Homework Assignment No. 13 - Solutions

Problem 1 - (10 points)

Problem 7.4-3 - Derive Eq. (17). If $A = 2$, at what value of v_{in}/nV_t will $i_{out} = 5I_5$ or $5I_b$ if b=1 ?

Solution

Start with the following relationships:

$$i_1 + i_2 = I_5 + A(i_2 - i_1) \qquad \text{Eq. (15)}$$

and $$\frac{i_2}{i_1} = \exp\left(\frac{v_{in}}{nV_t}\right) \qquad \text{Eq. (16)}$$

Defining $i_{out} = b(i_2 - i_1)$ solve for i_2 and I_1.

$$i_1 + i_1 \exp\left(\frac{v_{in}}{nV_t}\right) = I_5 + Ai_1 \exp\left(\frac{v_{in}}{nV_t}\right) - Ai_1$$

or $$i_1[(1+A) + (1-A) \exp\left(\frac{v_{in}}{nV_t}\right)] = I_5 \quad \rightarrow \quad i_1 = \frac{I_5}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

Similarly for i_2,

$$i_1 = \frac{I_5 \exp\left(\frac{v_{in}}{nV_t}\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

$$\therefore \quad i_{out} = b(i_2 - I_1) = i_{out} = (i_2 - I_1) = \frac{I_5\left(\exp\left(\frac{v_{in}}{nV_t}\right)-1\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)} \qquad \text{Eq. (17)}$$

Setting $i_{out} = 5I_5$ and solving for $\frac{v_{in}}{nV_t}$ gives,

$$5[3- \exp\left(\frac{v_{in}}{nV_t}\right)] = \exp\left(\frac{v_{in}}{nV_t}\right) -1 \quad \rightarrow \quad 16 = 6 \exp\left(\frac{v_{in}}{nV_t}\right) \quad \rightarrow \quad \exp\left(\frac{v_{in}}{nV_t}\right) = 2.667$$

$$\therefore \quad \frac{v_{in}}{nV_t} = \ln(2.667) = \underline{\underline{0.9808}}$$

Problem 2 - (10 points)

Problem 7.5-5. Find the equivalent rms noise voltage of the op amp designed in Example 6.5-2 over a bandwidth of 1Hz to 100kHz. Use the values for *KF* of Example 7.5-1.

Solution

The circuit for this amplifier is shown.

The *W*/*L* ratios in microns are:

$S_1 = S_2 = 12/1$

$S_3 = S_4 = 16/1$

$S_5 = 7/1$

$S_5 = 8.75/1$

$S_6 = S_7 = S_8 = S_{14} = S_{15} = 40/1$

$S_9 = S_{10} = S_{11} = S_{12} = 18.2/1$

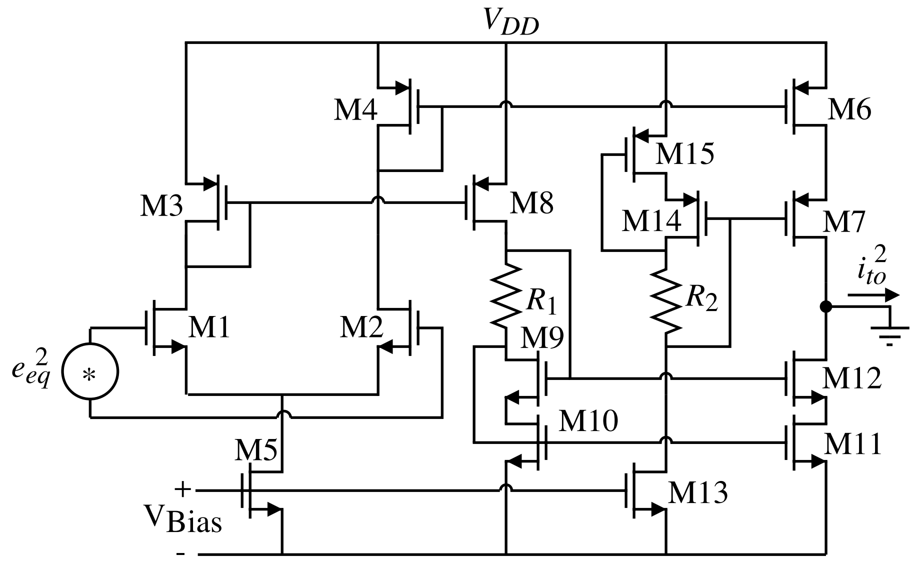

Fig. S7.5-05

Find the short circuit noise current at the output, i_{to}^2, due to each noise-contributing transistor in the circuit (we will not includeM7, M9, M12 and M14 because they are cascodes and their effective g_m is small. The result is,

$$i_{to}^2 = 2g_{m1}^2 e_{n1}^2 \left(\frac{g_{m8}^2}{g_{m3}^2}\right) + 2g_{m8}^2 e_{n3}^2 + \ 2g_{m8}^2 e_{n8}^2 + + 2g_{m11}^2 e_{n10}^2$$

where we have assumed that g_{m1}=g_{m2}, g_{m3}=g_{m4}, g_{m6}=g_{m8}, and g_{m10}=g_{m11} and e_{n1}=e_{n2}, e_{n3}=e_{n4}, e_{n6}=e_{n8}, and e_{n10}=e_{n11}. Dividing i_{to}^2 by the transconductance gain gives

$$e_{eq}^2 = \frac{i_{to}^2}{g_{m1}^2 g_{m8}^2 / g_{m3}^2} = 2e_{n1}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n3}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n8}^2 + 2\left(\frac{g_{m3}^2 g_{m11}^2}{g_{m1}^2 g_{m8}^2}\right)e_{n10}^2$$

The values of the various parameters are:

$g_{m1} = 251\mu\text{S}$, $g_{m3} = 282.5\mu\text{S}$, $g_{m8} = 707\mu\text{S}$, and $g_{m11} = 707\mu\text{S}$.

$$\therefore \qquad e_{eq}^2 = 2e_{n1}^2\left[1 + 1.266\left(\frac{e_{n3}^2}{e_{n1}^2} + \frac{e_{n8}^2}{e_{n1}^2} + \frac{e_{n10}^2}{e_{n1}^2}\right)\right]$$

Problem 7.5-5 – Continued

1/f Noise:

Using the results of Ex. 7.5-1 we get B_N = 7.36x10^{-22}(V·m)^2 and B_p = 2.02x10^{-22}(V·m)^2

$$e_{n1}^2 = \frac{B_N}{fW_1L_1} = \frac{7.36\text{x}10^{-22}}{f\cdot 12\text{x}10^{-12}} = \frac{6.133\text{x}10^{-11}}{f} \; V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_3L_3} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 16} = 0.2058$$

$$\frac{e_{n8}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_8L_3} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 40} = 0.0823$$

$$\frac{e_{n10}^2}{e_{n1}^2} = \frac{B_N\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_{10}L_{10}} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{12}{18.2} = 0.6593$$

$$\therefore \quad e_{eq}^2 = 2\,\frac{6.133\text{x}10^{-11}}{f}[1+1.266(0.2058+0.0823+0.6593)] = 2\,\frac{6.133\text{x}10^{-11}}{f}2.1995$$

$$e_{eq}^2 = \frac{2.1995\text{x}10^{-10}}{f} \; V^2/\text{Hz}$$

Thermal noise:

$$e_{n1}^2 = \frac{8kT}{3g_{m1}} = \frac{8\cdot 1.38\text{x}10^{-23}\cdot 300}{3\cdot 251\text{x}10^{-6}} = 4.398\text{x}10^{-17} \; V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m3}} = \frac{251}{282.4} = 0.8888 \quad \text{and} \quad \frac{e_{n8}^2}{e_{n1}^2} = \frac{e_{n10}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m8}} = \frac{251}{707} = 0.0.355$$

The corner frequency is f_c = 2.698x10^{-10}/2.66x10^{-16} = 1.01x10^{6} Hz. Therefore in a 1Hz to 100kHz band, the noise is 1/*f*. Solving for the *rms* value gives,

$$V_{eq}^2\text{ (rms)} = \int_1^{100,000} \frac{2.698\text{x}10^{-10}}{f}df = 2.698\text{x}10^{-10}[\ln(100,000) - \ln(1)]$$

$$= 3.1062\text{x}10^{-9}\text{ V}^2\text{(rms)}$$

$\therefore \quad V_{eq}$(rms) = <u>55.73μV(rms)</u>

Problem 3 - (10 points)

Problem 7.6-1 - If the W and L of all transistor in Fig. 7.6-3 are 100μm and 1μm, respectively, find the lowest supply voltage that gives a zero value of *ICMR* if the dc current in M5 is 100μA.

Solution:

$$I_5 = 100\ \ \mu A\text{, and } \left(\frac{W}{L}\right) = 100$$

$$V_{IC}(\max) = V_{DD} + V_{T1}(\min) - V_{dsat3}$$

and, $$V_{IC}(\min) = V_{dsat1} + V_{T1}(\max) + V_{dsat5}$$

The input common-mode range is

$$ICMR = V_{IC}(\max) - V_{IC}(\min)$$

For ICMR=0

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\max) - V_{T1}(\min)$$

or, $$V_{DD} = \sqrt{\frac{2I_1}{K_N^{'}S_1}} + \sqrt{\frac{2I_5}{K_N^{'}S_5}} + \sqrt{\frac{2I_3}{K_P^{'}S_3}} + V_{T1}(\max) - V_{T1}(\min)$$

or, $$V_{DD} = \underline{\underline{0.671\text{V}}}$$

Problem 4 - (10 points)

Problem 7.6-2 - Repeat Problem 7.6-1 if M1 and M2 are natural MOSFETs with a V_T = 0.1V and the other MOSFET parameters are given in Table 3.1-2.

Solution

$$V_{T1} = 0.1\text{V},\ I_5 = 100\ \mu A\text{, and } \left(\frac{W}{L}\right) = 100$$

Let, the variation in the threshold voltage be $\pm 20\%$

or, $\Delta V_{T1} = \pm 0.02\ \text{V}$

$$V_{IC}(\text{max}) = V_{DD} + V_{T1}(\text{min}) - V_{dsat3}$$

and, $V_{IC}(\text{min}) = V_{dsat1} + V_{T1}(\text{max}) + V_{dsat5}$

The input common-mode range is

$$ICMR = V_{IC}(\text{max}) - V_{IC}(\text{min})$$

For ICMR=0

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

or, $V_{DD} = \sqrt{\frac{2I_1}{K'_N S_1}} + \sqrt{\frac{2I_5}{K'_N S_5}} + \sqrt{\frac{2I_3}{K'_P S_3}} + V_{T1}(\text{max}) - V_{T1}(\text{min})$

or, $V_{DD} = \underline{\underline{0.411\text{V}}}$

Problem 5 - (10 points)

Problem 7.6-3 - Repeat Problem 7.6-1 if M1 and M2 are depletion MOSFETs with a V_T = -1V and the other MOSFET parameters are given in Table 3.1-2.

Solution

$$V_{T1} = -1\text{ V},\ I_5 = 100\ \ \mu A\text{, and } \left(\frac{W}{L}\right) = 100$$

Let, the variation in the threshold voltage be $\pm 20\%$

or, $\Delta V_{T1} = \mp 0.2\ \text{V}$

$$V_{IC}(\max) = V_{DD} + V_{T1}(\min) - V_{dsat3}$$

and, $V_{IC}(\min) = V_{dsat1} + V_{T1}(\max) + V_{dsat5}$

The input common-mode range is

$$ICMR = V_{IC}(\max) - V_{IC}(\min)$$

For ICMR=0

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\max) - V_{T1}(\min)$$

or, $V_{DD} = \sqrt{\frac{2I_1}{K_N' S_1}} + \sqrt{\frac{2I_5}{K_N' S_5}} + \sqrt{\frac{2I_3}{K_P' S_3}} + V_{T1}(\max) - V_{T1}(\min)$

or, $V_{DD} = \underline{\underline{0.771\text{V}}}$