Problem 1 - (50 points)

The CMOS op amp shown uses a complementary differential input stages to achieve a wider input voltage common mode range. Assume that all transistors are scaled from a X1 NMOS and PMOS that have been designed to have a small-signal transconductance of 100µS and a channel conductance of 1µS at 25µA of current. Use what you have learned in class to give your best estimate of the slew rate (V/µs), output resistance, $R_{out}$, small-signal voltage gain ($v_{out}/v_{id}$), and the gainbandwidth, $GB$, in MHz.

Solution

The dc currents for $v_{id} = 0$ are shown above. One can show that the maximum amount of current available to the output capacitor is twice the 50µA current sink/source or 100µA. Therefore, the slew rate is $SR = 100µA/5pF = 20V/µs$.

The small-signal voltage gain can be written by inspection as (note the M13-M14-M17-M18 combination is used to recover the full differential output of both complementary input stages),

$$\frac{v_{out}}{v_{id}} = (g_{m1} + g_{m2})R_{out} \text{ where } g_{m1} = g_{m2} = 100µS$$

$$R_{out} \approx [(r_{ds9}||r_{ds4})g_{m11}r_{ds11}]||(r_{ds18}g_{m14}r_{ds14})||(r_{ds3}||r_{ds19})g_{m15}r_{ds15}]$$

Scaling $r_{ds}$ for the currents gives,

$r_{ds9} = 1000kΩ/6 = 166.7kΩ$, $r_{ds11} = 1000kΩ/5 = 200kΩ$,

$r_{ds18} = r_{ds14} = r_{ds19} = 1000kΩ/3 = 333.3kΩ$, $r_{ds15} = 1000kΩ/2 = 500kΩ$.
Problem 1 – Continued

Scaling $g_m$ for the currents gives,

\[ g_{m11} = \sqrt{5} \ 100\,\mu\text{S} = 223.6\,\mu\text{S}, \ g_{m14} = \sqrt{3} \ 100\,\mu\text{S} = 173\,\mu\text{S}, \ g_{m15} = \sqrt{2} \ 100\,\mu\text{S} = 141\,\mu\text{S} \]

\[ \therefore \ R_{out} \approx \ [(167\|1000)(0.224)(200\,\Omega)]\|[(333)(0.173)(333.3\,\text{k}\Omega)]\|[(333\|1000)(0.173)(500\,\text{k}\Omega)] \]

\[ R_{out} = 6.390\,\text{M}\Omega\|19.18\,\text{M}\Omega\|17.62\,\text{M}\Omega = 3.768\,\text{M}\Omega \]

Now,

\[ \frac{v_{out}}{v_{id}} = 200\,\mu\text{S}(3.768\,\text{M}\Omega) = 769 \ \text{V/V} \]

The gainbandwidth is,

\[ GB = \frac{g_{m11} + g_{m2}}{C_L} = \frac{200\,\mu\text{S}}{5\,\text{pF}} = 40 \times 10^6 \ \text{rads/sec or 6.28MHz} \]
**Problem 2 - (25 points)**

If a two-stage, Miller compensated CMOS op amp has a RHP zero at \(5GB\), a dominant pole due to the Miller compensation, and a second pole at \(-p_2\), find the value of the first stage transconductance \((g_mI)\), the second stage transconductance \((g_{mII})\), and the value of the Miller capacitor, \(C_c\), if \(GB = 10\text{MHz}\), the load capacitor is 10pF, and the phase margin is to be 50\(^\circ\). Assume that the unity gain magnitude frequency is \(GB\).

**Solution**

1.) The phase margin gives \(p_2\) which will give \(g_{mII}\).

\[
180^\circ - 90^\circ - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}(0.2) = 50^\circ \quad \rightarrow \quad \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 28.69^\circ
\]

\[
|p_2| = \frac{GB}{0.544} = \frac{20\pi\text{MHz}}{0.544} = 115.5 \times 10^6 \text{ rads/sec.}
\]

We know that,

\[
|p_2| = \frac{g_{mII}}{C_L} \quad \rightarrow \quad g_{mII} = |p_2|C_L = (115.5 \times 10^6 \text{ rads/sec.})(10\text{pF}) = 1.155\text{mS}
\]

2.) The Miller capacitor is found from the RHP zero location.

\[
\frac{g_{mII}}{C_c} = z_1 \quad \rightarrow \quad C_c = \frac{g_{mII}}{z_1} = \frac{1.155\text{mS}}{5GB} = \frac{1.115\text{mS}}{10\pi\times10^7} = 3.55\text{pF}
\]

3.) Finally, the input stage transconductance is given by,

\[
GB = \frac{g_{mI}}{C_c} \quad \rightarrow \quad g_{mI} = GB \cdot C_c = (2\pi\times10^7)(3.55\text{pF}) = 223\mu\text{S}
\]
Problem 3 - (25 points)

The CMOS equivalent of a 741 op amp input stage is shown. If the transistor model parameters are \( K_N' = 300\mu A/V^2, V_{TN} = 0.5V, \lambda_N = 0.02V^{-1} \) and \( K_P' = 70\mu A/V^2, V_{TP} = -0.5V, \lambda_P = 0.04V^{-1} \) find the numerical values of \( R_{i1}, G_{m1}, \) and \( R_{o1} \) for this input stage if all W/L’s of every transistor are 10.

Solution

The small-signal model for this problem is shown. First find the small-signal model parameters:

\[
g_{m1} = g_{m2} = \sqrt{2 \times 300 \times 10^{-15}} = 300\mu S
\]
\[
g_{m3} = g_{m4} = \sqrt{2 \times 70 \times 10^{-15}} = 145\mu S
\]
\[
r_{ds1} = r_{ds2} = r_{ds5} = r_{ds6} = 50/15\mu A = 3.33M\Omega \quad \text{and} \quad r_{ds3} = r_{ds4} = 25/15\mu A = 1.67M\Omega
\]

Summing currents:

\[g_{m1}v_{gs1} + g_{m3}v_{gs3} + \frac{v_{gs3}}{r_{ds1}} + \frac{g_{m3}v_{gs3}}{r_{ds3}} = 300v_{gs1} + 0.3v_{gs1} + 0.6v_{gs3} + 145v_{gs3} = 0\]

\[300.3v_{gs1} + 145.6v_{gs3} = 0 \quad \Rightarrow \quad v_{gs1} = -0.485v_{gs3}\]

Voltage loop through M1 and M3:

\[0.5g_{m1}v_{id} = v_{gs1} - v_{gs3} = -1.485v_{gs3} \quad \Rightarrow \quad v_{gs3} = -0.337v_{id}\]

\[i_{d3} = -g_{m3}v_{gs3} = 0.337 \times 145\mu S v_{id} = 48.82\mu S v_{id}\]

\[G_{m1}v_{id} = (i_{d3} + i_{d4}) = 97.65\mu S v_{id} \quad \therefore \quad G_{m1} = 97.65\mu S \quad R_{i1} = \infty\]

Output resistance:

\[R_{o1} = r_{ds6} || [(1/g_{m2})g_{m4}r_{ds4}] = 3.33M\Omega || 0.807M\Omega = 0.650M\Omega\]