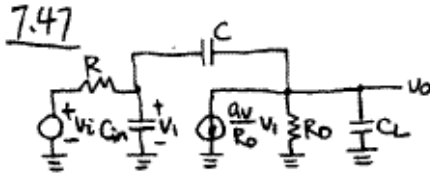


Homework Assignment No. 9 - Solutions

Problem 1 - (10 points)



The zero value time constants

$$C_{in}R = 0.2 \times 10^{-12} \times 20 \times 10^3 = 4 \times 10^{-9} \text{ s}$$

$$C(R + R_o + \frac{a_v}{R_o} R R_o) = C(R + R_o + a_v R)$$

$$= C[(1 + a_v)R + R_o]$$

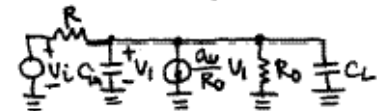
$$= 50 \times 10^{-12} [(1 + 1000)20 \times 10^3 + 5 \times 10^3]$$

$$= 1 \times 10^{-3} \text{ s}$$

$$C_L R_o = 0.5 \times 10^{-12} \times 5 \times 10^3 = 2.5 \times 10^{-9} \text{ s}$$

$$P_1 = \frac{1}{4 \times 10^{-9} + 1 \times 10^{-3} + 2.5 \times 10^{-9}} = -1 \times 10^3 \text{ rad/s}$$

$1 \times 10^3 \text{ s} \gg 4 \times 10^{-9} \text{ s}, 2.5 \times 10^{-9} \text{ s}$. C is shorted.



$$(C_{in} + C)(R \parallel \frac{R_o}{a_v} \parallel R_o)$$

$$\approx (C_{in} + C) \frac{R_o}{a_v}$$

$$= (0.2 + 0.5) \times 10^{-12} \frac{5 \times 10^3}{1000}$$

$$= 3.5 \times 10^{-12} \text{ s}$$

$$P_2 = -\frac{1}{3.5 \times 10^{-12}} = -2.9 \times 10^{11} \text{ rad/s}$$

Note that there is a zero at

$$z = -\frac{a_v/R_o}{C} = -\frac{1000/5 \times 10^3}{50 \times 10^{-12}} = -4 \times 10^9 \text{ rad/s}$$

It is between the two poles and confirmed by SPICE.

```

INTEGRATOR
VI 1 0 AC 1
R 1 2 20K
RO 3 4 5K
CIN 2 0 0.2P
C 2 3 50P
CL 3 0 0.5P
E 4 0 0 2 1000
    
```

```

.OPTIONS NOMOD
.AC DEC 5 10 1000
.PLOT AC VM(3)
.WIDTH OUT=80
.OPTIONS SPICE
.END
    
```

```

***** OPERATING POINT INFORMATION      TNUM= 27.000  TEND= 27.000

+0:1      = 0.      0:2      = 0.      0:3      = 0.
+0:4      = 0.

***** AC ANALYSIS                      TNUM= 27.000  TEND= 27.000
    
```

FREQ	VM(3)	1.000E-04	1.000E-03	1.000E+00	1.000E+02	1.000E+04
1.000E+01	9.983E-02					
1.584E+01	9.958E-02					
2.512E+01	9.878E-02					
3.982E+01	9.700E-02					
6.309E+01	9.298E-02					
1.000E+02	8.140E-02					
1.584E+02	7.008E-02					
2.512E+02	5.348E-02					
3.982E+02	3.700E-02					
6.309E+02	2.448E-02					
1.000E+03	1.570E-02					
1.584E+03	9.970E-03					
2.512E+03	6.330E-03					
3.982E+03	3.990E-03					
6.309E+03	2.530E-03					
1.000E+04	1.580E-03					
1.584E+04	1.000E-03					
2.512E+04	6.330E-04					
3.982E+04	3.990E-04					
6.309E+04	2.530E-04					
1.000E+05	1.580E-04					
1.584E+05	1.000E-04					
2.512E+05	6.330E-05					
3.982E+05	3.990E-05					
6.309E+05	2.530E-05					
1.000E+06	1.580E-05					
1.584E+06	1.000E-05					
2.512E+06	6.330E-06					
3.982E+06	3.990E-06					
6.309E+06	2.530E-06					
1.000E+07	1.580E-06					
1.584E+07	1.000E-06					
2.512E+07	6.330E-07					
3.982E+07	3.990E-07					
6.309E+07	2.530E-07					
1.000E+08	1.580E-07					
1.584E+08	1.030E-07					
2.512E+08	6.800E-08					
3.982E+08	4.710E-08					
6.309E+08	3.030E-08					
1.000E+09	2.000E-08					
1.584E+09	1.600E-08					
2.512E+09	1.100E-08					
3.982E+09	7.500E-09					
6.309E+09	5.000E-09					
1.000E+10	3.400E-09					
1.584E+10	2.400E-09					
2.512E+10	1.700E-09					
3.982E+10	1.200E-09					
6.309E+10	8.000E-10					
1.000E+11	5.600E-10					

Problem 2 – (10 points)8.6(a)

From (8.66)

$$Z_{ia} = \frac{R_F Z_i}{R_F + Z_i} = \frac{100 \times 500}{600} = 83.3 \text{ k}\Omega$$

From (8.68)

$$\begin{aligned} Z_{oa} &= Z_o \parallel R_F \parallel R_L \\ &= 200 \parallel 100\text{k} \parallel 15\text{k} \\ &\approx 200 \Omega \end{aligned}$$

From (8.70)

$$\begin{aligned} T &= \left(\frac{10^5 \times 15 \times 10^3}{10^5 \times 15 \times 10^3 + 200 \times 10^5 + 200 \times 15 \times 10^3} \right) \times 75,000 \\ &\quad \times \frac{500}{600} \\ &= 61,560 \end{aligned}$$

Thus with feedback

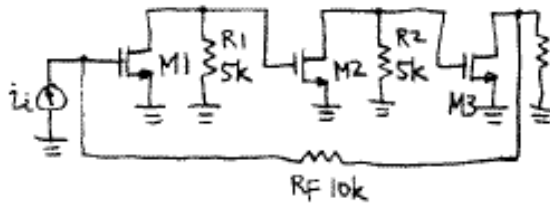
$$Z_i = \frac{83.3 \text{ k}}{61,560} = 1.4 \Omega$$

$$Z_o = \frac{200}{61,560} = 0.0032 \Omega$$

$$\begin{aligned} A &= \frac{1}{f} \frac{1}{1 + \frac{1}{T}} = \frac{100 \text{ k}\Omega}{1 + \frac{1}{61,560}} \\ &= 99.998 \text{ k}\Omega \end{aligned}$$

Problem 3 - (10 points)

8.7

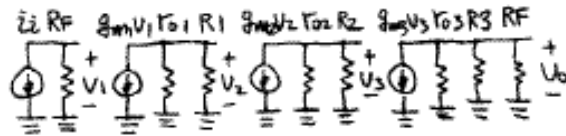


$$g_m = \sqrt{2k' \frac{W}{L} I_D} = \sqrt{2 \times 60 \times 10^{-6} \times 100 \times 10^{-3}}$$

$$= 3.5 \times 10^{-3} \text{ A/V}$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{50}{10^{-3}} = 50 \text{ k}\Omega$$

(a)



$$a = \frac{v_o}{i_i} \Big|_{f=0} = R_F (-g_{m1})(r_{o1} \parallel R_1) (-g_{m2})(r_{o2} \parallel R_2)$$

$$(-g_{m3})(r_{o3} \parallel R_3 \parallel R_F)$$

$$= -g_{m1}^3 (r_{o1} \parallel R_1)^2 R_F (r_{o3} \parallel R_3 \parallel R_F)$$

$$= -(3.5 \times 10^{-3})^3 (50 \text{ k} \parallel 5 \text{ k})^2 10 \text{ k} (50 \text{ k} \parallel 5 \text{ k} \parallel 10 \text{ k})$$

$$= -2.76 \times 10^7 \Omega$$

$$f = -\frac{z_{FB}}{v_o} = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega}$$

$$a_f = 2.76 \times 10^3$$

$$\frac{v_o}{i_i} = \frac{a}{1+a_f} = \frac{-2.76 \times 10^7}{1+2.76 \times 10^3} = -10 \text{ k}\Omega$$

$$R_i = \frac{R_F}{1+a_f} = \frac{10 \text{ k}}{1+2.76 \times 10^3} = 3.7 \Omega$$

$$R_o = \frac{r_{o3} \parallel R_3 \parallel R_F}{1+a_f} = \frac{50 \text{ k} \parallel 5 \text{ k} \parallel 10 \text{ k}}{1+2.76 \times 10^3} = 1.13 \Omega$$

(b)

$$a = -2.76 \times 10^7 \frac{R_F \parallel R_3}{R_F} = -2.76 \times 10^7 \frac{10 \text{ k} \parallel 10 \text{ k}}{10 \text{ k}}$$

$$= -2.51 \times 10^6 \Omega$$

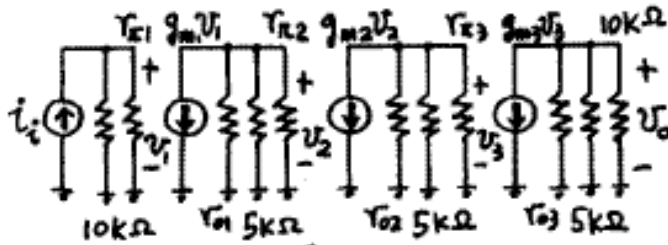
$$a_f = 251$$

$$R_o = \frac{r_{o3} \parallel R_3 \parallel R_F}{1+a_f} = \frac{50 \text{ k} \parallel 5 \text{ k} \parallel 10 \text{ k}}{1+251} = 12.4 \Omega$$

Problem 4 - (10 points)

8.8

(a) Basic amplifier



$$r_{\pi 1} = r_{\pi 2} = r_{\pi 3} = \frac{\beta}{g_m} = 5.2 \text{ k}\Omega$$

$$g_m = \frac{1}{26} v, r_o = 50 \text{ k}\Omega$$

$$R_1 = 10 \text{ k}\Omega \parallel r_{\pi 1} = \frac{5.2 \times 10}{15.2} = 3.42 \text{ k}\Omega$$

$$R_2 = r_{o1} \parallel 5 \text{ k}\Omega \parallel r_{\pi 2} = 2.42 \text{ k}\Omega$$

$$R_3 = r_{o2} \parallel 5 \text{ k}\Omega \parallel r_{\pi 3} = 2.42 \text{ k}\Omega$$

$$R_4 = r_{o3} \parallel 5 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 3.13 \text{ k}\Omega$$

$$\begin{aligned} \therefore \frac{v_o}{i_i} &= -R_1 g_{m1} R_2 g_{m2} R_3 g_{m3} R_4 \\ &= -3.42 \frac{2420}{26} \frac{2420}{26} \frac{3130}{26} \text{ k}\Omega \\ &= -3.57 \times 10^9 \Omega = a \end{aligned}$$

$$f = -\frac{df_b}{\omega} = -\frac{1}{10 \text{ k}\Omega}$$

$$\begin{aligned} \therefore \text{overall } \frac{v_o}{i_i} &= \frac{a}{1+af} = \frac{-3.57 \times 10^9}{1+3.57 \times 10^5} \\ &= -10 \text{ k}\Omega \end{aligned}$$

$$\text{loop gain} = af = 3.57 \times 10^5$$

$$R_i = \frac{R_1}{1+af} = \frac{3420}{1+3.57 \times 10^5} = 0.0096 \Omega$$

$$R_o = \frac{R_4}{1+af} = \frac{3130}{1+3.57 \times 10^5} = 0.0088 \Omega$$

(b)

$$\text{New value of } R_1 = 3.42 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 774 \Omega$$

$$\therefore a = -3.57 \times 10^9 \frac{774}{3420} = -808 \text{ M}\Omega$$

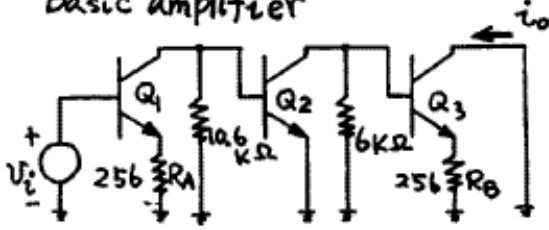
$$\therefore af = 808 \times 10^6 \times 10^{-4} = 8.08 \times 10^4$$

$$R_o = \frac{R_4}{1+af} = \frac{3130}{1+8.08 \times 10^4} = 0.0387 \Omega$$

Problem 5 - (10 points)

8.11

Basic amplifier



$$R_{E1} \parallel (R_F + R_{E2}) = 290 \parallel 2.19 \text{ k} = 256 \Omega$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = 52 \times 120 = 6.24 \text{ k}\Omega$$

$$r_{o1} = 80 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{26 \times 120}{0.77} = 4.05 \text{ k}\Omega$$

$$r_{o2} = \frac{40}{0.77} = 52 \text{ k}\Omega$$

$$r_{\pi 3} = \frac{26 \times 120}{0.73} = 4.27 \text{ k}\Omega$$

$$r_{o3} = \frac{40}{0.73} = 54.8 \text{ k}\Omega$$

In forward gain calculation, neglect r_{o1} and r_{o3} .

For the basic amplifier,

$$\frac{i_o}{v_i} = \frac{g_{m1}}{1 + g_{m1} R_A} R_1 g_{m2} R_2 \frac{g_{m3}}{1 + g_{m3} R_B}$$

$$R_1 = 10.6 \text{ k} \parallel r_{\pi 2} = 2.93 \text{ k}\Omega$$

$$R_2 = r_{o2} \parallel 6 \text{ k} \parallel R_{i3}$$

$$R_{i3} = r_{\pi 3} (1 + g_{m3} R_B)$$

$$= 4.27 \left(1 + \frac{0.73}{26} \times 256 \right)$$

$$= 35 \text{ k}\Omega$$

$$\therefore R_2 = 52 \text{ k} \parallel 6 \text{ k} \parallel 35 \text{ k} = 4.66 \text{ k}\Omega$$

$$\therefore \frac{i_o}{v_i} = \frac{1}{52} \frac{1}{1 + \frac{256}{52}} 2930 \frac{0.77}{26} 4660$$

$$\times \frac{0.73}{26} \frac{1}{8.19}$$

$$\therefore a = 4.5 \text{ A/V}$$

From (8.95)

$$f = \frac{1}{\alpha_3} \frac{R_{E1} R_{E2}}{R_{E1} + R_{E2} + R_F}$$

$$= \frac{1}{0.99} \frac{290 \times 290}{290 + 290 + 1900}$$

$$= 34.25 \Omega$$

$$\therefore \text{loop gain} = af = 4.5 \times 34.25 = 154$$

Overall gain with feedback

$$= \frac{a}{1 + af} = \frac{4.5}{1 + 154} = \frac{4.5}{155} \text{ A/V}$$

$$\therefore \frac{i_o}{v_i} = 29 \text{ mA/V}$$

For the basic amplifier

Input resistance

$$r_{ia} = r_{\pi 1} (1 + g_{m1} R_A)$$

$$= 6.24 \left(1 + \frac{256}{52} \right) = 36.96 \text{ k}\Omega$$

Output resistance

$$r_{oa} = r_{o3} \left(1 + g_{m3} R_B \frac{r_{\pi 3}}{r_{\pi 3} + R_{S3}} \right)$$

$$R_{S3} = r_{o2} \parallel 6 \text{ k} = 5.38 \text{ k}\Omega$$

$$\therefore r_{oa} = 54.8 \left(1 + \frac{0.73}{26} 256 \frac{4.27}{4.27 + 5.38} \right)$$

$$= 229 \text{ k}\Omega$$

For the feedback amplifier

$$R_i = r_{ia} (1 + af) = 36.96 \times 155 = 5.73 \text{ M}\Omega$$

$$R_o = r_{oa} (1 + af) = 229 \times 155 = 35.5 \text{ M}\Omega$$