Homework No. 5 - Solutions

Problem 1 - (10 points) (Problem 6.2-8 of A&H)

A two-stage, Miller-compensated CMOS op amp has a RHP zero at 20GB, a dominant pole due to the Miller compensation, a second pole at \( p_2 \) and a mirror pole at -3GB.  (a) If \( GB \) is 1MHz, find the location of \( p_2 \) corresponding to a 45° phase margin.  (b) Assume that in part (a) that \( |p_2| = 2GB \) and a nulling resistor is used to cancel \( p_2 \).  What is the new phase margin assuming that \( GB = 1MHz \)?  (c) Using the conditions of (b), what is the phase margin if \( C_L \) is increased by a factor of 4?

Solution

a.) Since the magnitude of the op amp is unity at \( GB \), then let \( \omega = GB \) to evaluate the phase.

\[
\text{Phase margin} = PM = 180^\circ - \tan^{-1}\left(\frac{GB}{|p_1|}\right) - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|z_1|}\right)
\]

But, \( p_1 = GB/A_o, \) \( p_3 = -3GB \) and \( z_1 = -20GB \) which gives

\[
PM = 45^\circ = 180^\circ - \tan^{-1}(A_o) - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}(0.33) - \tan^{-1}(0.05)
\]

\[
45^\circ = 90^\circ - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}(0.33) - \tan^{-1}(0.05) = 90^\circ - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - 18.26^\circ - 2.86^\circ
\]

\[
\therefore \quad \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 45^\circ - 18.26^\circ - 2.86^\circ = 23.48^\circ \quad \Rightarrow \quad \frac{GB}{|p_2|} = \tan(23.84^\circ) = 0.442
\]

\( p_2 = -2.26\cdot GB = -14.2\times10^6 \text{ rads/sec} \)

b.) The only roots now are \( p_1 \) and \( p_3 \). Thus,

\[
\text{PM} = 180^\circ - 90^\circ - \tan^{-1}(0.33) = 90^\circ - 18.3^\circ = 71.7^\circ
\]

c.) In this case, \( z_1 \) is at -2GB and \( p_2 \) moves to -0.5GB. Thus the phase margin is now,

\[
\text{PM} = 90^\circ - \tan^{-1}(2) + \tan^{-1}(0.5) - \tan^{-1}(0.33) = 90^\circ - 63.43^\circ + 26.57^\circ - 18.3^\circ = 34.4^\circ
\]
Problem 2 – (Problem 6.2-10 of A&H)
For the two-stage op amp of Fig. 6.2-8, find \( W_1/L_1, W_6/L_6, \) and \( C_c \) if \( GB = 1 \, \text{MHz}, \) \( |p_2| = 5 \, GB, \) \( z = 3 \, GB \) and \( C_L = C_2 = 20 \, \text{pF}. \) Use the parameter values of Table 3.1-2 and consider only the two-pole model of the op amp. The bias current in M5 is 40 \( \mu \text{A} \) and in M7 is 320 \( \mu \text{A}. \)

**Solution**

Given

\( GB = 1 \, \text{MHz}. \)

\( p_2 = 5 GB \)

\( z = 3GB \)

\( C_L = C_2 = 20 \, \text{pF} \)

Now, \( p_2 = \frac{g_{m6}}{C_2} \)

or,

\( g_{m6} = 628.3 \mu \text{S} \)

or,

\[
\left( \frac{W}{L} \right)_6 = \frac{g_{m6}^2}{2K_p I_{D6}} \approx 12.33
\]

RHP zero is given by

\( z = \frac{g_{m6}}{C_c} \)

or,

\( C_c = \frac{g_{m6}}{z} = 33.3 \text{pF} \)

Finally, Gain-bandwidth is given by

\( GB = \frac{g_{m1}}{C_c} \)

or,

\( g_{m1} = 209.4 \, \mu \text{S} \)

or,

\[
\left( \frac{W}{L} \right)_1 = \frac{g_{m1}^2}{2K_n I_{D1}} \approx 10
\]

Figure 6.2-8  A two-stage op amp with various parasitic and circuit capacitances shown.
Problem 3 - (10 points) (Problem 6.2-11 of A&H)

In the figure shown, assume that $R_I = 150 \, \text{k}\Omega$, $R_{II} = 100 \, \text{k}\Omega$, $g_{mII} = 500 \, \mu\text{S}$, $C_I = 1 \, \text{pF}$, $C_{II} = 5 \, \text{pF}$, and $C_c = 30 \, \text{pF}$. Find the value of $R_z$ and the locations of all roots for (a) the case where the zero is moved to infinity and (b) the case where the zero cancels the highest pole.

![Inverting High-Gain Stage](image)

**Solution**

(a.) Zero at infinity.

$$R_z = \frac{1}{g_{mII}} = \frac{1}{500 \, \mu\text{S}}$$

$$R_z = 2 \, \text{k}\Omega$$

Check pole due to $R_z$.

$$p_4 = \frac{-1}{R_z C_I} = \frac{-1}{2 \, \text{k}\Omega \cdot 1 \, \text{pF}} = -500 \times 10^6 \, \text{rps or 79.58 MHz}$$

The pole at $p_2$ is

$$p_2 = \frac{-g_{mII} C_c}{C_I C_{II} + C_c C_{II}} = \frac{-500 \, \mu\text{S}}{5 \, \text{pF}} = 100 \times 10^6 \, \text{rps or 15.9 MHz}$$

Therefore, $p_2$ is the next highest pole.

(b.) Zero at $p_2$.

$$R_z = \left( \frac{C_c + C_{II}}{C_c} \right) \left( \frac{1}{g_{mII}} \right) = \left( \frac{30 + 5}{30} \right) \frac{1}{500 \, \mu\text{S}} = 2.33 \, \text{k}\Omega$$

$$R_z = 2.33 \, \text{k}\Omega$$
Problem 4 – (10 points)
The poles and zeros of a Miller compensated, two-stage op amp are shown below.

(a.) If the influence of \( p_3 \) and \( z_1 \) are ignored, what is the \( GB \) in MHz of this op amp for 60° phase margin?

(b.) What is the value of \( A_v(0) \)? What is the value of \( C_c \) if \( g_{m1}=g_{m2}=500\mu S \)?

(c.) If \( p_2 \) is moved to \( p_3 \), what is the new \( GB \) in MHz for 60° phase margin? What is the new \( C_c \) if the input transconductances are the same as in (b.)?

Solution

(a.) The phase margin, PM, can be written as

\[
PM = 180 - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_3|}\right) - \tan^{-1}\left(\frac{GB}{|z_1|}\right) = 90° - \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 60°
\]

\[
\therefore \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 30° \quad \rightarrow \quad GB = 0.5774 \cdot |p_2| = 5.774MHz
\]

(b.) \( A_v(0) = \frac{GB}{|p_1|} = \frac{5.774MHz}{1kHz} = 5.774\text{V/V} \)

\[
\frac{g_{m1}}{C_c} = GB \quad \rightarrow \quad C_c = \frac{g_{m1}}{GB} = \frac{500\mu S}{2\pi \cdot 5.774 \times 10^6} = 13.78\text{pF}
\]

(c.) The phase margin, PM, can be written as

\[
PM = 180 - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_3|}\right) - \tan^{-1}\left(\frac{GB}{|z_1|}\right) = 90° - 3 \cdot \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 60°
\]

\[
\therefore \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 10° \quad \rightarrow \quad GB = 0.1763 \cdot |p_2| = 0.01763 \cdot 100MHz = 17.63MHz
\]

\[
C_c = \frac{g_{m1}}{GB} = \frac{500\mu S}{2\pi \cdot 17.63 \times 10^6} = 4.514\text{pF}
\]
Problem 5

A self-compensated op amp has three higher order poles grouped closely around $-1 \times 10^9$ radians/sec. What should be the $GB$ of this op amp in Hz to achieve a 60° phase margin? If the low frequency gain of the op amp is 80dB, where is the location of the dominant pole, $p_1$? If the output resistance of this amplifier is 10MΩ, what is the value of $C_L$ that will give this location for $p_1$? (Ignore any other capacitance at the output for this part of the problem).

Solution

The key to this problem is to assume that the three closely grouped poles around $-1 \times 10^9$ radians/sec. can be approximated as three poles at $-1 \times 10^9$ radians/sec. Therefore,

$$\text{Phase margin} = \text{PM} = 180^\circ - \tan^{-1}\left(\frac{GB}{|p_1|}\right) - 3 \tan^{-1}\left(\frac{GB}{|p_H|}\right) = 60^\circ$$

where $p_H$ is a pole at $-1 \times 10^9$ radians/sec. Assuming that $GB/|p_1|$ is large then, we can write the above as,

$$180^\circ - 90^\circ - 3 \tan^{-1}\left(\frac{GB}{|p_H|}\right) = 60^\circ \implies 30^\circ = -3 \tan^{-1}\left(\frac{GB}{|p_H|}\right) \implies \frac{GB}{|p_H|} = \tan(10^\circ) = 0.1763$$

$\therefore$ $GB = 0.1763|p_H| = 176.3$ Mradians/sec. $\implies$ $GB = 28.06$ MHz

80dB $\implies$ 10,000 which gives

$$|p_1| = \frac{GB}{A_v} = \frac{176.3 \times 10^6}{10^4} = 17,630 \text{ radians/sec.} \implies |p_1| = 2.806$ kHz

The expression for $p_1$ is

$$|p_1| = \frac{1}{R_{out}C_L} \implies C_L = \frac{1}{R_{out}|p_1|^2} = \frac{1}{1.763 \times 10^4 \times 1.17} = 5.672 \text{ pF}$$