Problem 1 - (25 points)
An emitter follower, push-pull output stage is shown. Assume that $\beta_N = \beta_P = 100$, $V_t = 25\text{mV}$, and $I_s = 10\text{fA}$.

a.) If the emitter areas of Q1 and Q2 are $10\mu\text{m}^2$, find the emitter area of Q3 and Q4 so that the collector current in Q3 and Q4 is 1mA when $v_{IN} = v_{OUT} = 0$.

b.) What is the ±peak output voltage of this amplifier? Assume the 100µA sources can have a minimum voltage across them of 0.2V.

c.) What is the ±slew rate of this amplifier in $V/\mu\text{s}$?

d.) What is the small-signal input and output resistance of this amplifier when $v_{IN} = v_{OUT} = 0$? (Do not include the load resistance in the output resistance.)

Solution

a.) $V_{EB1} + V_{BE2} = V_{BE4} + V_{EB3} \rightarrow \frac{I_{C1}^2}{I_{s1} I_{s2}} = \frac{I_{C3}^2}{I_{s3} I_{s4}} \rightarrow I_{s3} = I_{s4} = \frac{I_{C3} I_{s1}}{I_{C1}} = 10I_{s1}$

$\therefore A_{E3} = A_{E4} = 10A_{E1} = \frac{100\mu\text{m}^2}{100\mu\text{m}^2}$

b.) $V_{peak} = \pm(100\mu\text{A})(1+\beta_o)R_L = \pm100\mu\text{A} \cdot 101 \cdot 100\Omega = \pm1.01\text{V}$

Check to make sure this answer is okay. $V_{BE4} = V_t \ln\left(\frac{10.1\text{mA}}{10\text{fA}}\right) = 0.691\text{V}$

$\therefore$ Maximum swing is $2 \cdot 0.691 - 0.2 = 1.109\text{V}$ so $V_{peak} = \pm1.01\text{V}$

c.) $\pm SR = \left(\frac{10.1\text{mA}}{100\mu\text{F}}\right) = 101\text{V/µs}$

d.) Small signal model:

$$V_t = \frac{I_t}{2(g_{m2} + r_{\pi4})} + R_L I_t (1+\beta_o)$$

$$g_{m2} = \frac{100\mu\text{A}}{25\text{mV}} = \frac{1}{250\Omega}$$

$$g_{m4} = \frac{1\text{mA}}{25\text{mV}} = \frac{1}{25\Omega}$$

$$R_{in} = 0.5[250 + 25(101)] + 101(100) = 11,487\text{k}\Omega$$

$$R_{out} = 0.5 \left[ \frac{1}{g_{m4}} + \frac{1/g_{m2}}{1+\beta_o} \right] = 0.5[25 + (250/101)] = 13.37\Omega$$
Problem 2 - (25 points)

Find the value for the small-signal output resistance $R_{out}$ ignoring $R_L$ and the value of the small-signal input resistance for the amplifier shown. Let the dc currents through $M1$ and $M2$ be 500µA, $W1/L1 = 100\mu m/1\mu m$ and $W2/L2 = 200\mu m/1\mu m$. Assume the parameters of the NMOS transistors are $K_N' = 110V/\mu A^2$, $V_{TN} = 0.7V$, and for the PMOS transistors are $K_P' = 50V/\mu A^2$, $V_{TP} = -0.7V$. Ignore $r_{ds1}$ and $r_{ds2}$.

Solution

Calculating the small-signal parameters gives,

$$g_{m1} = \sqrt{2 \cdot 110 \cdot 500 \cdot 100} = 3.316mS, \quad g_{m2} = \sqrt{2 \cdot 50 \cdot 500 \cdot 200} = 3.162mS$$

The small-signal model is given as,

For $R_{out}$, sum the currents at the output (with the LH $v_t = 0$) to get,

$$i_t = v_t \left[ \frac{1}{R_1 + R_2} + \frac{g_{m1} + g_{m2}}{2} \right] \quad \rightarrow \quad R_{out} = \frac{v_t}{i_t} = \frac{1}{\frac{1}{R_1 + R_2} + \frac{g_{m1} + g_{m2}}{2}} = \frac{1}{308\Omega}$$

For $R_{in}$, remove the RH $v_t$ and write a loop equation at the input to get,

$$v_t = i_t(R_1 + R_2) + (i_t - g_{m1}v_{gs} - g_{m2}v_{gs})R_L = i_t(R_1 + R_2 + R_L) - (g_{m1} + g_{m2})v_{gs}$$

But $v_{gs} = v_t - i_t R_1$ which gives,

$$R_{in} = \frac{v_t}{i_t} = \frac{R_1 + R_2 + R_L + (g_{m1} + g_{m2})R_L R_1}{1 + (g_{m1} + g_{m2})R_L} = \frac{201k\Omega + (3.316 + 3.162)(1)(100k\Omega)}{1 + (3.316 + 3.162)(1)}$$

$$R_{in} = 113.5k\Omega$$
Problem 3 - (25 points)

Find the midband voltage gain and the –3dB frequency in Hertz for the circuit shown.

Solution

The midband voltage gain can be expressed as,

\[
\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{2}} \frac{V_{2}}{V_{1}} \frac{V_{1}}{V_{in}} = (1) \left( \frac{-R_2}{R_2 + 1000} \right) \rightarrow \text{-0.99V/V}
\]

Finding the open-circuit, time constants:

\[R_{C1O} = R_1 = 1k\Omega \rightarrow R_{C1O}C_1 = 10\text{ns}\]

\[R_{C2O} = \frac{V_t}{i_t} = \frac{R_1 + R_2 + 0.002R_1R_2}{1 + 0.001R_2} = \frac{1k\Omega + 100k\Omega + 200k\Omega}{1 + 100} = 2.98k\Omega \rightarrow R_{C2O}C_2 = 2.98\text{ns}\]

\[R_{C3O} = R_2 || 1k\Omega = 0.99k\Omega \rightarrow R_{C3O}C_3 = 4.95\text{ns}\]

\[R_{C4O} = R_3 = 1k\Omega \rightarrow R_{C4O}C_4 = 10\text{ns}\]

\[\Sigma T_{oc} = (10 + 2.98 + 4.95 + 10)\text{ns} = 27.93\text{ns}\]

\[\omega_{3dB} = \frac{1}{\Sigma T_{oc}} = 35.8 \times 10^6 \rightarrow f_{3dB} = 5.698 \text{ MHz}\]
Problem 4 - (25 points)

On page 514 of the text, the statement is made that “the common base input impedance is low at low frequencies and becomes inductive at high frequencies”...

Find the small-signal input impedance to the common base amplifier and express the values of the equivalent circuit, $R_1$, $R_2$, and $L$ in terms of the parameters of the BJT small signal model ($r_b$, $r_\pi$, $C_\pi$, and $\beta_o$). Ignore $r_o$ and assume that $R_1 > R_2$.

Solution

Use the following small signal model for this problem.

\[
I_t + \frac{V_{\pi}}{Z_{\pi}} + g_m V_\pi = 0 \rightarrow I_t = -V_\pi \left( g_m + \frac{1}{Z_{\pi}} \right)
\]

and

\[
V_t = -V_\pi \left( \frac{V_{\pi}}{Z_{\pi}} r_b \right) \rightarrow V_t = -V_\pi \left( 1 + \frac{r_b}{Z_{\pi}} \right)
\]

\[Z_{in} = \frac{V_t}{I_t} = \left( \frac{Z_{\pi} + r_b}{1 + g_m Z_{\pi}} \right) \text{ where } Z_{\pi} = \frac{r_\pi}{sC_\pi r_\pi + 1}\]

Now,

\[
Z_{in} = \frac{r_b + \frac{r_\pi}{sC_\pi r_\pi + 1}}{1 + \frac{g_m R_{\pi}}{sC_\pi r_\pi + 1}} = \frac{r_b (1 + sC_\pi r_\pi) + r_\pi}{1 + \frac{g_m}{sC_\pi r_\pi} + sC_\pi r_\pi} = \frac{(r_b + r_\pi + sC_\pi r_\pi r_b) \beta_o + sC_\pi r_\pi r_b}{(r_b + r_\pi) \beta_o + sC_\pi r_\pi r_b}
\]

\[
= \frac{(r_b + r_\pi) \beta_o + sC_\pi r_\pi r_b}{1 + \frac{r_b + sC_\pi r_\pi r_b}{\beta_o}} = \frac{r_b + sC_\pi r_\pi r_b}{\beta_o}
\]

\[Z_{in} = \frac{R_1 (R_2 + sL)}{R_1 + R_2 + sL} \approx \frac{R_1 (R_2 + sL)}{R_1 + sL} \text{ if } R_1 > R_2
\]

Equating the two expressions for $Z_{in}$ gives,

\[
R_1 = r_b, \quad R_2 = \frac{(r_b + r_\pi) \beta_o}{\beta_o}, \text{ and } L = \frac{C_\pi r_\pi r_b}{\beta_o}
\]
Design the circuit for \( V_{OH} = 1.2 \text{ V} \) and \( V_{OL} = 0 \text{ V} \) in 0.1 \( \mu \text{m} \) technology.

**Problem 2 – P4.9**

* Resistive Load inverter:

\[
\frac{V_{DD} - V_{OL}}{R_L} = \frac{V_{OH}}{L}
\]

\[
= \frac{W_N}{L} \mu_{NC_{ox}} \left[ \frac{1}{1 + \frac{V_{OL}}{E_{iL}}} \right] \left[ \frac{2(V_{OH} - V_{OL})V_{OL}}{E_{iL}} \right]
\]

\[
= \frac{1.2 - 0.1}{10k} \frac{W_N (270)(1.6 \times 10^{-6})}{0.1} \frac{1}{1 + \frac{0.1}{0.6}} \frac{1}{1 + \frac{0.1}{0.6}} \frac{(1.2 - 0.4)0.1 - 0.1^2}{2}
\]

\[= \frac{0.2 \mu m}{1.2 V} \]

\[= 0.039 E_{\text{um}} \]

* Saturated Enhancement Load inverter (ignoring body-effect):

\[
I_L = \left( 1 + \frac{V_{out}}{E_{CN}L_i} \right)
\]

\[
= \frac{W_N}{L_i} \mu_{NC_{ox}} \left[ \frac{V_{in} - V_{TH}}{V_{out} - V_{TH}^2} \right]
\]

\[
= \frac{W_N (270)(1.6 \times 10^{-6})}{0.1} \frac{(1.2 - 0.4)0.1 - 0.1^2}{2} = \frac{0.1(10^{-4})(8)(1.6)(1.2 - 0.1 - 0.4)}{(1.2 - 0.1 - 0.4) + 0.6}
\]

\[= 0.174 E_{\text{um}} \]

* Linear Enhancement Load inverter (ignoring body-effect):

\[
I_L = \left( 1 + \frac{V_{out}}{E_{CN}L_i} \right)
\]

\[
= \frac{W_N}{L_i} \mu_{NC_{ox}} \left[ \frac{V_{in} - V_{TH}}{V_{out} - V_{TH}^2} \right]
\]

\[
= \frac{W_N (270)(1.6 \times 10^{-6})}{0.1} \frac{(1.2 - 0.4)0.1 - 0.1^2}{2} = \frac{0.1(10^{-4})(8)(1.6)(1.6 - 0.1 - 0.4)}{(1.6 - 0.1 - 0.4) + 0.6}
\]

\[= 0.38 E_{\text{um}} \]

The linear enhancement load inverter requires the largest pull-down device since it has the strongest pull-up device. The resistive load inverter is next and the saturated enhancement load requires the smallest pull-down device.

**Problem 3 – P4.10**

We will illustrate the process and estimate the solutions for this problem.

We already know that \( V_{OH} = 1.2 \text{ V} \) and \( V_{OL} = 0 \text{ V} \). For \( V_S \) use: