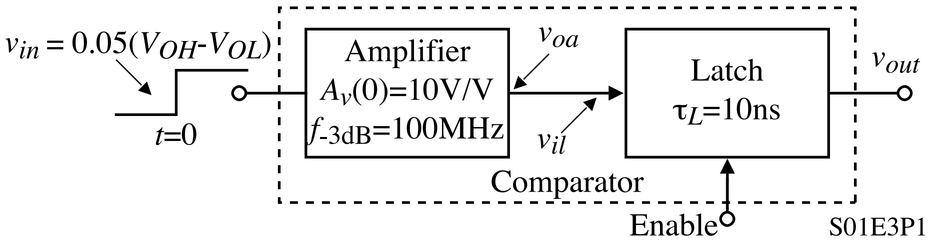
FINAL EXAMINATION - SOLUTIONS

(Average score = 89/100)

Problem 1 - (20 points - This problem is required)

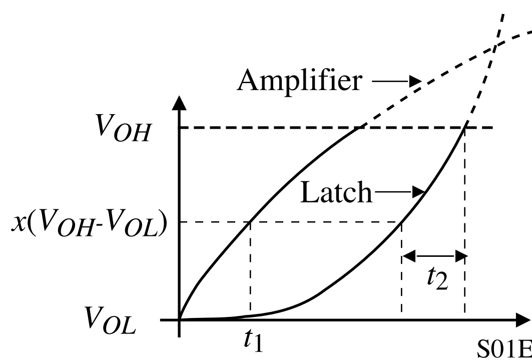
A comparator consists of an amplifier cascaded with a latch as shown below. The amplifier has a voltage gain of 10V/V and f_{-3dB} = 100MHz and the latch has a time constant of 10ns. The maximum and minimum voltage swings of the amplifier and latch are V_{OH} and V_{OL}. When should the latch be enabled after the application of a step input to the amplifier of $0.05(V_{OH}-V_{OL})$ to get minimum overall propagation time delay? What is the value of the minimum propagation time delay? It may useful to recall that the propagation time delay of the latch is given as $t_p = \tau_L \, ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right)$ where v_{il} is the latch input (ΔV_i of the text).



S01E3P1

Solution

The solution is based on the figure shown.



S01E3S1

We note that,

$$v_{oa}(t) = 10[1-e^{-\omega_{-3dB}t}]0.05(V_{OH}-V_{OL}).$$

If we define the input voltage to the latch as,

$$v_{il} = x\cdot(V_{OH}-V_{OL})$$

then we can solve for t_1 and t_2 as follows:

$$x\cdot(V_{OH}-V_{OL}) = 10[1-e^{-\omega_{-3dB}t_1}]0.05(V_{OH}-V_{OL}) \rightarrow x = 0.5[1-e^{-\omega_{-3dB}t_1}]$$

This gives,

$$t_1 = \frac{1}{\omega_{-3dB}} ln\left(\frac{1}{1-2x}\right)$$

From the propagation time delay of the latch we get,

$$t_2 = \tau_L \, ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right) = \tau_L \, ln\left(\frac{1}{2x}\right)$$

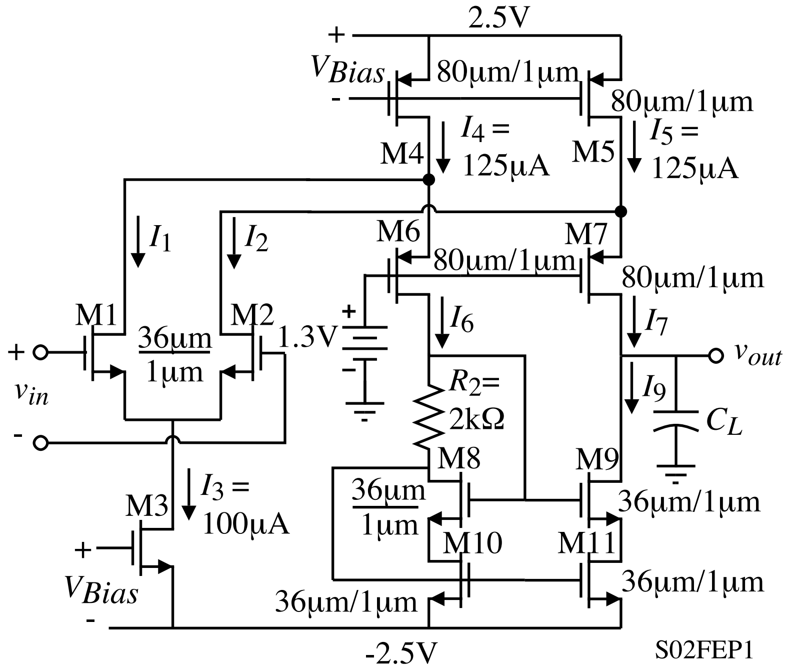
$$\therefore \quad t_p = t_1 + t_2 = \frac{1}{\omega_{-3dB}} ln\left(\frac{1}{1-2x}\right) + \tau_L \, ln\left(\frac{1}{2x}\right) \rightarrow \frac{dt_p}{dx} = 0 \text{ gives } x = \frac{\pi}{1+2\pi} = 0.4313$$

$$t_1 = \frac{10\text{ns}}{2\pi} ln\,(1+2\pi) = 1.592\text{ns}\cdot 1.9856 = \underline{\underline{3.16\text{ns}}} \text{ and } t_2 = 10\text{ns}\, ln\left(\frac{1+2\pi}{2\pi}\right) = 1.477\text{ns}$$

$$\therefore \quad t_p = t_1 + t_2 = 3.16\text{ns}+1.477\text{ns} = \underline{\underline{4.637\text{ns}}}$$

Problem 2 - (20 points - This problem is required)

If the folded-cascode op amp shown having a small-signal voltage gain of 7464V/V is used as a comparator, find the dominant pole if C_L = 5pF. If the input step is 10mV, determine whether the response is linear or slewing and find the propagation delay time. Assume the parameters of the NMOS transistors are K_N'=110V/μA^2, V_{TN} = 0.7V, λ_N=0.04V^{-1} and for the PMOS transistors are K_P'=50V/μA^2, V_{TP} = -0.7V, λ_P=0.05V^{-1}.



Solution

V_{OH} and V_{OL} can be found from many approaches. The easiest is simply to assume that V_{OH} and V_{OL} are 2.5V and –2.5V, respectively. However, no matter what the input, the values of V_{OH} and V_{OL} will be in the following range,

$$(V_{DD}\text{-}2V_{ON}) < V_{OH} < V_{DD} \quad \text{and} \quad V_{DD} < V_{OH} < (V_{SS}+2V_{ON})$$

The reasoning is as follows, suppose V_{in} >0. This gives I_1>I_2 which gives I_6<I_7 which gives I_9<I_7. V_{out} will increase until I_7 equals I_9. The only way this can happen is for M5 and M7 to leave saturation. The same reasoning holds for V_{in} <0.

Therefore assuming that V_{OH} and V_{OL} are 2.5V and –2.5V, respectively, we get

$$V_{in}(\text{min}) = \frac{5\text{V}}{7464} = 0.67\text{mV} \quad \rightarrow \quad k = \frac{10\text{mV}}{0.67\text{mV}} = 14.93$$

Problem 2 – Continued

The folded-cascode op amp as a comparator can be modeled by a single dominant pole. This pole is found as,

$$p_1 = \frac{1}{R_{out}C_L} \text{ where } R_{out} \approx g_{m9}r_{ds9}r_{ds11}||[g_{m7}r_{ds7}(r_{ds2}||r_{ds5})]$$

$g_{m9}=\sqrt{2\cdot 75\cdot 110\cdot 36} = 771\mu S$, $g_{ds9}=g_{ds11}=75\text{x}10^{-6}\cdot 0.04 = 3\mu S$, $g_{ds2}=50\text{x}10^{-6}(0.04)=2\mu S$

$g_{m7}=\sqrt{2\cdot 75\cdot 50\cdot 80} = 775\mu S$, $g_{ds5}=125\text{x}10^{-6}\cdot 0.05 = 6.25\mu S$, $g_{ds7}=50\text{x}10^{-6}(0.05)=3.75\mu S$

$$g_{m9}r_{ds9}r_{ds11} = (771\mu S)\left(\frac{1}{3\mu S}\right)\left(\frac{1}{3\mu S}\right) = 85.67M\Omega$$

$$g_{m7}r_{ds7}(r_{ds2}||r_{ds5}) \approx (775\mu S)\left(\frac{1}{3.75\mu S}\right)\left(\frac{1}{2\mu S}||\frac{1}{6.25\mu S}\right) = 25.05M\Omega\ ,$$

$$R_{out} \approx 85.67M\Omega||25.05M\Omega = 19.4M\Omega$$

The dominant pole is found as, $p_1 = \frac{1}{R_{out}C_L} = \frac{1}{19.4\text{x}10^6 5pF} = 10{,}318$ rps

The time constant is $\tau_1 = 96.9\mu s$.

For a dominant pole system, the step response is, $v_{out}(t) = A_{vd}(1-e^{-t/\tau_1})V_{in}$

The slope is the largest at $t = 0$. Evaluating this slope gives,

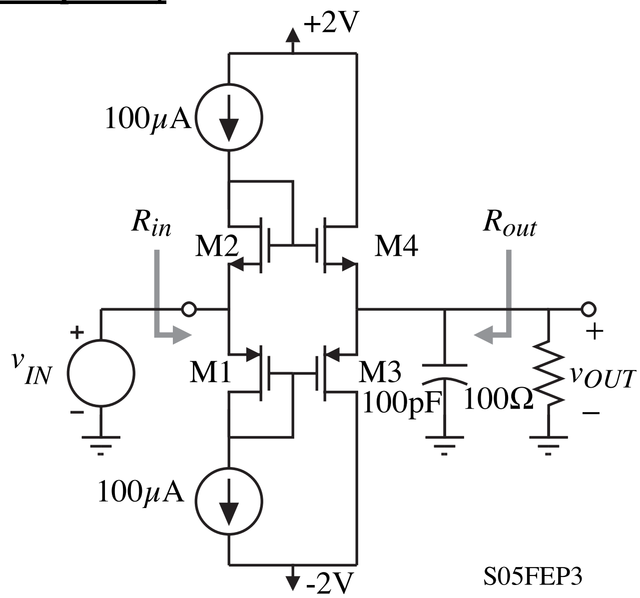
$\frac{dv_{out}}{dt} = \frac{A_{vd}}{\tau_1} e^{-t/\tau_1}V_{in}$ For $t = 0$, the slope is $\frac{A_{vd}}{\tau_1} V_{in} = \frac{7464}{96.9\mu s}(10mV) = 0.77V/\mu s$

The slew rate of this op amp/comparator is $SR = \frac{I_3}{C_L} = \frac{100\mu A}{5pF} = 20V/\mu s$

Therefore, the comparator does not slew and its propagation delay time is found from the linear response as,

$$t_P = \tau_1 \ln\left(\frac{2k}{2k-1}\right) = 96.9\mu s\cdot\ln\left(\frac{2\cdot 14.93}{2\cdot 14.93-1}\right) = (96.9\mu s)(0.0341) = \underline{\underline{3.3\mu s}}$$

Problem 3 – (20 points – This problem is optional)



An source follower, push-pull output stage is shown. Assume the parameters of the NMOS transistors are K_N'=110V/μA^2, V_{TN} = 0.7V, λ_N=0.04V^{-1} and for the PMOS transistors are K_P'=50V/μA^2, V_{TP} = -0.7V, λ_P=0.05V^{-1}.

a.) If $W_1/L_1 = W_2/L_2 = 10$, find the W_3/L_3 and W_4/L_4 so that the drain current in M3 and M4 is 1mA when $v_{IN} = v_{OUT} = 0$.

b.) What is the ±peak output voltage of this amplifier? Assume the 100µA sources can have a minimum voltage across them of 0.2V.

c.) What is the ±slew rate of this amplifier in V/µs?

d.) What is the small-signal input and output resistance of this amplifier when $v_{IN} = v_{OUT}$ = 0? (Do not include the load resistance in the output resistance.)

Solution

a.) With $v_{IN} = v_{OUT} = 0$, the W/L ratios of M3 and M4 are given by the current ratios. Thus, $W_3/L_3 = W_4/L_4 = \underline{\underline{100}}$.

b.) The current limit due to 1mA is ±1V. Check to see if voltage limit is less.

$$V_{GS4}(1\text{mA}) = \sqrt{\frac{2\cdot 1\text{mA}}{110\cdot 100}} + 0.7 = 0.426 + 0.7 = 1.126\text{V}$$

$$V_{out}(\text{max}) = 2 - 0.2 - 1.126 = \underline{\underline{0.674\text{V}}}$$

$$V_{GS3}(1\text{mA}) = \sqrt{\frac{2\cdot 1\text{mA}}{50\cdot 100}} + 0.7 = 0.632 + 0.7 = 1.332\text{V}$$

$$V_{out}(\text{min}) = 2 - 0.2 - 1.332 = \underline{\underline{0.467\text{V}}}$$

c.) The slew rate is

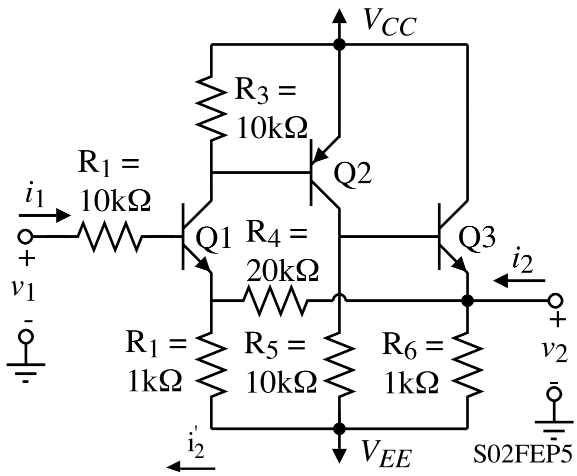
$$\pm SR = \frac{1\text{mA}}{100\text{pF}} = \underline{\underline{10\text{V}/\mu\text{s}}}$$

d.) $R_{in} = \infty$. $R_{out} = \dfrac{1}{g_{m3} + g_{m4}}$ $\qquad g_{m4} = \sqrt{2\cdot 110\cdot 1000\cdot 100}\ \mu\text{S} = 4.69\text{mS}$

$$g_{m3} = \sqrt{2\cdot 50\cdot 1000\cdot 100}\ \mu\text{S} = 3.162\text{mS} \qquad R_{out} = \frac{1000}{3.162 + 4.69} = \underline{\underline{127\Omega}}$$

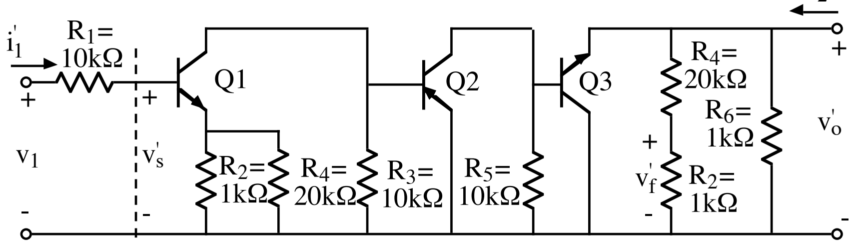
Problem 4 - (20 points - This problem is optional)

The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $\beta = 100$, $r_\pi = 5\text{k}\Omega$ and $r_o = \infty$.

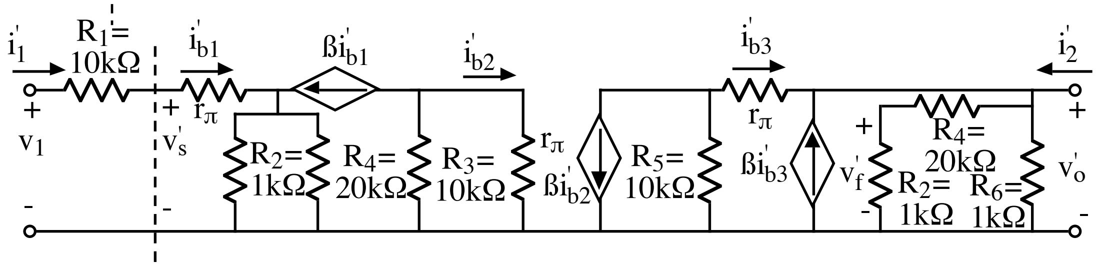


Solution

Open-loop, quasi-ac model:



Small-signal, open-loop model:



$$R_i = \frac{v_s'}{i_{b1}'} = r_\pi + (1+\beta)(R_2||R_4) = 5\text{k}\Omega + (101)(1\text{k}\Omega||20\text{k}\Omega) = 101.19\text{k}\Omega$$

$$a = \frac{v_o'}{v_s'} = \left(\frac{v_o'}{i_{b3}'}\right)\left(\frac{i_{b3}'}{i_{b2}'}\right)\left(\frac{i_{b2}'}{i_{b1}'}\right)\left(\frac{i_{b1}'}{v_s'}\right)$$

$$= (1+\beta)[R_6||(R_2+R_4)]\left(\frac{-\beta R_5}{R_5+r_\pi+(1+\beta)[R_6||(R_2+R_4)]}\right)\left(\frac{-\beta R_3}{R_3+r_\pi}\right)\left(\frac{1}{R_i}\right)$$

$$= [(101)(954.55)](-8.976)(-66.67)(1/101.19\text{k}\Omega) = 570.16 \text{ V/V}$$

$$f = \frac{R_2}{R_2+R_4} = \frac{1}{21} = 0.0476 \quad \text{and } R_o = \frac{v_o'}{i_2'} = R_6||(R_2+R_4)||\left(\frac{r_\pi+R_5}{1+\beta}\right) = 128.5\Omega$$

Closed-loop quantities are:

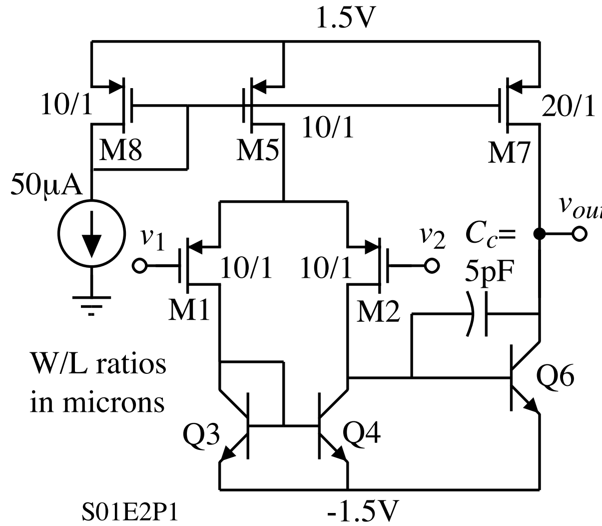
$$A_{vf} = \frac{v_o}{v_s} = \frac{a}{1+af} = \frac{570.16}{1+27.15} = 20.25,\ R_{if} = (1+af)R_i = 2.848\text{M}\Omega$$

$$\therefore \quad \boxed{R_{out} = R_{of} = \frac{R_o}{1+A_v\beta_f} = \frac{128.5\Omega}{28.15} = 4.565\Omega}$$

$$\boxed{R_{in} = R_1+R_{if} = 2.858\text{M}\Omega} \quad \text{and} \quad \boxed{\frac{v_2}{v_1} = A_{vf}\frac{R_{if}}{R_1+R_{if}} = 20.18 \text{ V/V}}$$

Problem 5 - (20 points - This problem is optional)

A two-stage, BiCMOS op amp is shown. For the PMOS transistors, the model parameters are K_P'=50μA/V^2, V_{TP} = -0.7V and λ_P = 0.05V^{-1}. For the NPN BJTs, the model parameters are β_F = 100, V_{CE}(sat) = 0.2V, V_A = 25V, V_t = 26mV, I_s = 10fA and n=1. (a.) Identify which input is positive and which input is negative. (b.) Find the numerical values of differential voltage gain, $A_v(0)$, GB (in Hertz), the slew rate, SR, and the location of the RHP zero. (c.) Find the numerical value of the maximum and minimum input common mode voltages.



Solution

(a.) The plus and minus signs on the schematic show which input is positive and negative.

(b.) The differential voltage gain, $A_v(0)$, is given as

$$A_v(0) = \frac{g_{m1}}{g_{ds2}+g_{o4}+g_{\pi 6}} \cdot \frac{g_{m6}}{g_{ds7}+g_{o6}} \qquad g_{m1} = g_{m2} = \sqrt{2\cdot 50\cdot 25\cdot 10} = 158.1\mu\text{S}$$

$$r_{ds2} = \frac{1}{\lambda_P I_D} = \frac{20}{25\mu\text{A}} = 0.8\text{M}\Omega,\ r_{o4} = \frac{V_A}{I_C} = \frac{25\text{V}}{25\mu\text{A}} = 1\text{M}\Omega,\ g_{m6} = \frac{I_C}{V_t} = \frac{100\mu\text{A}}{26\text{mV}} = 3846\mu\text{S}$$

$$r_{\pi 6} = \frac{\beta_F}{g_{m6}} = 26\text{k}\Omega, \quad r_{ds7} = \frac{1}{\lambda_P I_D} = \frac{20}{100\mu\text{A}} = 0.2\text{M}\Omega \text{ and } r_{o6} = \frac{V_A}{I_C} = \frac{25\text{V}}{100\mu\text{A}} = 0.25\text{M}\Omega$$

$$\therefore \quad |A_v(0)| = [158.1(0.8||1||0.026)][3846(0.2||0.25)] = 3.888\cdot 427.36 = \underline{\underline{1{,}659.6\text{V/V}}}$$

$$GB = \frac{g_{m1}}{C_c} = \frac{158.1\mu\text{S}}{5\text{pF}} = 31.62\text{x}10^6 \text{ rads/sec} \rightarrow \underline{\underline{GB = 5.0325\text{MHz}}}$$

$$SR = \frac{50\mu\text{A}}{5\text{pF}} = \underline{\underline{10\text{V}/\mu\text{s}}}$$

$$\text{RHP zero} = \frac{g_{m6}}{C_c} = \frac{3.846\text{mS}}{5\text{pF}} = \underline{\underline{769.24\text{x}10^6 \text{ rads/sec}}}. \text{ (122MHz)}$$

(c.) The maximum input common mode voltage is given as

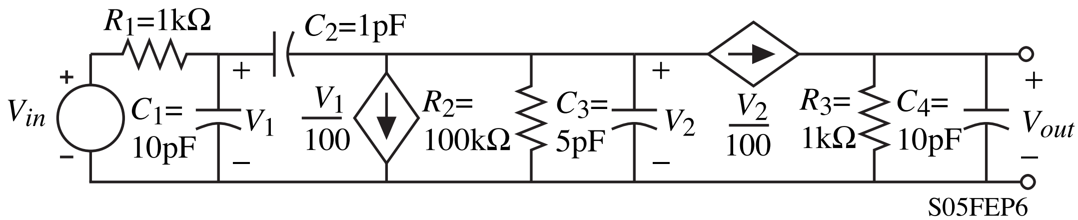
$$v_{icm}+ = V_{CC}\text{-}V_{DS5}(\text{sat}) - V_{SG1} = 1.5 - \sqrt{\frac{2\cdot 50}{50\cdot 10}} - 0.7 - \sqrt{\frac{2\cdot 25}{50\cdot 10}} = 0.8 - 0.447 - 0.316 =$$

$$\therefore \quad v_{icm}+ = \underline{\underline{0.0367\text{V}}}$$

$$v_{icm}- = -1.5 + V_{BE3} - V_{T1} = -1.5 + V_t \ln\left(\frac{25\mu\text{A}}{10\text{fA}}\right) - 0.7 = -2.2 + 0.5626 = \underline{\underline{-1.6374\text{V}}}$$

Problem 6 - (20 points - This problem is optional)

Find the midband voltage gain and the –3dB frequency in Hertz for the circuit shown.



Solution

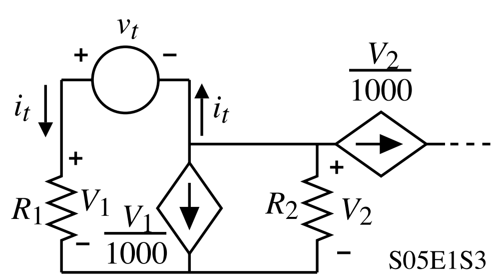
The midband voltage gain can be expressed as,

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_2}\frac{V_2}{V_1}\frac{V_1}{V_{in}} = (10)\left(\frac{-R_2}{R_2+1000}\right)(1) = \underline{\underline{-9.9\text{V/V}}}$$

Finding the open-circuit, time constants:

R_{C1O}: $R_{C1O} = R_1 = 1\text{k}\Omega \quad\rightarrow\quad R_{C1O}C_1 = 10\text{ns}$

R_{C2O}:



$$v_t = R_1 i_t + R_2\left[i_t + \frac{V_1}{100} + \frac{V_2}{100}\right]$$

But $v_t = V_1 - V_2$ and $V_1 = R_1 i_t$,

$$\therefore \quad v_t = R_1 i_t + R_2 i_t + \frac{2R_1R_2 i_t}{100} - \frac{R_2 v_t}{100}$$

$$R_{C2O} = \frac{v_t}{i_t} = \frac{R_1+R_2+0.02R_1R_2}{1+0.01R_2}$$

$$= \frac{1\text{k}\Omega+100\text{k}\Omega+2000\text{k}\Omega}{1+1000} = 2.099\text{k}\Omega \quad\rightarrow\quad R_{C2O}C_2 = 2.1\text{ns}$$

R_{C3O}: $R_{C3O} = R_2 \| 100\Omega = 99.9\Omega \quad\rightarrow\quad R_{C3O}C_3 = 0.5\text{ns}$

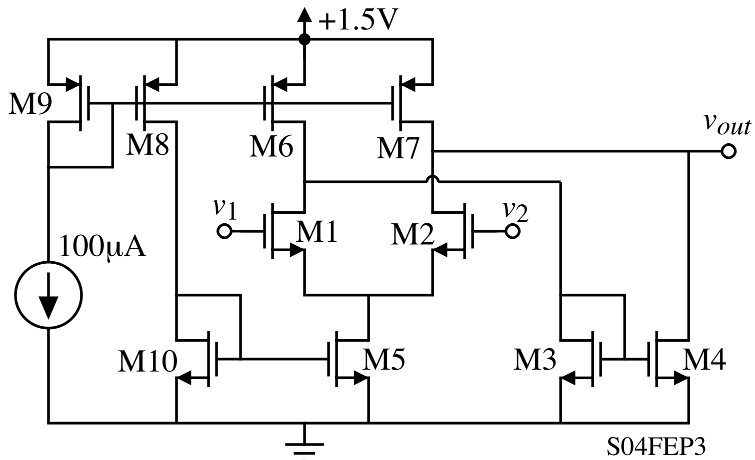
R_{C4O}: $R_{C4O} = R_3 = 1\text{k}\Omega \quad\rightarrow\quad R_{C4O}C_4 = 10\text{ns}$

$$\Sigma T_{oc} = (10+2.1+0.5+10)\text{ns} = 22.6\text{ns}$$

$$\omega_{-3\text{dB}} \approx \frac{1}{\Sigma T_{oc}} = 44.25\text{x}10^6 \quad\rightarrow\quad f_{-3\text{dB}} = \underline{7.04\text{ MHz}}$$

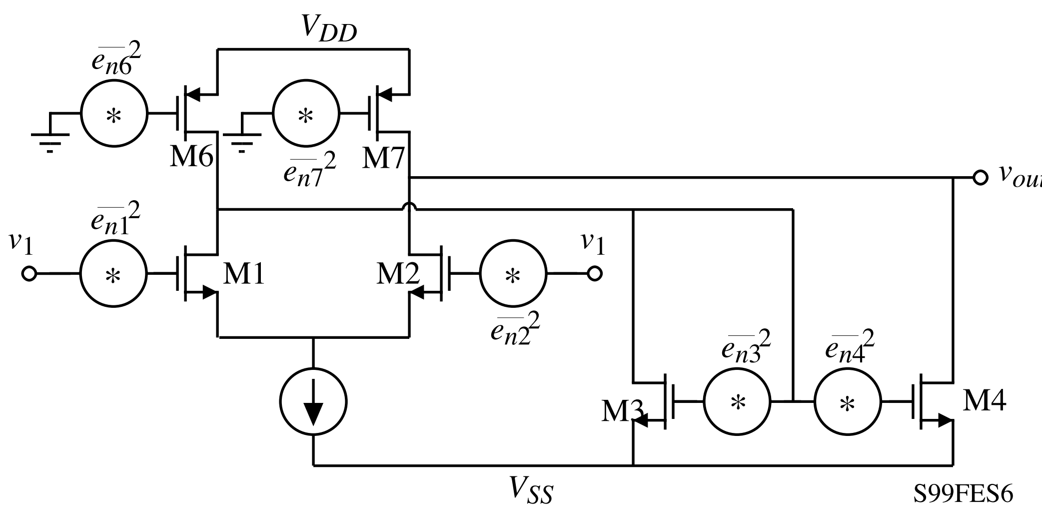
<u>Problem 7 – (20 points – This problem is optional)</u>

Find an expression for the equivalent input noise voltage of the circuit in the previous problem, $\overline{e^2_{eq}}$, in terms of the small signal model parameters and the individual equivalent input noise voltages, $\overline{e^2_{ni}}$, of each of the transistors (i = 1 through 7). Assume M1 and M2, M3 and M4, and M6 and M7 are matched.



<u>Solution</u>

Equivalent noise circuit:



$$\overline{e^2_{out}} = (g_{m1}{}^2\overline{e^2_{n1}} + g_{m2}{}^2\overline{e^2_{n2}} + g_{m3}{}^2\overline{e^2_{n3}} + g_{m4}{}^2\overline{e^2_{n4}} + g_{m6}{}^2\overline{e^2_{n6}} + g_{m7}{}^2\overline{e^2_{n7}})R_{out}{}^2$$

$$\overline{e^2_{eq}} = \frac{\overline{e^2_{out}}}{(g_{m1}R_{out})^2} = \overline{e^2_{n1}} + \overline{e^2_{n2}} + \left(\frac{g_{m3}}{g_{m1}}\right)^2(\overline{e^2_{n3}} + \overline{e^2_{n4}}) + \left(\frac{g_{m6}}{g_{m1}}\right)^2(\overline{e^2_{n6}} + \overline{e^2_{n7}})$$

If M1 through M2 are matched then $g_{m1} = g_{m2}$ and we get

$$\overline{e^2_{eq}} = 2\overline{e^2_{n1}} + 2\left(\frac{g_{m3}}{g_{m3}}\right)^2\overline{e^2_{n3}} + 2\left(\frac{g_{m6}}{g_{m1}}\right)^2\overline{e^2_{n6}}$$