Homework Assignment No. 2 - Solutions

Problem 1 - (10 points)

a) For the emitter follower output stage shown below, find the value of R_1 for maximum efficiency and find the value of that efficiency. $V_{CC}=-V_{EE}=2.5V$, $V_{CE}(sat)=0.2V$, $R_L=10k\Omega$, $V_{BE}(on)=0.7V$.

b) The load resistance R_L is replaced with a capacitor of 100pF. If the input voltage suddenly drops from 2.5V to -2.5V, explain what happens at the output and accurately sketch the output voltage as a function of time, specifying its initial and final values and time interval.

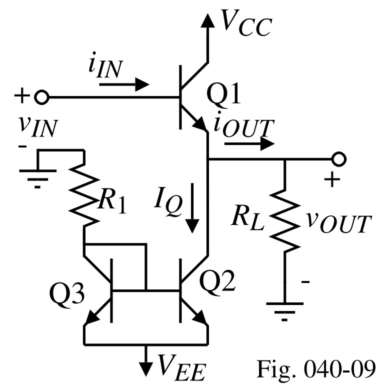


Fig. 040-09

Solution

The I_Q for maximum efficiency is found as,

$$I_Q=\left(\frac{V_{CC}-V_{CE}(sat)}{R_L}\right)=230\mu A$$

$$R_1=\left(\frac{-V_{EE}-V_{BE}}{I_Q}\right)=7.826k\Omega$$

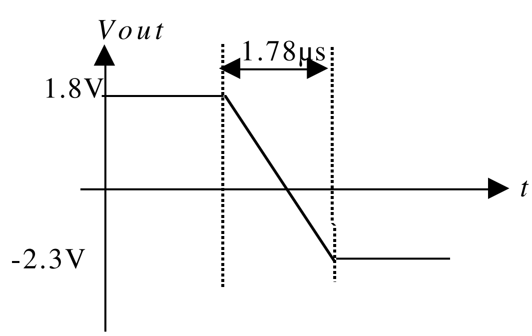
$$P_L(\max)=\left(\frac{V_{CC}-V_{CE}(sat)}{\sqrt{2}}\right)\left(\frac{I_Q}{\sqrt{2}}\right)=0.5(2.3V)(0.23mA)=0.2645mW$$

$$P_{supply}=2V_{CC}I_Q=2(2.5)(0.23mA)=1.15mW$$

$$\eta=\frac{P_{L(\max)}}{P_{\sup ply}}=\frac{1}{4}\left(1-\frac{V_{CE(sat)}}{V_{CC}}\right)=23\%$$

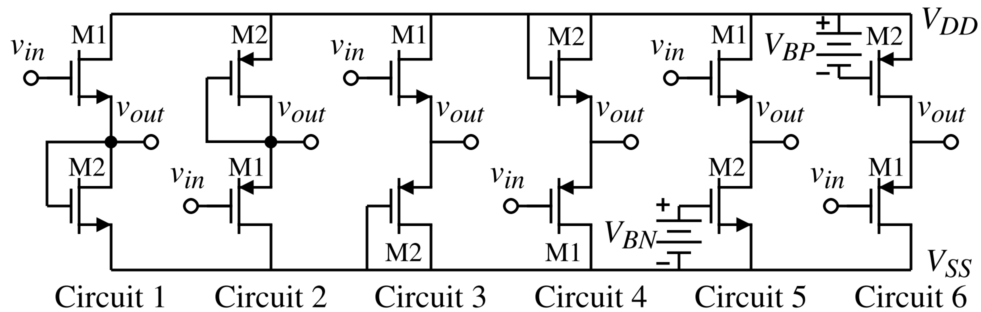
b) The output would slew under such condition. The current will be limited by the bias current:

Slew rate=0.23mA/100pF=2.3V/µs



Problem 2 - (10 points)

Six versions of a source follower are shown below. Assume that $K'_N = 2K'_P$, $\lambda_P = 2\lambda_N$, all W/L ratios of all devices are equal, and that all bias currents in each device are equal. Neglect bulk effects in this problem and assume no external load resistor. Identify which circuit or circuits have the following characteristics: (a.) highest small-signal voltage gain, (b.) lowest small-signal voltage gain, (c.) the highest output resistance, (d.) the lowest output resistance, (e.) the highest v_{out}(max) and (f.) the lowest v_{out}(max).



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Solution

(a.) and (b.) - Voltage gain. Small signal model:

The voltage gain is found as: $\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m+G_L}$

where G_L is the load conductance. Therefore we get:

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{g_{mN}}{g_{mN}+g_{mN}}$	$\frac{g_{mP}}{g_{mP}+g_{mP}}$	$\frac{g_{mN}}{g_{mN}+g_{mP}}$	$\frac{g_{mP}}{g_{mP}+g_{mN}}$	$\frac{g_{mN}}{g_{mN}+g_{dsN}+g_{dsN}}$	$\frac{g_{mP}}{g_{mP}+g_{dsP}+g_{dsP}}$

But $g_{mN} = \sqrt{2}\, g_{mP}$ and $g_{dsN} = 0.5 g_{dsP}$, therefore

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{1}{2}$	$\frac{1}{2}$	0.5858	0.4142	$\frac{g_{mP}}{g_{mP}+(g_{dsN}+g_{dsN})/\sqrt{2}}$	$\frac{g_{mP}}{g_{mP}+g_{dsP}+g_{dsP}}$

Thus, circuit 5 has the highest gain and circuit 4 the lowest gain

(c.) and (d.) - Output resistance.

The denominators of the first table show the following:

Ckt.6 has the highest output resistance and Ckt. 1 the lowest output resistance.

(e.) Assuming no current has to be provided by the output, circuits 2, 4, and 6 can pull the output to V_{DD}. ∴ **Circuits 2, 4 and 6 have the highest output swing.**

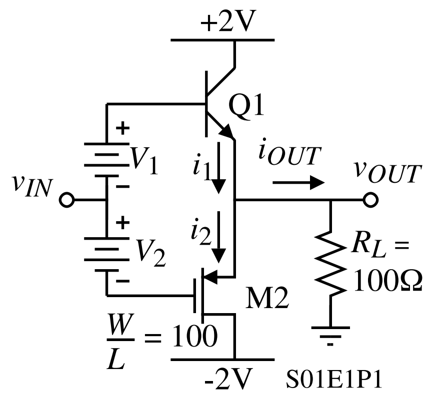
(f.) Assuming no current has to be provided by the output, circuits 1, 3, and 5 can pull the output to ground. ∴ **Circuits 1, 3 and 5 have lowest output swing.**

Summary

(a.) Ckt. 5 has the highest voltage gain
(b.) Ckt. 4 has the lowest voltage gain
(c.) Ckt. 6 has the highest output resistance
(d.) Ckt. 1 has the lowest output resistance
(e.) Ckts. 2,4 and 6 have the highest output
(f.) Ckts. 1,3 and 5 have the lowest output

Problem 3 - (10 points)

A push-pull follower is shown which uses an NPN BJT and a p-channel MOSFET. In this problem, ignore the bulk effect, the channel length modulation, and the Early voltage. The parameters for the NPN BJT are β_F = 100, I_s = 10fA and V_t = 25.9mV. The model parameters for the PMOS are K_P' = 50μA/V^2 and V_T = -0.7V. (a.) Find the value of the dc batteries, V_1 and V_2, which will cause 100μA to flow in Q1 and M2 when the dc value of v_{IN} = 0VDC. (b.) Find the small-signal input resistance, output resistance and voltage gain when the dc value of v_{IN} = 0VDC.

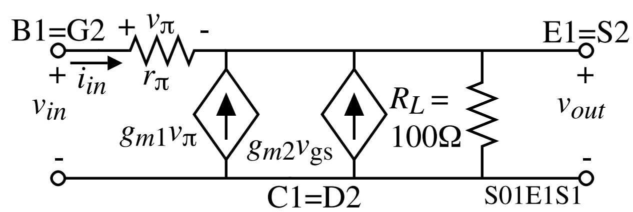


Solution

(a.) $V_1 = V_{BE1} = V_t \ln\left(\frac{i_C}{I_s}\right) = 0.0259\ln\left(\frac{100\mu A}{10fA}\right) = 0.5964V$ _ $\boxed{V_1 = 0.5964V}$

$V_2 = V_{SG2} = \sqrt{\frac{2I_D}{K_P'(W/L)}} + |V_{TP}| = \sqrt{\frac{2\cdot 100}{50\cdot 100}} + 0.7 = 0.9V$ __ $\boxed{V_2 = 0.9V}$

(b.) Small-signal model (simplified):



$g_{m1} = \frac{I_{C1}}{V_t} = \frac{100\mu A}{25.9mV} = 3.86mS$

$r_{\pi 1} = \frac{1+\beta_F}{g_{m1}} = 26.159k\Omega$

$g_{m2} = \sqrt{\frac{2K_P' W_2 I_{D2}}{L_2}} = \sqrt{2\cdot 50\cdot 100\cdot 100} = 1mS$

R_{in}: $v_{in} = r_{\pi 1} i_{in} + (i_{in} + g_{m1}v_\pi + g_{m2}\, v_{gs2})R_L = r_{\pi 1} i_{in} + (i_{in} + g_{m1} r_{\pi 1} i_{in} + g_{m2}\, r_{\pi 1} i_{in})R_L$

$$R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi 1} + R_L + g_{m1} r_{\pi 1} R_L + g_{m2}\, r_{\pi 1} R_L = r_{\pi 1} + R_L(1+\beta_F) + g_{m2}\, r_{\pi 1} R_L$$

$\therefore R_{in} = 26.159k\Omega + 101\cdot 100\Omega + 1\cdot 26.159k\Omega \cdot 0.1 = 38.875k\Omega$ $\boxed{R_{in} = 38.875k\Omega}$

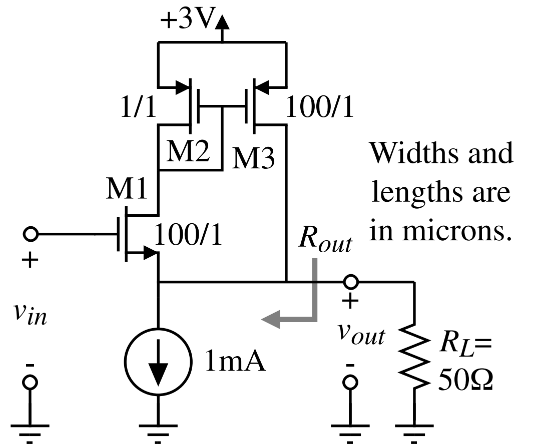
R_{out}: $R_{out} = \frac{1}{g_{m1}} || \frac{1}{g_{m2}} = \frac{1}{3.86mS+1mS} = 205.8k$_ __$\boxed{R_{out} = 205.8kW}$

$\frac{v_{out}}{v_{in}}$: $\frac{v_{out}}{v_{in}} = \frac{v_{out}}{i_{in}}\frac{i_{in}}{v_{in}} = \frac{R_L(1+b_F)+g_{m2}r_{p1}R_L}{r_{p1}+R_L(1+b_F)+g_{m2}r_{p1}R_L} = \frac{12.716}{38.875} = 0.3271$

$\boxed{\frac{v_{out}}{v_{in}} = 0.3271V/V}$

Problem 4 - (10 points)

Find an algebraic expression for the voltage gain, v_{out}/v_{in}, and the output resistance, R_{out}, of the source follower shown in terms of the small-signal model parameters, g_m and R_L (ignore r_{ds}). If the bias current is 1mA find the numerical value of the voltage gain and the output resistance. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $K_P' = 50\mu A/V^2$, $V_{TP} = -0.7V$.



Widths and lengths are in microns.

S03E1P1

Solution

A small-signal model for this circuit is shown below neglecting r_{ds} of the transistors.

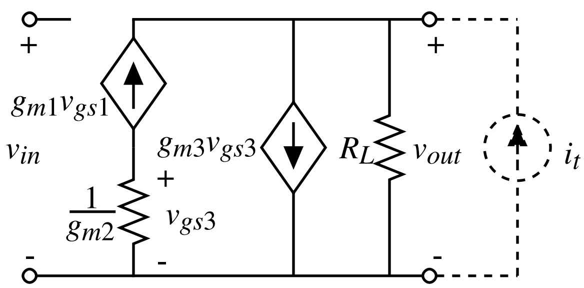


Fig. S03E1S1

Summing currents at the output node gives,

$$g_{m1}v_{gs1} = g_{m3}v_{gs3} + G_L v_{out}$$

Also, $v_{gs3} = -\,g_{m1}v_{gs1}(1/g_{m2})$

$$\therefore \quad g_{m1}v_{gs1} = g_{m3}\left(-\frac{g_{m1}}{g_{m2}}\right)v_{gs1} + G_L v_{out}$$

$$g_{m1}v_{gs1}\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_L v_{out} \rightarrow$$

$$g_{m1}(v_{out} - v_{in})\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_L v_{out}$$

$$\therefore \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right) + G_L}}$$

Setting $v_{in} = 0$ and applying i_t and solving for v_{out} and ignoring R_L gives,

$$i_t = g_{m3}v_{gs3} + g_{m1}v_{out} = g_{m3}\left(\frac{g_{m1}}{g_{m2}}\right)v_{out} + g_{m1}v_{out}$$

$$\therefore \quad \frac{v_{out}}{i_t} = \boxed{R_{out} = \frac{1}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}}$$

Note that the 1mA splits between M1(M2) and M3 in a ratio of 1 to 100. Therefore, $I_{D1} = I_{D2} = 9.9\mu A$ and $I_{D3} = 990.1\mu A$.

$$\therefore \quad g_{m1} = \sqrt{2\cdot 110\cdot 100\cdot 9.9} = 466.71\mu S, \quad g_{m2} = \sqrt{2\cdot 50\cdot 1\cdot 9.9} = 31.47\mu S$$

and $g_{m3} = \sqrt{2\cdot 110\cdot 100\cdot 990.1} = 3146.7\mu S$

$$\frac{v_{out}}{v_{in}} = \frac{466.71\cdot 101}{466.71\cdot 101 + 1/50} = \frac{47.137}{47.137 + 20} = \underline{\underline{0.702\ V/V}}$$

$$R_{out} = \frac{1000}{47.137} = \underline{\underline{21.2\Omega}}$$