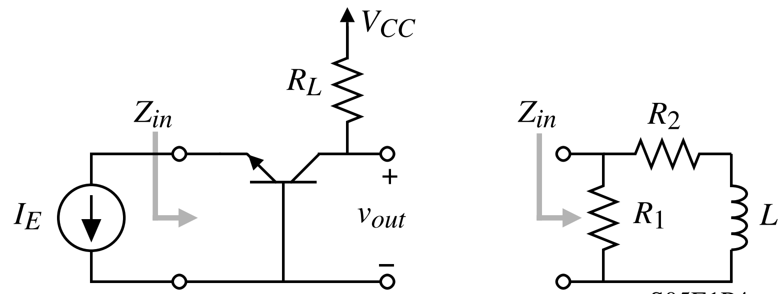
R_{C4O}: $\quad R_{C4O} = R_3 = 1\text{k}\Omega \quad \rightarrow \quad R_{C4O}C_4 = 10\text{ns}$

$$\Sigma T_{oc} = (10+2.98+4.95+10)\text{ns} = 27.93\text{ns}$$

$$\omega_{-3\text{dB}} \approx \frac{1}{\Sigma T_{oc}} = 35.8\text{x}10^6 \quad \rightarrow \quad f_{-3\text{dB}} = \underline{\underline{5.698\text{ MHz}}}$$

Problem 4 - (25 points)

On page 514 of the text, the statement is made that "the common base input impedance is low at low frequencies and becomes inductive at high frequencies"... Find the small-signal input impedance to the common base amplifier and express the values of the equivalent circuit, R_1, R_2, and L in terms of the parameters of the BJT small signal model (r_b, r_π, C_π, and β_o). Ignore r_o and assume that $R_1 > R_2$.



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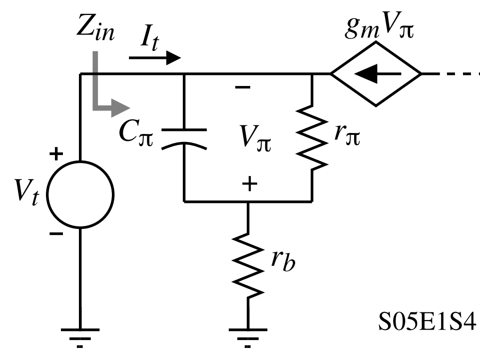
Solution

Use the following small signal model for this problem.



S05E1S4

$$I_t + \frac{V_\pi}{Z_\pi} + g_m V_\pi = 0 \ \rightarrow\ I_t = -V_\pi\left(g_m + \frac{1}{Z_\pi}\right)$$

and

$$V_t = -V_\pi - \frac{V_\pi}{Z_\pi} r_b \ \rightarrow\ V_t = -V_\pi\left(1 + \frac{r_b}{Z_\pi}\right)$$

$$\therefore\quad Z_{in} = \frac{V_t}{I_t} = \left(\frac{Z_\pi + r_b}{1 + g_m Z_\pi}\right) \quad \text{where} \quad Z_\pi = \frac{r_\pi}{sC_\pi r_\pi + 1}$$

Now,

$$Z_{in} = \frac{r_b + \dfrac{r_\pi}{sC_\pi r_\pi + 1}}{1 + \dfrac{g_m r_\pi}{sC_\pi r_\pi + 1}} = \frac{r_b(1 + sC_\pi r_\pi) + r_\pi}{1 + g_m r_\pi + sC_\pi r_\pi} = \frac{(r_b + r_\pi) + sC_\pi r_\pi r_b}{1 + \beta_o + sC_\pi r_\pi}$$

$$= \frac{\dfrac{(r_b + r_\pi)}{\beta_o} + \dfrac{sC_\pi r_\pi r_b}{\beta_o}}{1 + \dfrac{1}{\beta_o} + \dfrac{sC_\pi r_\pi}{\beta_o}} = \frac{\left(\dfrac{(r_b + r_\pi)}{\beta_o} + \dfrac{sC_\pi r_\pi r_b}{\beta_o}\right) r_b}{r_b + \dfrac{r_b}{\beta_o} + \dfrac{sC_\pi r_\pi r_b}{\beta_o}} \approx \frac{\left(\dfrac{(r_b + r_\pi)}{\beta_o} + \dfrac{sC_\pi r_\pi r_b}{\beta_o}\right) r_b}{r_b + \dfrac{sC_\pi r_\pi r_b}{\beta_o}}$$

$$Z_{in} = \frac{R_1(R_2 + sL)}{R_1 + R_2 + sL} \approx \frac{R_1(R_2 + sL)}{R_1 + sL} \quad \text{if } R_1 > R_2$$

Equating the two expressions for Z_{in} gives,

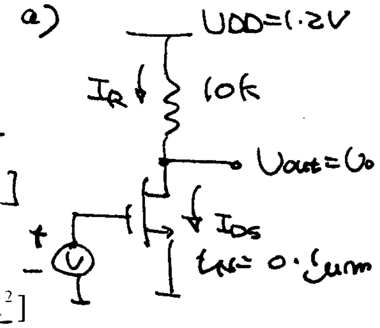
$$\boxed{R_1 = r_b,\ R_2 = \frac{(r_b + r_\pi)}{\beta_o},\ \text{and } L = \frac{C_\pi r_\pi r_b}{\beta_o}}$$

⟨HW#5 Solution⟩

Design the circuit for $V_{OL}=0.1V$, $L=0.1\mu m$ in $0.13\mu m$ technology.

Problem 2 – P4.9

a)



* Resistive Load inverter:

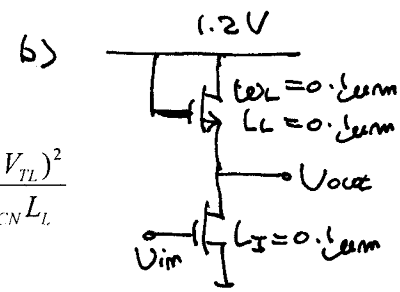
$E_CL=0.6V$
$V_{OH}=1.2V$
$V_{OL}=0.1V$
$C_{ox}=1.6\mu F/cm^2$
$\mu_n=270\ \frac{cm^2}{V\cdot s}$
$V_{sat}=8\times10^6$

$$\frac{V_{DD}-V_{OL}}{R_L}=\frac{W_N}{L_N}\frac{\mu_n C_{ox}}{\left(1+\frac{V_{OL}}{E_C L}\right)}\cancel{[2(V_{OH}-V_T)V_{OL}-V_{OL}^2]}\left[(V_{OH}-V_T)V_{OL}-\frac{V_{OL}^2}{2}\right]$$

$$\frac{1.2-0.1}{10k}=\frac{W_N}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]$$

$\therefore W_N=0.2\mu m$ $\boxed{W=0.398\mu m}$ ✓

b)



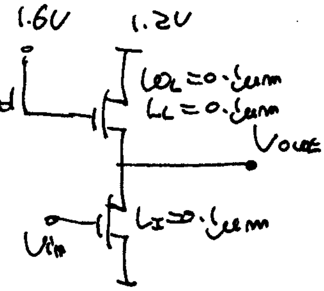
* Saturated Enhancement Load inverter (ignoring body-effect):

$I_I(lin)=I_L(sat)$

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[(V_{in}-V_{TI})V_{out}-\frac{V_{out}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]=\frac{0.1(10^{-4})(8)(1.6)(1.2-0.1-0.4)^2}{(1.2-0.1-0.4)+0.6}$$ ✓

$\therefore W_N=0.1\mu m$ $\boxed{W=0.174\mu m}$



* Linear Enhancement Load inverter (ignoring body-effect): Load is saturated

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[(V_{in}-V_{TI})V_{out}-\frac{V_{out}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]=\frac{0.1(10^{-4})(8)(1.6)(1.6-0.1-0.4)^2}{(1.6-0.1-0.4)+0.6}$$ ✓

$\therefore W_N=0.6\mu m$ $\boxed{W=0.328\mu m}$

The linear enhancement load inverter requires the largest pull-down device since it has the strongest pull up device. The resistive load inverter is next and the saturated enhancement load requires the smallest pull-down device.

Problem 3 – P4.10

We will illustrate the process and estimate the solutions for this problem.

We already know that V_{OH}=1.2 V and V_{OL}=0 V. For V_S use: