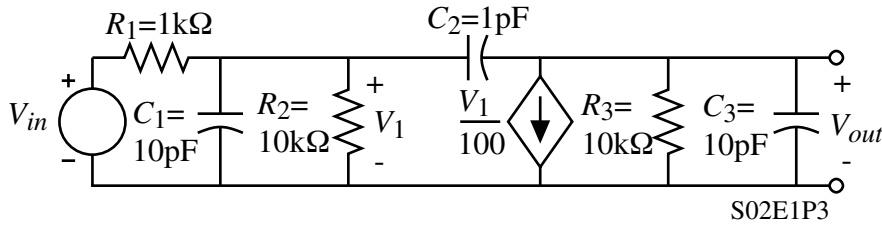


Homework Assignment No. 4 - Solutions

Problem 1

Find the midband voltage gain and the -3dB frequency in Hertz for the circuit shown.



Solution

The midband gain is given as,

$$\frac{V_{out}}{V_{in}} = - \left(\frac{10\text{k}\Omega}{100} \right) \left(\frac{10\text{k}\Omega}{11\text{k}\Omega} \right) = \underline{\underline{-90.91\text{V/V}}}$$

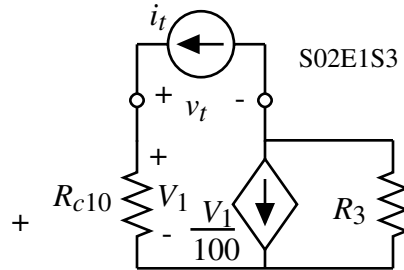
To find the -3dB frequency requires finding the 3 open-circuit time constants.

R_{C10} :

$$R_{C10} = 1\text{k}\Omega \parallel 10\text{k}\Omega = 0.9091\text{k}\Omega \quad \rightarrow \quad R_{C10}C_1 = 0.9091 \cdot 10\text{ns} = 9.09\text{ns}$$

R_{C20} :

$$\begin{aligned} v_t &= i_t R_{C10} + R_3(i_t + 0.01V_1) \\ &= i_t(R_{C10} + R_3 + 0.01R_{C10}R_3) \\ \therefore R_{C20} &= R_{C10} + R_3 + 0.01R_{C10}R_3 \\ &= 0.9091 \\ 10(1 + 0.01 \cdot 909.1)\text{k}\Omega &= 101.82\text{k}\Omega \\ R_{C20}C_2 &= 101.82 \cdot 1\text{ns} = 101.82\text{ns} \end{aligned}$$



R_{C30} :

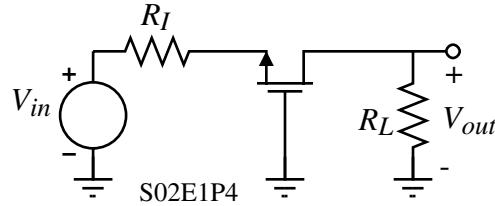
$$R_{C30} = 10\text{k}\Omega \quad \rightarrow \quad R_{C30}C_3 = 10 \cdot 10\text{ns} = 100\text{ns}$$

$$\Sigma T_0 = (9.091 + 101.82 + 100)\text{ns} = 210.91\text{ns} \quad \rightarrow \quad \omega_{-3\text{dB}} = \frac{1}{\Sigma T_0} = 4.74 \times 10^6 \text{ rad/s}$$

$$f_{-3\text{dB}} = \frac{4.74 \times 10^6}{2\pi} = \underline{\underline{754.6\text{kHz}}}$$

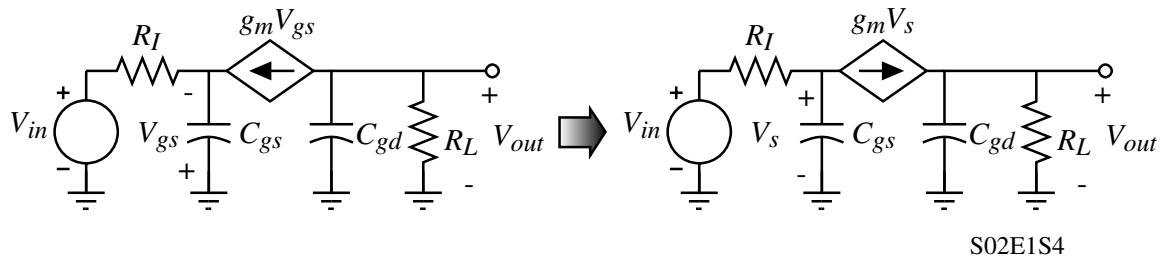
Problem 2 – (10 points)

Find the midband voltage gain and the exact value of the two poles of the voltage transfer function for the circuit shown. Assume that $R_I = 1\text{k}\Omega$, $R_L = 10\text{k}\Omega$, $g_m = 1\text{mS}$, $C_{gs} = 5\text{pF}$ and $C_{gd} = 1\text{pF}$. Ignore r_{ds} .

**Solution**

The best approach to this problem is a direct analysis.

Small-signal model:



$$V_{out} = g_m Z_L V_s \quad \text{where} \quad Z_L = \frac{1}{sR_L C_{gd} + 1} \quad \text{and} \quad \frac{V_{in} - V_s}{R_I} = g_m V_s + sC_{gs} V_s$$

Solving for V_s from the second equation gives,

$$V_s = \frac{V_{in}}{1 + g_m R_I + sC_{gs} R_I}$$

Substituting V_s in the first equation gives,

$$\begin{aligned} V_{out} &= g_m Z_L \frac{V_{in}}{1 + g_m R_I + sC_{gs} R_I} \rightarrow \frac{V_{out}}{V_{in}} = g_m \left(\frac{1}{sR_L C_{gd} + 1} \right) \left(\frac{1}{1 + g_m R_I + sC_{gs} R_I} \right) \\ &= \left(\frac{g_m R_L}{1 + g_m R_I} \right) \left(\frac{1}{sR_L C_{gd} + 1} \right) \left(\frac{1}{\frac{sC_{gd} R_I}{1 + g_m R_I} + 1} \right) = \text{MBG} \left(\frac{1}{1 - \frac{s}{p_1}} \right) \left(\frac{1}{1 - \frac{s}{p_2}} \right) \end{aligned}$$

$$\therefore \text{MBG} = \left(\frac{g_m R_L}{1 + g_m R_I} \right) = \left(\frac{1 \cdot 10}{1 + 1 \cdot 1} \right) = \underline{\underline{5V/V}}$$

$$p_1 = -\frac{1}{R_L C_{gd}} = -\frac{1}{10 \cdot 1\text{ns}} = \underline{\underline{-10^8 \text{ rad/s}}} \quad \text{and} \quad p_2 = -\frac{1 + g_m R_I}{R_I C_{gs}} = -\frac{1 + 1}{1 \cdot 5\text{ns}} = \underline{\underline{-4 \times 10^8 \text{ rad/s}}}$$

