Homework Assignment No. 9 - Solutions
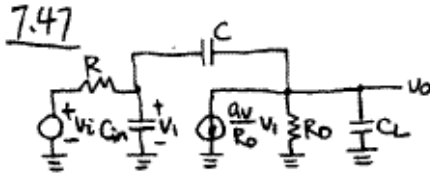
Problem 1 - (10 points)

7.47

The zero value time constants

$C_{in}R = 0.2 \times 10^{-12} \times 20 \times 10^{3} = 4 \times 10^{-9} S$

$C(R + R_0 + \frac{a_v}{R_0} R R_0) = C(R + R_0 + a_v R)$

$= C[(1 + a_v)R + R_0]$
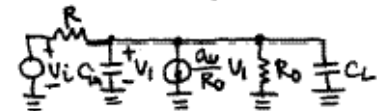
$= 50 \times 10^{-12}[(1 + 1000)20 \times 10^{3} + 5 \times 10^{3}]$

$= 1 \times 10^{-3} S$

$C_L R_0 = 0.5 \times 10^{-12} \times 5 \times 10^{3} = 2.5 \times 10^{-9} S$

$P_1 = -\dfrac{1}{4 \times 10^{-9} + 1 \times 10^{-3} + 2.5 \times 10^{-9}} = -1 \times 10^{3} \, rad/s$

$1 \times 10^{-3} S \gg 4 \times 10^{-9} S, \ 2.5 \times 10^{-9} S, \ C \text{ is shorted.}$

$(C_{in} + C)(R \| \frac{R_0}{a_v} \| R_0)$

$\approx (C_{in} + C) \dfrac{R_0}{a_v}$

$= (0.2 + 0.5) \times 10^{-12} \dfrac{5 \times 10^{3}}{1000}$

$= 3.5 \times 10^{-12} S$

$P_2 = -\dfrac{1}{3.5 \times 10^{-12}} = -2.9 \times 10^{11} \, rad/s.$

Note that there is a zero at

$Z = -\dfrac{a_v / R_0}{C} = -\dfrac{1000/5 \times 10^{3}}{50 \times 10^{-12}} = -4 \times 10^{9} \, rad/s$

It is between the two poles and confirmed by SPICE.

```
INTEGRATOR
VI   1   0   AC   1
R    1   2   20K
RO   3   4   5K
CIN  2   0   0.2P
C    2   3   50P
CL   3   0   0.5P
E    4   0   0   2   1000
.OPTIONS NOMOD
.AC DEC 5 10 100G
.PLOT AC VM(3)
.WIDTH OUT=80
.OPTIONS SPICE
.END
```

****** OPERATING POINT INFORMATION   TNOM= 27.000 TEMP= 27.000

```
+0:1   =  0.     0:2   =  0.     0:3   =  0.
+0:4   =  0.
```

****** AC ANALYSIS   TNOM= 27.000 TEMP= 27.000

Problem 2 – (10 points)

__8.6__ (a)

From (8.66)

$$Z_{ia} = \frac{R_F Z_i}{R_F + Z_i} = \frac{100 \times 500}{600} = 83.3 \; K\Omega$$

From (8.68)

$$Z_{oa} = Z_o \| R_F \| R_L$$
$$= 200 \| 100k \| 15K$$
$$\cong 200\Omega$$

From (8.70)

$$T = \left(\frac{10^5 \times 15 \times 10^3}{10^5 \times 15 \times 10^3 + 200 \times 10^5 + 200 \times 15 \times 10^3} \right) \times 75,000$$
$$\times \frac{500}{600}$$
$$= 61,560$$

Thus with feedback
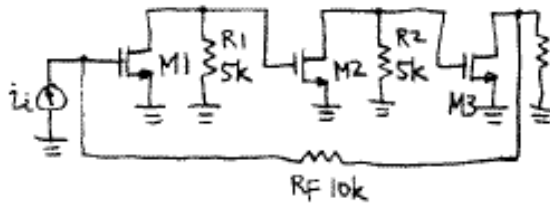
$$Z_i = \frac{83.3 K}{61,560} = 1.4 \; \Omega$$

$$Z_o = \frac{200}{61,560} = 0.0032 \; \Omega$$

$$A = \frac{1}{f} \frac{1}{1 + \frac{1}{T}} = \frac{100 k \Omega}{1 + \frac{1}{61,560}}$$
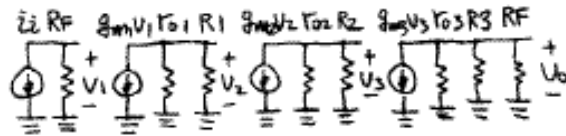$$= 99.998 \; K\Omega$$

Problem 3 – (10 points)

8.7

$$g_m = \sqrt{2k'\frac{W}{L}I_D} = \sqrt{2\times60\times10^{-6}\times100\times10^{-3}}$$

$$= 3.5\times10^{-3} \text{A/V}$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{50}{10^{-3}} = 50k\Omega$$

(b)

$$a = -2.76\times10^7 \frac{R_F\|R_S}{R_F} = -2.76\times10^7 \frac{10k\|1k}{10k}$$

$$= -2.51\times10^6 \Omega$$

$$af = 251$$

$$R_o = \frac{r_{o3}\|R_3\|R_F}{1+af} = \frac{50k\|5k\|10k}{1+251} = 12.4\Omega$$

(a)

$$a = \frac{v_o}{i_i}\Big|_{f=0} = R_F(-g_m)(r_{o1}\|R_1)(-g_{m2})(r_{o2}\|R_2)$$

$$(-g_{m3})(r_{o3}\|R_3\|R_F)$$

$$= -g_{m1}^3(r_{o1}\|R_1)^2 R_F(r_{o3}\|R_3\|R_F)$$

$$= -(3.5\times10^{-3})^3(50k\|5k)^2 10k(50k\|5k\|10k)$$

$$= -2.76\times10^7 \Omega$$

$$f = -\frac{i_{f0}}{v_o} = -\frac{1}{R_F} = -\frac{1}{10k\Omega}$$

$$af = 2.76\times10^3$$

$$\frac{v_o}{i_i} = \frac{a}{1+af} = \frac{-2.76\times10^7}{1+2.76\times10^3} = -10k\Omega$$

$$R_i = \frac{R_F}{1+af} = \frac{10k}{1+2.76\times10^3} = 3.7\Omega$$

$$R_o = \frac{r_{o3}\|R_3\|R_F}{1+af} = \frac{50k\|5k\|10k}{1+2.76\times10^3} = 1.13\Omega$$

Problem 4 – (10 points)

8.8

(a) Basic amplifier
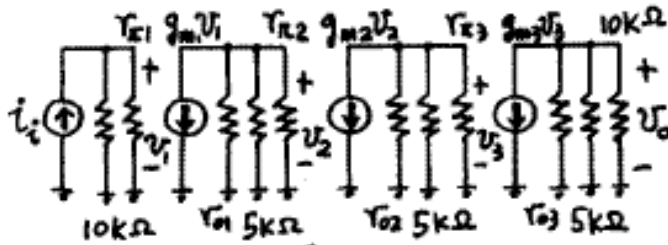
$r_{\pi 1} = r_{\pi 2} = r_{\pi 3} = \dfrac{\beta}{g_m} = 5.2\,K\Omega$

$g_m = \dfrac{1}{26}\mho, \quad r_0 = 50\,K\Omega$

$R_1 = 10k \| r_{\pi 1} = \dfrac{5.2 \times 10}{15.2} = 3.42\,K\Omega$

$R_2 = r_{01} \| 5k \| r_{\pi 2} = 2.42\,K\Omega$

$R_3 = r_{02} \| 5k \| r_{\pi 3} = 2.42\,K\Omega$

$R_4 = r_{03} \| 5k \| 10K = 3.13\,K\Omega$

$\therefore \dfrac{v_o}{i_i} = -R_1 g_{m1} R_2 g_{m2} R_3 g_{m3} R_4$

$= -3.42 \dfrac{2420}{26} \dfrac{2420}{26} \dfrac{3130}{26}\,K\Omega$

$= -3.57 \times 10^9\,\Omega = a$

$f = -\dfrac{i_{fb}}{v_o} = -\dfrac{1}{10k\Omega}$

$\therefore$ overall $\dfrac{v_o}{i_i} = \dfrac{a}{1+af} = \dfrac{-3.57 \times 10^9}{1 + 3.57 \times 10^5}$

$= -10\,K\Omega$

loop gain $= af = 3.57 \times 10^5$

$R_i = \dfrac{R_1}{1+af} = \dfrac{3420}{1+3.57 \times 10^5} = 0.0096\,\Omega$

$R_o = \dfrac{R_4}{1+af} = \dfrac{3130}{1+3.57 \times 10^5} = 0.0088\,\Omega$

(b)

New value of $R_1 = 3.42 \| 1^k = 774\,\Omega$

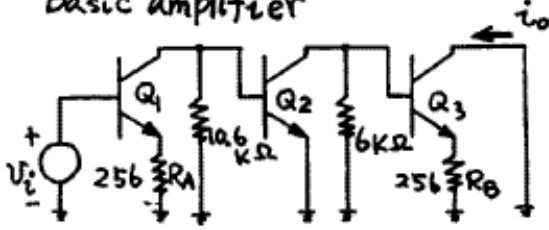$\therefore a = -3.57 \times 10^9 \dfrac{774}{3420} = -808\,M\Omega$

$\therefore af = 808 \times 10^6 \times 10^{-4} = 8.08 \times 10^4$

$R_o = \dfrac{R_4}{1+af} = \dfrac{3130}{1+8.08 \times 10^4} = 0.0387\,\Omega$

Problem 5 – (10 points)

8.11
Basic amplifier

$R_{E1} \| (R_F + R_{E2}) = 290 \| 2.19k = 256\Omega$

$r_{\pi 1} = \dfrac{\beta}{g_{m1}} = 52 \times 120 = 6.24 \, k\Omega$

$r_{o1} = 80 k\Omega$

$r_{\pi 2} = \dfrac{26 \times 120}{0.77} = 4.05 \, k\Omega$

$r_{o2} = \dfrac{40}{0.77} = 52 \, k\Omega$

$r_{\pi 3} = \dfrac{26 \times 120}{0.73} = 4.27 \, k\Omega$

$r_{o3} = \dfrac{40}{0.73} = 54.8 \, k\Omega$

In forward gain calculation, neglect r_{o1} and r_{o3}.

For the basic amplifier,

$\dfrac{i_o}{v_i} = \dfrac{g_{m1}}{1 + g_{m1}R_A} R_1 g_{m2} R_2 \dfrac{g_{m3}}{1 + g_{m3}R_B}$

$R_1 = 10.6k \| r_{\pi 2} = 2.93 \, k\Omega$

$R_2 = r_{o2} \| 6k \| R_{i3}$

$R_{i3} = r_{\pi 3} (1 + g_{m3}R_B)$

$\quad = 4.27 \left(1 + \dfrac{0.73}{26} \times 256\right)$

$\quad = 35 \, k\Omega$

$\therefore R_2 = 52k \| 6k \| 35k = 4.66 \, k\Omega$

$\therefore \dfrac{i_o}{v_i} = \dfrac{1}{52} \dfrac{1}{1 + \frac{256}{52}} 2930 \dfrac{0.77}{26} 4660$

$\qquad \times \dfrac{0.73}{26} \dfrac{1}{8.19}$

$\therefore a = 4.5 \, A/v$

From (8.95)

$f = \dfrac{1}{\alpha_3} \dfrac{R_{E1} R_{E2}}{R_{E1} + R_{E2} + R_F}$

$\quad = \dfrac{1}{0.99} \dfrac{290 \times 290}{290 + 290 + 1900}$

$\quad = 34.25 \, \Omega$

$\therefore$ loop gain $= af = 4.5 \times 34.25 = 154$

Overall gain with feedback

$= \dfrac{a}{1 + af} = \dfrac{4.5}{1 + 154} = \dfrac{4.5}{155} \, A/v$

$\therefore \dfrac{i_o}{v_s} = .29 \, mA/v$

For the basic amplifier
Input resistance.

$r_{ia} = r_{\pi 1}(1 + g_{m1}R_A)$

$\quad = 6.24 \left(1 + \dfrac{256}{52}\right) = 36.96 \, k\Omega$

Output resistance

$r_{oa} = r_{o3}\left(1 + g_{m3}R_B \dfrac{r_{\pi 3}}{r_{\pi 3} + R_{s3}}\right)$

$R_{s3} = r_{o2} \| 6k = 5.38 \, k\Omega$

$\therefore r_{oa} = 54.8\left(1 + \dfrac{0.73}{26} 256 \dfrac{4.27}{4.27 + 5.38}\right)$

$\quad = 229 \, k\Omega$

For the feedback amplifier

$R_i = r_{ia}(1 + af) = 36.96 \times 155 = 5.73 \, M\Omega$

$R_o = r_{oa}(1 + af) = 229 \times 155 = 35.5 \, M\Omega$