

Homework No. 1 - Solutions

Problem 1 - (10 points)

A top view of a MOS transistor is shown. (a) Identify the type of transistor (NMOS or PMOS) and its value of W and L .

(b.) Draw the cross-section A-A' approximately to scale.

(c) Assume that dc voltage of terminal 1 is 5V, terminal 2 is 3V and terminal 3 is 0V. Find the numerical value of the capacitance between terminals 1 and 2, 2 and 3, and 1 and 3. Assume that the dc value of the output voltage is 2.5V and that the voltage dependence for pn junction capacitances is for both transistors is -0.5 (this is called MJ in SPICE).

Solution

(a.) This transistor is an NMOS transistor with the drain as terminal 1, the gate as terminal 2, and the bulk and source connected together to terminal 3. The $W = 13\mu\text{m}$ and $L = 2\mu\text{m}$.

(b.) The approximate cross-section is shown (vertical scale is magnified by 4 times).

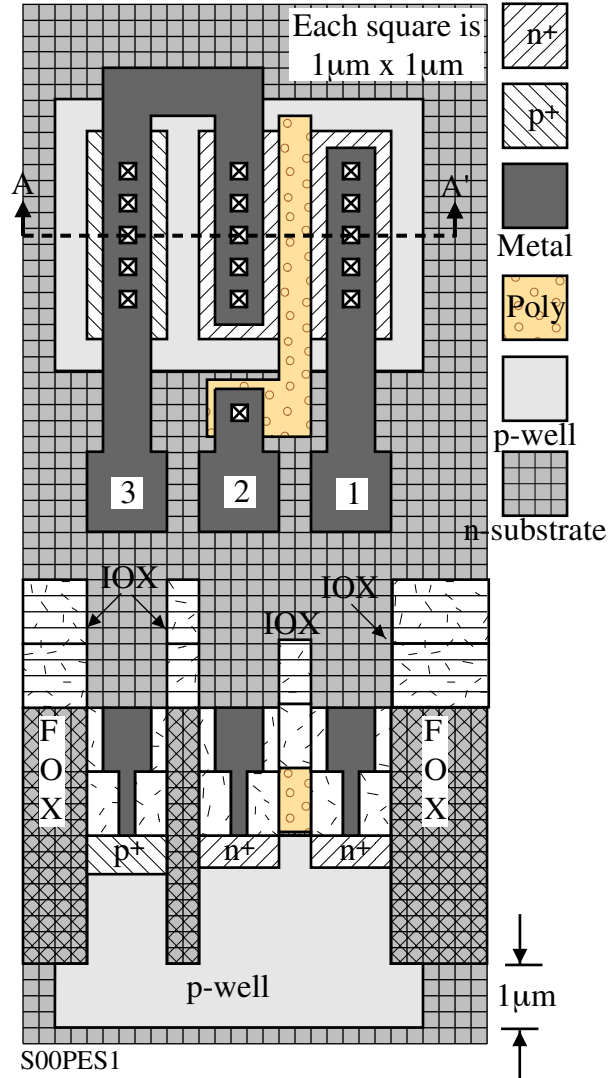
(c.) With $V_{DS} = 5\text{V}$, $V_{GS} = 3\text{V}$ and $V_T = 0.75\text{V}$, the transistor is in saturation. Therefore, the capacitors are:

$$C_{12} = C_{GD} = LD(\text{NMOS}) \times W \times C_{ox} \\ = 0.45\mu\text{m} \cdot 13\mu\text{m} \cdot 0.7\text{fF}/\mu\text{m}^2 = \underline{4.095\text{fF}}$$

$$C_{23} = C_{GS} = LD(\text{NMOS}) \times W \times C_{ox} + 0.67(W \times L) \times C_{ox} = 4.095\text{fF} + 12.133\text{fF} \\ = \underline{16.228\text{fF}}$$

C_{13} requires the area of the drain (AD) and the perimeter of the drain (PD). These values are $AD = 13\mu\text{m} \times 5\mu\text{m} = 65\mu\text{m}^2$ and $PD = 2(5+13) = 36\mu\text{m}$.

$$C_{13} = C_{BD} = \frac{[AD \cdot 0.33\text{fF}/\text{m}^2 + PD \cdot 0.9\text{fF}/\mu\text{m}]}{\sqrt{1 + \frac{5}{0.6}}} = \frac{[65\mu\text{m}^2 \cdot 0.33\text{fF}/\text{m}^2 + 36\mu\text{m} \cdot 0.9\text{fF}/\mu\text{m}]}{\sqrt{1 + \frac{5}{0.6}}} \\ = \underline{17.63\text{fF}}$$



Problem 2 - (10 points)

Find the numerical values of I_1 , I_2 , V_D , V_E , and V_C to within $\pm 5\%$ accuracy.

Solution

First find I_1 . This is done by solving the equations $I_1 = \frac{K'W}{2L} (V_{GS4} - V_T)^2$

and $5V = I_1 100k\Omega + V_{GS4}$

Solving quadratically gives

$$V_{GS4}^2 - V_{GS4} \left(2V_T - \frac{1}{12} \right) + \left(V_T^2 - \frac{5}{12} \right) = 0$$

$$V_{GS}^2 - 1.41667V_{GS} + 0.145833 = 0$$

This gives $V_{GS} = 0.708335 \pm 0.5965 = 1.305V \quad \therefore V_D = -2.5 + 1.305 = \underline{\underline{-1.195V}}$

This value of V_{GS} gives $I_1 = \frac{5 - 1.195}{100k\Omega} = \underline{\underline{36.95\mu A}}$

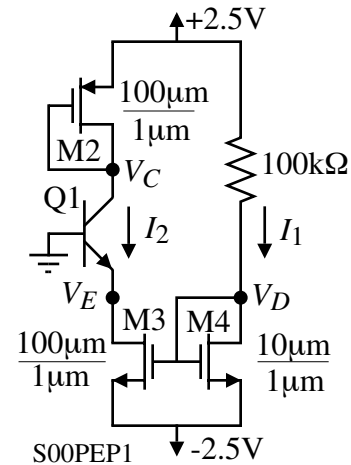
Neglecting the lambda effects, let $I_2 = 10I_1 = \underline{\underline{369.5\mu A}}$

The base-emitter voltage of Q1 is found as

$$V_E = -V_{BE1} = -V_T \ln \left(\frac{I_2}{I_s} \right) = -0.026 \ln \left(\frac{369.5\mu A}{10fA} \right) = \underline{\underline{-0.633V}}$$

Finally, the value of $V_{GS2} = \sqrt{\frac{2I_2}{K'W_2/L_2}} + V_T = \sqrt{\frac{2 \cdot 369.5}{800}} + 0.75 = 1.711V$

$\therefore V_C = 2.5V - 1.711V = \underline{\underline{+0.7889V}}$

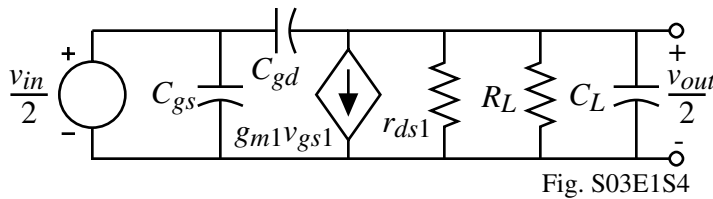
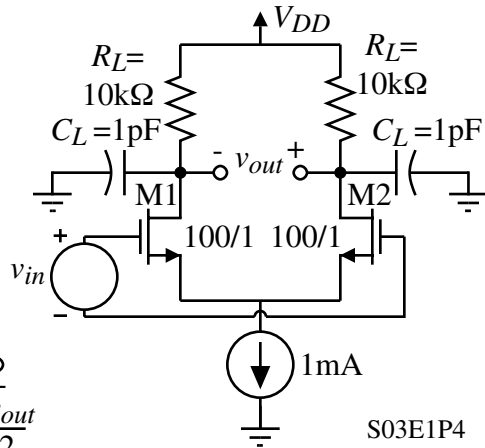


Problem 3

Find the numerical values of all roots and the midband gain of the transfer function v_{out}/v_{in} of the differential amplifier shown. Assume that $K_N' = 110\mu\text{A}/\text{V}^2$, $V_{TN} = 0.7\text{V}$, and $\lambda_N = 0.04\text{V}^{-1}$. The values of $C_{gs} = 0.2\text{pF}$ and $C_{gd} = 20\text{fF}$.

Solution

A small-signal model appropriate for this circuit is shown.



Summing the currents at the output nodes gives,

$$g_{m1}v_{gs1} + sC_{gd}(v_{out}-v_{in}) + (g_{ds1} + G_L)v_{out} + sC_L v_{out} = 0$$

(Note: we are ignoring the fact that v_{out} and v_{in} should be divided by two since it makes no difference in the results and is easier to write.) Replacing v_{gs1} by v_{in} gives

$$-(g_{m1} - sC_{gd})v_{in} = [(g_{ds1} + G_L) + sC_L + sC_{gd}] v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{-(g_{m1} - sC_{gd})}{s(C_L + C_{gd}) + (g_{ds1} + G_L)} = \left(\frac{-g_{m1}}{g_{ds1} + G_L} \right) \left(\frac{1 - \frac{sC_{gd}}{g_m}}{1 + s \frac{C_L + C_{gd}}{g_{ds1} + G_L}} \right)$$

$$\therefore \text{MGB} = -g_{m1}(r_{ds} \parallel R_L), \quad \text{Zero} = \frac{g_m}{C_{gd}} \quad \text{and} \quad \text{Pole} = -\frac{g_{ds} + G_L}{C_{gd} + C_L}$$

$$g_m = \sqrt{2 \cdot 110 \cdot 100 \cdot 500} = 3316.7\mu\text{S} \quad \text{and} \quad r_{ds} = \frac{1}{\lambda I_D} = \frac{25}{500\mu\text{A}} = 50 \text{ k}\Omega$$

$$\therefore \text{MGB} = -3.3167\text{mS} \cdot (10\text{k}\Omega \parallel 50\text{k}\Omega) = \underline{\underline{-27.64 \text{ V/V}}}$$

$$\text{Zero} = \frac{3.3167 \times 10^{-3}}{20 \times 10^{-15}} = \underline{\underline{1.658 \times 10^{11} \text{ radians/sec.}}}$$

$$\text{Pole} = \frac{-1}{1.02 \times 10^{-12} (10\text{k}\Omega \parallel 50\text{k}\Omega)} = \underline{\underline{-1.1176 \times 10^8 \text{ radians/sec.}}}$$

Problem 4

Find the voltage transfer function of the common-gate amplifier shown. Identify the numerical values of the small-signal voltage gain, v_{out}/v_{in} , and the poles and zeros. Assume that $I_D = 500\mu\text{A}$, $K_N' = 100\mu\text{A}/\text{V}^2$, $V_{TN} = 0.5\text{V}$, and $K_P' = 50\mu\text{A}/\text{V}^2$, $V_{TP} = -0.5\text{V}$, $\lambda \approx 0\text{V}^{-1}$, $C_{gs} = 0.5\text{pF}$ and $C_{gd} = 0.1\text{pF}$.

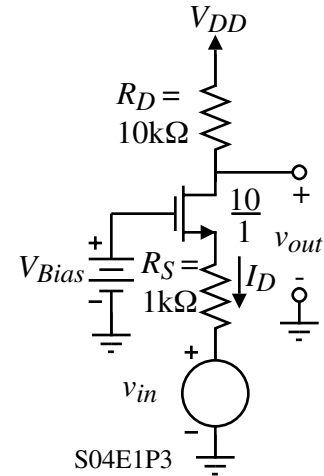
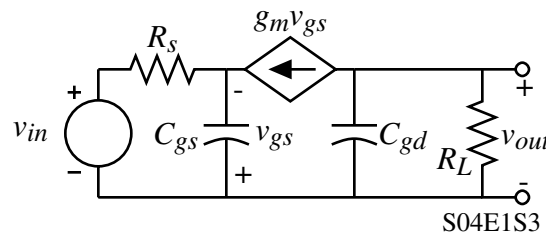
Solution

The small signal transconductance is,

$$g_m = \sqrt{2 \cdot K_N' \cdot (W/L) I_D} = \sqrt{2 \cdot 100 \cdot 10 \cdot 500} = 1\text{mS}$$

$$r_{ds} = \infty$$

The small signal model is,



The voltage gain can be expressed as follows,

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_{gs}} \right) \left(\frac{V_{gs}}{V_{in}} \right), \quad \frac{V_{out}}{V_{gs}} = -g_m \left(\frac{R_L (1/sC_{gd})}{R_L + (1/sC_{gd})} \right)$$

Sum currents at the source to get,

$$\frac{V_{in} + V_{gs}}{R_s} + g_m V_{gs} + sC_{gs} V_{gs} = 0 \quad \rightarrow \quad \frac{V_{gs}}{V_{in}} = \frac{-G_s}{G_s + g_m + sC_{gs}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \left(\frac{g_m R_L}{1 + g_m R_L} \right) \left(\frac{1}{sC_{gd} R_L + 1} \right) \left(\frac{1}{\frac{sC_{gs}}{g_m + G_s} + 1} \right)$$

The various values are,

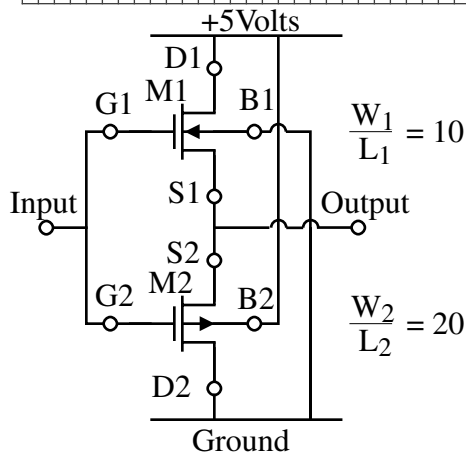
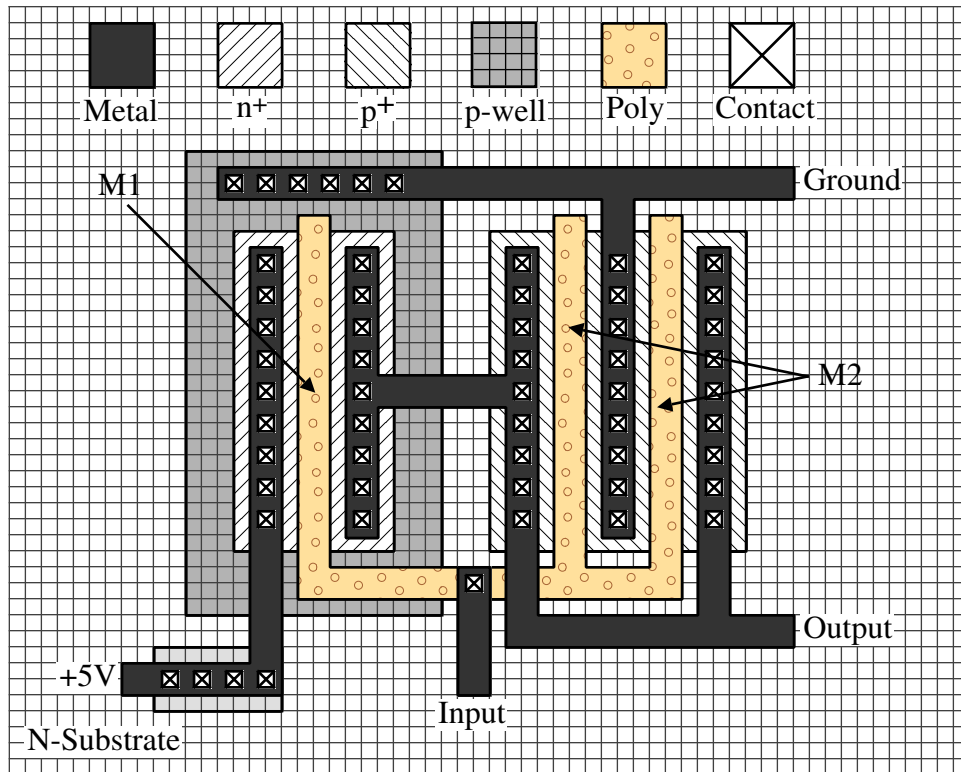
$$\text{Voltage gain} = \frac{g_m R_L}{1 + g_m R_L} = \frac{1 \cdot 10}{1 + 1} = \underline{\underline{5\text{V/V}}}$$

$$p_1 = \frac{-1}{C_{gd} R_L} = \frac{-1}{10^{-13} \cdot 10^4} = \underline{\underline{-10^9 \text{ radians/sec.}}}$$

$$p_2 = \frac{-(g_m + G_s)}{C_{gs}} = \frac{-10^{-3} + 10^{-3}}{0.5 \times 10^{-12}} = \underline{\underline{-4 \times 10^9 \text{ radians/sec.}}}$$

Problem 5

Draw the electrical schematic using the proper symbols for the transistors. Identify on your schematic the terminals which are +5V, ground, input, and output. Label the transistors on the layout as M1, M2, etc. and determine their W/L values. Assume each square in the layout is 1 micron by 1 micron. Find the area in square microns and periphery in microns for the source and drain of each transistor.



$$\frac{W_1}{L_1} = 10$$

$$A_{S1} = A_{D1} = 20 \times 4 = 80 \mu\text{m}^2$$

$$P_{S1} = P_{D1} = 4 + 4 + 20 + 20 = 48 \mu\text{m}^2$$

$$\frac{W_2}{L_2} = 20$$

$$A_{S2} = 2A_{S1} = 160 \mu\text{m}^2$$

$$A_{D2} = A_{D1} = 80 \mu\text{m}^2$$

$$P_{S2} = 2P_{S1} = 96 \mu\text{m}^2$$

$$P_{D2} = P_{D1} = 48 \mu\text{m}^2$$