Homework Assignment No. 11 - Solutions

Problem 1 – (10 points)
Problem 8.26 of GHLM

(a) $gm_2$ is the controlled source.

\[
\frac{V_x}{r_{01}} = g_{m2}U_x + \frac{V_x - U}{r_{02}}
\]

\[
R_{out}(a=0) = \frac{U}{i} = \frac{U}{V_x/r_{01}} = \frac{g_{m2}r_{01}r_{02} + r_01 - r_{02}}{r_{01}r_{02}} \approx g_{m2}r_{01}r_{02}
\]

The output is short

\[
U_x = g_{m2}(V_t - U_x)(r_{01}r_{02})
\]

\[
U_x = \frac{g_{m2}(r_{01}r_{02})}{1 + g_{m2}(r_{01}r_{02})} U_t
\]

R (short) = \frac{g_{m2}(r_{01}r_{02})}{1 + g_{m2}(r_{01}r_{02})}

R (open) = 0 (U_x = 0 when the output is open)

R_{out}(a=0) \approx g_{m2}r_{01}r_{02} \approx g_{m2}r_{01}r_{02}

(b) $a$ is the controlled source.

\[
\frac{U_x}{r_{01}} = g_{m2}U_x + \frac{U_x - U}{r_{02}}
\]

(c) The results are the same, as they should be, even though the terms $R_{out}(k=0)$, $R$ (open), and $R$ (short) differ in (a) and (b).
Problem 2 - (10 points)

The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find \( V_2/V_1 \), \( R_{in} = V_1/I_1 \), and \( R_{out} = V_2/I_2 \). Assume that all transistors are matched and that \( g_m = 1 \text{mA/V} \) and \( r_{ds} = \infty \).

**Solution**

This feedback circuit is shunt-series. We begin with the open-loop, quasi-ac model shown below.

The small-signal, open-loop model is:

\[
\frac{I'_o}{I'_s} = \left( \frac{I'_o}{I'_s} \right) \left( \frac{V_{gs2}}{V_{gs1}} \right) \left( \frac{V_{gs1}}{V_{gs2}} \right)
\]

\[
V_{gs2} = -g_m V_{gs1} R_2 - g_m V_{gs2} R_4
\]

Or

\[
\frac{V_{gs2}}{V_{gs1}} = -\frac{g_m R_2}{1 + g_m R_4} = \frac{-50}{2} = -25
\]

\[
\therefore \quad a = \frac{I'_o}{I'_s} = (g_m)(-25)\left( \frac{-1}{g_m} \right) = 25 \text{A/A}
\]

\[
f = \frac{I'_f}{I'_o} = \left( \frac{I'_f}{I'_o} \right) \left( \frac{V_{gs3}}{V_{gs3}} \right) = (g_m) \left( \frac{R_4}{1 + g_m R_1} \right) = (1 \text{mA/V})(0.5 \text{k\Omega}) = 0.5
\]

\[
\therefore \quad af = 25 \cdot 0.5 = 12.5
\]

\[
R_i = \frac{I'_i}{I'_s} = \frac{1}{g_m} = 1 \text{k\Omega} \quad \rightarrow \quad R_{in} = \frac{R_i}{1 + af} = \frac{1000}{13.5} = 74.07 \text{\Omega}
\]

**\( R_{out} = 50 \text{k\Omega} \) (\( R_3 \) is outside the feedback loop)**

\[
\frac{I'_o}{I'_s} = \frac{a}{1 + af} = \frac{25}{1 + 12.5} = 1.852 \text{ A/A} \quad \rightarrow \quad \frac{v_2}{v_1} = \frac{I'_o(-50 \text{k\Omega})}{I'_s(74.07 \text{\Omega})} = -1240.1 \text{ V/V}
\]
Problem 3 – (10 points)

(a) The basic amplifier without the feedback signal inserted at the inverting input of the op amp.

\[ g_m = \frac{2K}{l} I_0 = \frac{2 \times 8 \times 10^{-5} \times 100 \times 0.5 \times 10^3}{2} = 4.2 \times 10^3 \text{ A/V} \]

\[ g_{mb} = \frac{g_m}{2 \sqrt{2} a_f + V_{SB}} \]

\[ = \frac{0.3}{2 \sqrt{2} a_f} \]

\[ g_m = 4.2 \times 10^3 \times 8.1 \times 10^4 \text{ A/V} \]

\[ V_0 = g_m (V_c - V_o) \frac{1}{g_{mb}} \]

\[ a = \frac{V_o}{V_i} = a_v g_m \frac{g_m}{g_m + g_{mb}} \]

\[ f = 1 \]

\[ a_f = a_v g_m \frac{g_m}{g_m + g_{mb}} = 1000 \]

\[ 4.2 \times 10^4 = 838 \]

\[ A = \frac{a}{1 + a_f} = \frac{838}{1 + 838} = 0.999 \]

\[ R_{in} = R_i \]

\[ R_{in} = R_{in} (1 + a_f) = R_i (1 + a_f) = 1 \times (1 + 838) = 839 \text{ M} \Omega \]

\[ U_i = 0 \]

\[ R_a = \frac{1}{g_m} \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}} \]

\[ R_{out} = \frac{R_a}{1 + a_f} = \frac{1}{g_m + g_{mb}} \frac{1}{1 + a_f} \]

(b) Circuit diagram.

\[ V_0 = g_m (V_c - V_o) \frac{1}{g_{mb}} \]

\[ V_o = \frac{g_m}{R_i + \frac{g_m}{g_{mb}}} \]

\[ R = a_v \frac{g_m}{R_i + g_m + g_{mb}} \]

\[ = \frac{a_v g_m}{g_m + g_{mb}} \]

\[ = 838 \]

\[ A_{in} = \frac{V_o}{V_i} \mid a_v = 1 \times (V_i = 0 \text{ and } V_o = V_i) \]

\[ d = \frac{V_o}{V_i} \mid a_v = 0 \]

\[ = \frac{1}{g_m} \frac{1}{g_{mb}} \]

\[ R_{in} = \frac{1}{R_i + g_m + g_{mb}} \]

\[ = \frac{1}{g_m + g_{mb}} \frac{1}{R_i + \frac{g_m}{g_{mb}}} \]

\[ = \frac{1}{(4.2 \times 10^3 + 8.1 \times 10^4) 10^6} \]

\[ = 2.00 \times 10^4 \]

\[ A = A_{in} \frac{R}{1 + R} + \frac{d}{1 + R} \]

\[ = \frac{838}{1 + 838} + \frac{2.00 \times 10^4}{1 + 838} \]

\[ = 0.999 \]
Problem 3 – Continued

\[ a_w = 0 \]

\[ \begin{array}{c}
\text{Rin} (a_w = 0) = R_e + \frac{1}{g_m} \left( \frac{1}{g_{mb}} \right) \\
\text{R}(\text{short}) = R = 838 \\
\text{R}(\text{open}) = 0 \quad (V_i = 0) \\
\text{Rin} = \text{Rin} (a_w = 0) \frac{1 + R(\text{short})}{1 + R(\text{open})} \\
= R_e (1 + R) = 1M (1 + 838) = 839M \Omega \\
\end{array} \]

\[ \begin{array}{c}
\text{Rout} (a_w = 0) = \frac{1}{g_m} + \frac{1}{g_{mb}} \\
= g_m + g_{mb} \\
\text{R}(\text{short}) = 0 \quad (V_o = 0) \\
\text{R}(\text{open}) = R \\
\text{Rout} = \text{Rout} (a_w = 0) \frac{1 + R(\text{short})}{1 + R(\text{open})} \\
\approx \frac{1}{g_m + g_{mb}} \left( \frac{1}{1 + R} \right) \\
= 200 \frac{1}{1 + 838} = 0.238 \Omega \\
\end{array} \]
Problem 4 - (10 points)

Use the Blackman’s formula (see below) to calculate the small-signal output resistance of the stacked MOSFET configuration having identical drain-source drops for both transistors. Express your answer in terms of all the pertinent small-signal parameters and then simplify your answer if \( g_m > g_{ds} > (1/R) \). Assume the MOSFETs are identical.

\[
R_{out} = R_{out} (g_m=0) \left[ \frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})} \right]
\]

(You may use small-signal analysis if you wish but this circuit seems to be one of the rare cases where feedback analysis is more efficient.)

**Solution**

\[
R_{out} (g_m=0) = 2R \left( r_{ds1} + r_{ds2} \right)
\]

\[
RR(\text{port shorted}) = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}
\]

\[
RR(\text{port open}) = \frac{g_m 2r_{ds1} r_{ds2}}{r_{ds1}+r_{ds2}+2R}
\]

\[
R_{out} = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}} \left[ 1 + \frac{g_m 2r_{ds1} r_{ds2}}{r_{ds1}+r_{ds2}+2R} \right] = 2R \left( \frac{r_{ds1}+r_{ds2}+g_m 2r_{ds1} r_{ds2}}{r_{ds1}+r_{ds2}+2R} \right)
\]

Using the assumptions of \( g_m > g_{ds} > (1/R) \) we can simplify \( R_{out} \) as

\[
R_{out} \approx 2R \left( \frac{g_m 2r_{ds1} r_{ds2}}{g_{m2r_{ds2}r_{ds2}}} \right) = 2r_{ds1}
\]

What is the insight? Well there are two feedback loops, one a series (the normal cascode) and one a shunt. Apparently they are working against each other and the effective output resistance is pretty much what it would be if there were two transistors in series without any feedback.
Problem 5 – (10 points)

Find the dominant roots of the MOS follower and the BJT follower for the buffered, class-A op amp of Ex. 7.1-2. Use the capacitances of Table 3.2-1. Compare these root locations with the fact that \( GB = 5 \text{MHz} \)? Assume the capacitances of the BJT are \( C_\pi = 10 \text{pf} \) and \( C_\mu = 1 \text{pF} \).

\[ GB = 5 \text{MHz} \]

Solution

The model of just the output buffer of Ex. 7.1-2 is shown.

\[
\begin{align*}
V_{DD} & \quad 100 \mu A & \quad 1000 \mu A \\
M8 & \quad 10/1 & \quad \text{v}_i \\
M9 & \quad 90 \mu A & \quad 1467/1 \\
M10 & \quad C_L & \quad R_L \\
& \quad g_{m9}(V_i - V_1) \\
& \quad 100 \text{pF} & \quad 500 \Omega \\nV_{SS} & \quad \text{v}_o \\
Q10 & \quad R_1 & \quad C_1 \\
& \quad C_\pi & \quad r_\pi \\
& \quad g_{m10}(V_1 - V_o) & \quad V_o \\
\end{align*}
\]

The nodal equations can be written as,

\[
g_{m9}V_i = (g_{m9} + G_1 + g_{\pi 10} + sC_{\pi 10} + sC_1)V_1 - (g_{\pi 10} + sC_{\pi 10})V_o \\
0 = -(g_{m10} + g_{\pi 10} + sC_{\pi 10})V_1 + (g_{m10} + G_2 + g_{\pi 10} + sC_{\pi 10} + sC_2)V_o
\]

Solving for \( V_o/V_i \) gives,

\[
\frac{V_o}{V_i} = \frac{g_{m9}(g_{m10} + g_{\pi 10} + sC_{\pi 10})}{a_0 + sa_1 + s^2a_2}
\]

where

\[
\begin{align*}
a_0 &= g_{m9}g_{\pi 10} + g_{\pi 10}G_1 + g_{\pi 10}G_2 + g_{m9}g_{m10} + g_{m10}G_1 + g_{m9}G_2 + G_1G_2 \\
a_1 &= g_{m9}C_{\pi 10} + G_1C_{\pi 10} + G_2C_{\pi 10} + g_{\pi 10}C_1 + g_{\pi 10}C_2 + g_{m10}C_1 + g_{m10}C_2 + G_1C_2 \\
a_2 &= C_{\pi 10}C_1 + C_{\pi 10}C_2 + C_1C_2
\end{align*}
\]

The numerical value of the small signal parameters are:

\[
g_{m10} = \frac{1 \text{mA}}{25.9 \text{mV}} = 38.6 \text{mS}, \quad G_2 = 2 \text{mS}, \quad g_{\pi 10} = 386 \mu \text{S}, \quad g_{m9} = \sqrt{2 \cdot 50 \cdot 10 \cdot 90} = 300 \mu \text{S}, \quad G_1 = g_{ds8} + g_{ds9} = 0.05 \cdot 100 \mu \text{A} + 0.05 \cdot 90 \mu \text{A} = 9.5 \mu \text{S} \\
C_2 = 100 \text{pF}, \quad C_{\pi 10} = 10 \text{pF}, \quad C_1 = C_{gs9} + C_{bs9} + C_{bd8} + C_{gd8} + C_{\mu 10} \\
C_{gs9} = C_{ov} + 0.667C_{ox}W_9L_9 = (220 \times 10^{-12})(10 \times 10^{-6}) + 0.667(24.7 \times 10^{-4})(10 \times 10^{-12}) = 18.7 \text{fF}
\]
Problem 5 – Continued

\[ C_{bs9} = 560 \times 10^{-6} (30 \times 10^{-12}) + 350 \times 10^{-12} (26 \times 10^{-6}) = 25.9 \text{fF} \]

(Assumed area = 3µm x 10µm = 30µm and perimeter is 3µm + 10µm + 3µm + 10µm = 26µm)

\[ C_{bd8} = 560 \times 10^{-6} (438 \times 10^{-12}) + 350 \times 10^{-12} (298 \times 10^{-6}) = 349 \text{fF} \]

\[ C_{gd8} = C_{ov} = (220 \times 10^{-12}) (146 \times 10^{-6}) = 32.1 \text{fF} \]

∴ \[ C_1 = 18.7 \text{fF} + 25.9 \text{fF} + 349 \text{fF} + 32.1 \text{fF} + 1000 \text{fF} = 1.43 \text{pF} \]

(We have ignored any reverse bias influence on pn junction capacitors.)

The dominant terms of \( a_0 \), \( a_1 \), and \( a_2 \) based on these values are shown in boldface above.

\[ \frac{V_o}{V_i} \approx \frac{g_m g_{m10} g_{\pi10} C_{\pi10}}{g_m g_{m10} g_{\pi10} G_2 (s G_2 C_{\pi10} + s C_{\pi10} G_2) + s^2 C_{\pi10} C_2} \]

Assuming negative real axis roots widely spaced gives,

\[ p_1 = -\frac{1}{a} = \frac{-(g_m g_{m10} g_{\pi10} G_2)}{G_2 C_{\pi10} + g_{\pi10} C_2 + g_{m10} C_1 + g_{m9} C_2} = \frac{-1.235 \times 10^{-5}}{1.465 \times 10^{-13}} = -84.3 \times 10^6 \text{ rads/sec.} \]

= -13.4 MHz

\[ p_2 = -\frac{a}{b} = \frac{-G_2 C_{\pi10} + g_{\pi10} C_2 + g_{m10} C_1 + g_{m9} C_2}{C_{\pi10} C_2} = -\frac{1.235 \times 10^{-5}}{100 \times 10^{-12} \cdot 10 \times 10^{-12}} = -146.5 \times 10^6 \text{ rads/sec.} \rightarrow -23.32 \text{MHz} \]

\[ z_1 = -\frac{g_m}{C_{\pi10}} = -\frac{38.6 \times 10^{-3}}{10 \times 10^{-12}} = -3.86 \times 10^9 \text{ rads/sec.} \rightarrow -614 \text{MHz} \]

We see that neither \( p_1 \) or \( p_2 \) is greater than 10 GB if \( GB = 5 \text{MHz} \) so they will deteriorate the phase margin of the amplifier of Ex. 7.1-2.