EXAMINATION NO. 1 - SOLUTIONS

(Average Score = 71/100)

Problem 1 - (25 points)

Short questions. The point worth of the question is given in parentheses.

a.) (7) Explain clearly and concisely the difference between direct frequency synthesis and indirect frequency synthesis.

The main difference between direct frequency synthesis and indirect synthesis is that the output of a direct synthesizer is directly obtained from the reference oscillator by doing mathematical operations on its frequency while the output of an indirect synthesizer is produced by another oscillator (VCO)) that is forced to have a frequency that is related to the reference frequency. The VCO output frequency is usually a multiple of the reference frequency.

b.) (7) Why are direct synthesizers able to change frequencies much faster than indirect synthesizers? Explain.

The frequency-switching time of an indirect synthesizer is limited by the lock-up speed of the PLL. This can take tens of microseconds or even milliseconds. On the other hand, a direct synthesizer changes frequency by opening and closing switches. This can be very fast. The main source of delay in direct synthesizers is the delay and build-up time of the required filters.

c.) (6) Compare the characteristics, disadvantages and advantages of the linear multiplier phase detector, the EXOR phase detector, the JK phase detector, and the phase frequency detector.

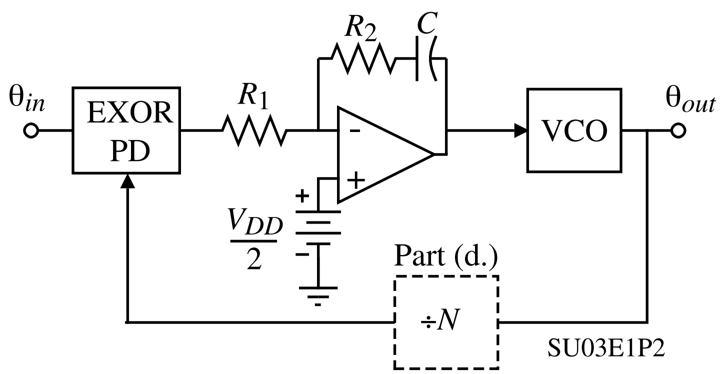
Type of PD	Advantages	Disadvantages
Linear Multiplier	Largest K_d, compatible with sinusoidal signals	Smallest pull-in range, complex circuits, nonlinear, harmonics must be filtered
EXOR	Simple digital circuit, not sensitive to fading	Limited pull-in range compared to EXOR and PFD
JK	Simple digital circuit, insensitive to asymmetry	Sensitive to fading
PFD	Simple digital circuits, largest pull-in range	Smallest K_d, sensitive to fading, dead zone

d.) (5) Compare the disadvantages and advantages of a charge pump and a filter in a PLL application.

Type of Filter	Advantages	Disadvantages
Filter	Passive is linear and uses no power, can have many different transfer functions	Requires off-chip components, op amps have nonidealities
Charge-Pump	Does not use op amps, can provide a pole at s=0, more compatible with IC technology	Leakage currents, mismatches, charge sharing, needs a zero for good phase margin

Problem 2 - (25 points)

Consider the PLL shown. Assume that: 1.) the phase detector is a simple CMOS EXOR whose logic levels are ground and V_{DD} = 5V, 2.) both the input to the loop and the VCO output are square waves that swing between ground and V_{DD}, and 3.) that the VCO has a perfectly linear relationship between the control voltage and output frequency of 10 MHz/V. The polarities are such that an increase in control voltage causes an increase in the VCO frequency.



(a.) Derive the expression for the open-loop transmission and the transfer function $\theta_{out}(s)/\theta_{in}(s)$.

(b.) Initially assume $R_2 = 0$ and $R_1 = 10\text{k}\Omega$. What value of C gives a loop crossover frequency of 100kHz? What is the phase margin. Assume the op amp is ideal.

(c.) With the value of C from part (b.), what value of R_2 will provide a phase margin of 45° while preserving a 100 kHz crossover frequency.

(d.) Now assume that a frequency divider of factor N is inserted into the feedback path. With the component values of part (c.), what is the largest value of N that can be tolerated without shrinking the phase margin below 14°?

Solution

(a.) $$\theta_{out}(s) = \frac{K_o}{s} F(s) K_d \left(\theta_{in}(s) + \frac{\theta_{out}(s)}{N}\right) = \frac{5K_o}{s\pi} F(s)\left(\theta_{in}(s) + \frac{\theta_{out}(s)}{N}\right)$$

$$K_d = \frac{5\text{V}}{\pi} \quad \text{and} \quad F(s) = -\frac{R_2 + (1/sC)}{sR_1C} = -\frac{sR_2C+1}{sR_1C} = -\frac{s\tau_2+1}{s\tau_1},\ \tau_1 = R_1C \ \text{ and } \ \tau_2 = R_2C$$

$$\therefore\ \theta_{out}(s) = -\frac{5K_o}{s\pi}\left(\frac{s\tau_2+1}{s\tau_1}\right)\left(\theta_{in}(s) + \frac{\theta_{out}(s)}{N}\right)$$

$$\theta_{out}(s)\left[1 + \frac{5K_o}{s\pi N}\left(\frac{s\tau_2+1}{s\tau_1}\right)\right] = \frac{5K_o}{s\pi}\left(\frac{s\tau_2+1}{s\tau_1}\right)\theta_{in}(s)$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{-\frac{5K_o}{\pi\tau_1}(s\tau_2+1)}{s^2 + \frac{5K_o}{\pi N}\frac{\tau_2}{\tau_1}s + \frac{5K_o}{\pi N\tau_1}}$$

Problem 2 – Continued

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{-\frac{5K_o}{\pi\tau_1}(s\tau_2+1)}{s^2+\frac{5K_o}{\pi N}\frac{\tau_2}{\tau_1}s + \frac{5K_o}{\pi N\tau_1}} \text{ and the loop gain } = LG = -\frac{5K_o}{sN\pi}\left(\frac{s\tau_2+1}{s\tau_1}\right)$$

Assume $N = 1$ to get the answer to part (a.).

(b.) With $R_2 = 0$, $\tau_2 = 0$ so that the loop gain becomes,

$$LG = -\frac{5K_o}{s^2\tau_1 N\pi} = \frac{5\cdot 2\pi x10^7}{s^2\tau_1\pi} = \frac{10^8}{\omega_c{}^2\tau_1} = 1 \quad \rightarrow \quad \tau_1 = \frac{10^8}{(2\pi\cdot 10^5)^2} = 253.3\mu\text{sec.}$$

$$\tau_1 = R_1C \rightarrow 253.3\mu\text{sec.} = 10\text{k}\Omega C \rightarrow \underline{C = 25.3\text{nF}}$$

The phase margin is 0°.

(c.) The phase margin is totally due to τ_2. It is written as,

$$\text{PM} = tan^{-1}(\omega_c\tau_2) = 45° \quad \rightarrow \quad \omega_c\tau_2 = 1 \rightarrow \tau_2 = \frac{1}{\omega_c} = \frac{1}{2\pi x10^5} = 1.5915\mu\text{s} = R_2C$$

$$\therefore \quad R_2 = \frac{1}{2\pi x10^5 25.3x10^{-9}} = \underline{62.83\Omega}$$

(d.) N does not influence the phase shift so we can write,

$$tan^{-1}(\omega_c\tau_2) = 14° \rightarrow \omega_c'\tau_2 = 0.2493 \rightarrow \omega_c' = 0.2493\omega_c = 156{,}657 \text{ rads/sec.}$$

Now the loop gain at ω_c' must be unity.

$$LG = -\frac{5K_o}{\omega_c' N\pi}\left(\frac{\sqrt{(\omega_c'\tau_2)^2+1}}{\omega_c'\tau_1}\right) = 1 \rightarrow N = \frac{5K_o}{(\omega_c')^2\pi\tau_1}\sqrt{(\omega_c'\tau_2)^2+1}$$

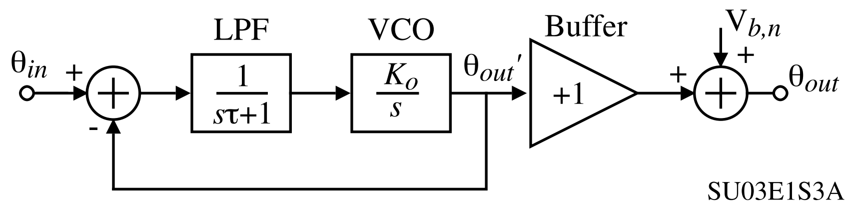
$$N = \frac{10^8}{(156{,}657)^2 253.3x10^{-6}}\sqrt{(0.2493)^2+1} = 16.58 = \underline{16}$$

Problem 3 - (25 points)

In many practical applications, it is necessary to buffer the VCO output signal. Assume that the buffer has a voltage gain of 1 and its output is corrupted by an additive white noise noise voltage of $V_{b,n}$.

(a.) Find the output phase, $\theta_{out}(s)$, as a function of the input phase $\theta_{in}(s)$ and the output noise of the buffer, $V_{b,n}$, for the PLL shown with the buffer outside of the PLL loop.

Give an approximate sketch for magnitude response of $\theta_{out}(j\omega)/V_{b,n}$ assuming $\zeta = 0.707$.



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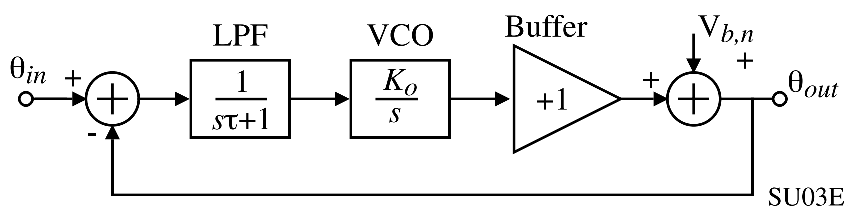
(b.) Find the output phase, $\theta_{out}(s)$, as a function of the input phase $\theta_{in}(s)$ and the output noise of the buffer, $V_{b,n}$, for the PLL shown with the buffer outside of the PLL loop.

Give an approximate sketch for magnitude response of $\theta_{out}(j\omega)/V_{b,n}$ assuming $\zeta = 0.707$.



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(c.) Which of the two PLL architectures leads to an output spectrum with less noise assuming that the input and VCO are noise free? How would your answer change if the input signal to the PLL was noisy? Why?

Solution

(a.) $\theta_{out}'(s) = \frac{K_o}{s} F(s)K_d[\theta_{in}(s)-\theta_{out}'(s)] \quad \rightarrow \quad \theta_{out}'(s) = \frac{K_vF(s)}{s+K_vF(s)}\theta_{in}(s)$

Substituting for $F(s)$ gives

$$\theta_{out}'(s) = \frac{K_v/\tau}{s^2+(s/\tau)+(K_v/\tau)}\theta_{in}(s) = \frac{\omega_n^2}{s^2+s2\zeta\omega_n+\omega_n^2}\theta_{in}(s)$$

$$\boxed{\theta_{out}(s) = \theta_{out}'(s) + V_{b,n} = \frac{\omega_n^2}{s^2+s2\zeta\omega_n+\omega_n^2}\theta_{in}(s) + V_{b,n}(s)}$$

(b.) $\theta_{out}(s) = \frac{K_o}{s} F(s)K_d[\theta_{in}(s)-\theta_{out}(s)] + V_{b,n}(s)$

$$\theta_{out}(s)\left[1+\frac{K_vF(s)}{s}\right] = \frac{K_vF(s)}{s}\theta_{in}(s) + V_{b,n}(s)$$

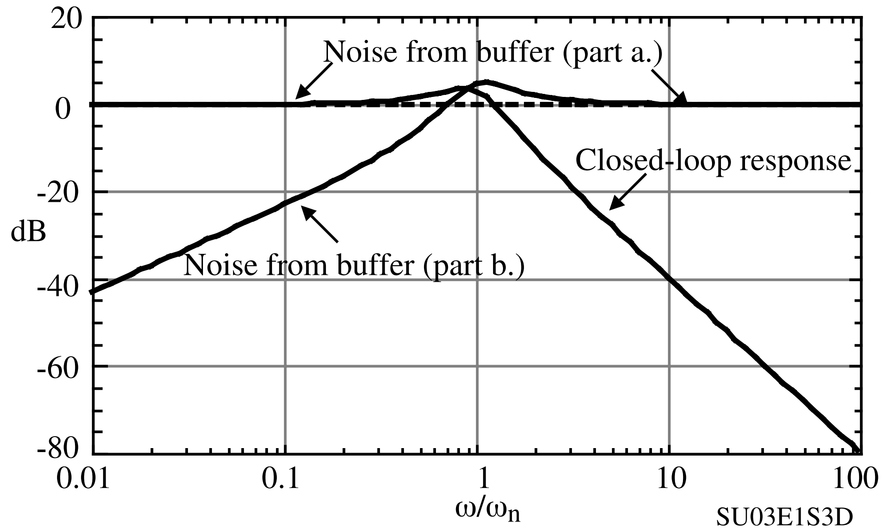
$$\therefore\ \theta_{out}(s) = \frac{K_vF(s)}{s+K_vF(s)}\theta_{in}(s) + \frac{s}{s+K_vF(s)}V_{b,n}(s)$$

$$\theta_{out}(s) = \frac{K_v/\tau}{s^2+(s/\tau)+(K_v/\tau)}\theta_{in}(s) + \frac{s^2+(s/\tau)}{s^2+(s/\tau)+(K_v/\tau)}V_{b,n}(s)$$

Problem 3 – Continued

$$\theta_{out}(s) = \frac{\omega_n^2}{s^2+s2\zeta\omega_n+\omega_n^2}\,\theta_{in}(s) + \frac{s^2+s2\zeta\omega_n}{s^2+s2\zeta\omega_n+\omega_n^2}\,V_{b,n}(s)$$

Sketch for both parts (a.) and (b.) are shown below.

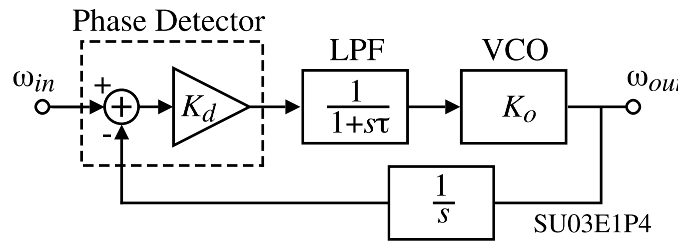


(c.) Obviously, part (b.) leads to an output spectrum with less noise. Part (a.) has the same noise contribution from the buffer regardless of the frequency. If the input is noisy then it will have a spectrum shown above similar to the closed-loop response. When the input noise is larger than the buffer noise, there is not much difference between the two architectures.

Problem 4 - (25 points)

A linear model of a PLL is shown. (a.) Solve for the closed-loop transfer function of $\omega_{out}(s)/\omega_{in}(s)$. Compare this transfer function with the following general transfer function and identify , H, ω_n, and ζ.

$$\frac{\omega_{out}(s)}{\omega_{in}(s)} = \frac{H\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



(b.) If $\zeta < 1$, the step response to $\omega_{in}(t) = \Delta\omega\cdot\mu(t)$ is given as

$$\omega_{out}(t) = H\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \theta\right)\right] \text{ where } \theta = \sin^{-1}\sqrt{1-\zeta^2}$$

Assume that $K_v = K_oK_d = 63.58\text{x}10^3$ rads/sec. and $\tau = 8\mu$sec. If the output frequency is to be changed from 901 MHz to 901.2 MHz, how long does the PLL output frequency take to settle with 100 Hz of its final value? Simplify your analysis by assuming worst case conditions (i.e. Maximum value of $\sin(x) = 1$).

Solution

Find the transfer function in terms of phase and then convert to frequency.

$$\theta_{out}(s) = \left(\frac{K_o}{s}\right)\left(\frac{1}{s\tau+1}\right) K_d[\theta_{in}(s)-\theta_{out}(s)] \rightarrow \theta_{out}(s)\left[1 + \frac{K_v}{s(s\tau+1)}\right] = \frac{K_v}{s(s\tau+1)}\theta_{in}(s)$$

where $K_v = K_oK_d$. Solving for the phase transfer function gives,

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{\omega_{out}(s)}{\omega_{in}(s)} = \frac{\frac{K_v}{\tau}}{s^2 + \frac{s}{\tau} + \frac{K_v}{\tau}} \rightarrow \boxed{\omega_n = \sqrt{\frac{K_v}{\tau}} = 89{,}148 \text{ rads/sec.}}$$

$$\boxed{\zeta = \frac{1}{2\sqrt{K_v\tau}} = 0.701} \text{ and } \boxed{H = 1}$$

(b.) The frequency response can be written as

$$f_{out}(t) = 200\text{kHz}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \theta\right)\right]\mu(t)$$

Setting $f_{out}(t_s) = 200\text{kHz} - 100\text{Hz}$, gives

$$200\text{x}10^3 - 100 = 200\text{x}10^3\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_s} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t_s + \theta\right)\right]$$

This equation simplifies to the following assuming the value of the $\sin(x)$ is 1.

$$\frac{100\text{Hz}}{200\text{kHz}} = \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_s} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t_s + \theta\right) \approx \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_s}$$

$$e^{\zeta\omega_n t_s} = \frac{2000}{\sqrt{1-\zeta^2}} = 2800 \rightarrow t_s = \frac{1}{\omega_n\zeta} ln(2800) = 2\tau(7.9375) = \underline{\underline{127\mu\text{sec}}}.$$