EXAMINATION NO. 2 - SOLUTIONS

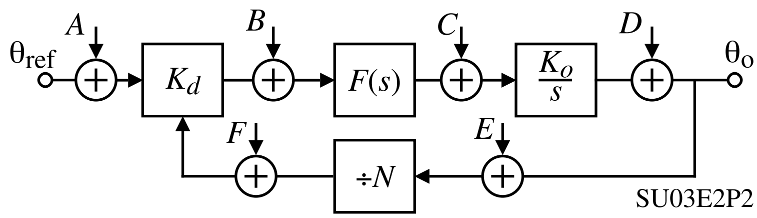
(Average Score = 80/100)

Problem 1 - (25 points)

A frequency synthesizer is shown below and has the following parameters:

$F(s) = \frac{1 + 0.01s}{s}$ $\quad K_o = 2\text{x}10^6$ (rads/V) $\quad K_d = 0.8$ (V/rad.)

$\beta = 2\pi$ $\quad N = 150$ $\quad f_{ref} = 120$ kHz



SU03E2P2

(a.) Where would you introduce the modulating voltage, v_p, if you wish to phase modulate the output of the synthesizer (A, B, C, D, E, or F)?

(b.) What is the peak amplitude of a 1kHz ac signal needed to produce an output peak phase deviation of 0.5 radians?

Solution

(a.) The modulating voltage should be introduced at B.

(b.) The transfer function between the input modulating voltage and the output phase is given as,

$$\theta_o(s) = \frac{K_o}{s} F(s)\left[V_p(s) - K_d \frac{\theta_o}{N}\right] \quad \rightarrow \quad \theta_o(s)\left[1 + \frac{K_o K_d F(s)}{sN}\right] = \frac{K_o F(s)}{s} V_p(s)$$

$$\frac{\theta_o(s)}{V_p(s)} = \frac{K_o F(s)}{s + \frac{K_v F(s)}{N}} = \frac{K_o(1 + 0.01s)}{s^2 + \frac{0.01 K_v}{N} s + \frac{K_v}{N}}$$

$$\therefore \omega_n = \sqrt{\frac{K_v}{N}} = \sqrt{\frac{1.6\text{x}10^6}{150}} = 103 \text{ rads/sec. (16.4 Hz) and } \zeta = \frac{K_v}{100N}\sqrt{\frac{N}{K_v}} \approx 1$$

Since, $f_n << 1$kHz, the transfer function can be approximated as,

$$\left|\frac{\theta_o(j\omega)}{V_p(j\omega)}\right| \approx \frac{0.01K_o}{\omega} = \frac{20{,}000}{2000\pi} = 3.183$$

∴ A phase deviation of 0.5 radians requires a modulating voltage of 0.5/3.183 or 0.157V

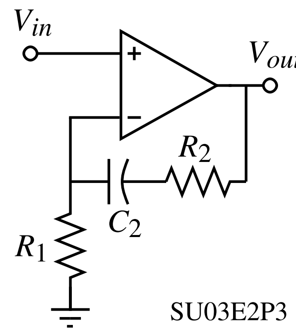
Peak deviation of the modulating voltage = 0.157V

Problem 2 - (25 points)

(a.) Find the transfer function of the filter shown assuming an ideal op amp.

(b.) Sketch a Bode plot for the magnitude of this filter if $R_1 = R_2 = 10\text{k}\Omega$ and $C_2 = 0.159\mu\text{F}$.

(c.) For the values in part (b.), find the single sideband spur at a reference frequency of 25 kHz if the op amp has an input offset current of $I_{os} = 50\text{nA}$ and an input offset voltage of $V_{io} = 100\mu\text{V}$. Assume that the spurious deviation due to the offset voltage at 25 kHz can be expressed as $\theta_d = 100V_{pm}$, where V_{pm} is the phase modulation caused by the offset voltage of the filter.



SU03E2P3

Solution

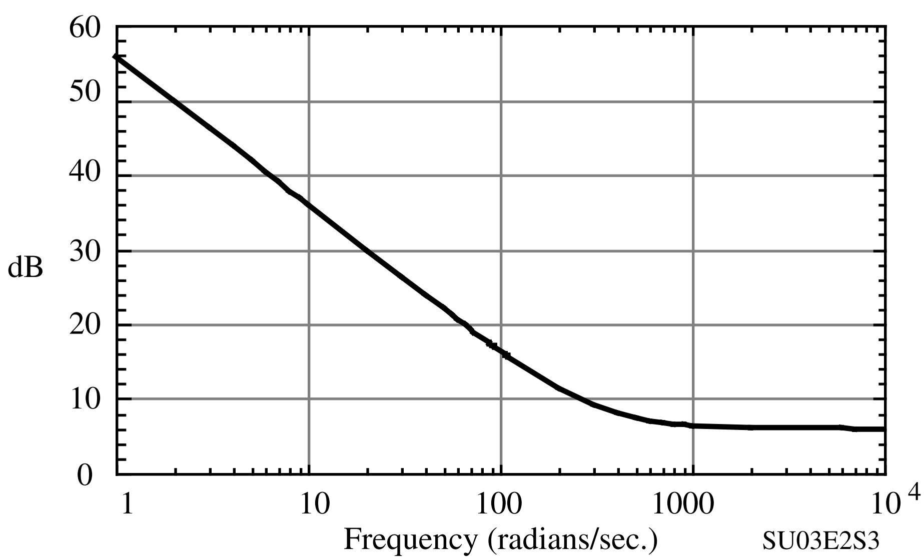
(a.) The transfer function assuming an ideal op amp can be found as,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_1+Z_2}{Z_1} = \frac{R_1 + R_2 + (1/sC_2)}{R_1} = \frac{s(R_1+R_2)C_2 + 1}{sC_2R_1}$$

(b.) If $R_1 = R_2 = 10\text{k}\Omega$ and $C_2 = 0.159\mu\text{F}$, then the filter transfer function becomes,

$$F(s) = \frac{s(R_1+R_2)C_2 + 1}{sC_2R_1} = \frac{s0.00318 + 1}{0.00159s} = \frac{\frac{s}{314.5} + 1}{\frac{s}{628.9}}$$

The sketch for the magnitude of this transfer function is below.



SU03E2S3

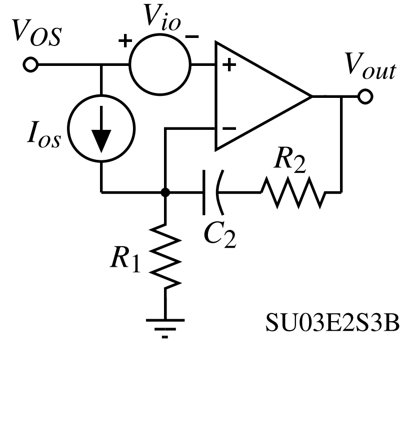
(c.) First, find the offset voltage at the input of the filter, V_{OS}, from the figure shown.

$$V_{OS} = V_{io} + I_{os}R_1 = 100\mu\text{V} + 50\text{nA}\cdot 10\text{k}\Omega$$

$$V_{OS} = 0.1\text{mV} + 0.5\text{mV} = 0.6\text{mV} = 600\mu\text{V}$$

$$\therefore\ \theta_d = 100V_{pm} = 100(2\cdot V_{OS}) = 0.12$$

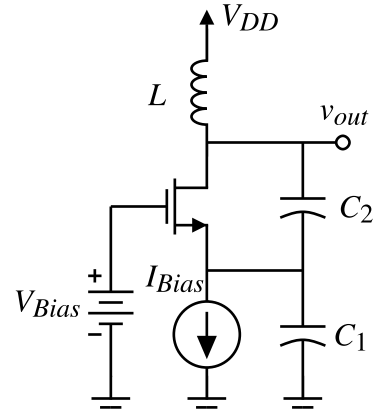
$$SSB = 20\log_{10}(\theta_d/2) = \underline{\underline{-24.44\ \text{dBc}}}$$



SU03E2S3B

Problem 3 - (25 points)

Find the oscillation frequency, ω_{osc} and the value of $g_m r_{ds}$ necessary to oscillate in terms of L, C_1, and C_2 for the LC oscillator shown.



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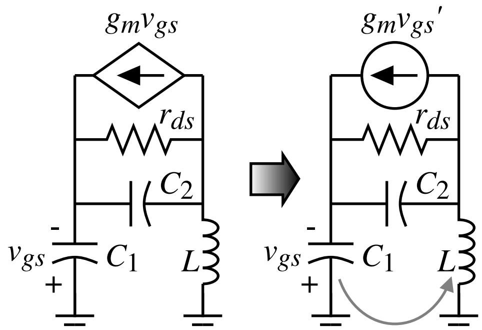
Solution

The small-signal model for solving this problem is shown below.



SU03E2S4

Note that the current from the independent source has two paths. One is through the parallel combination of r_{ds} and C_2, and the other is through C_1 and L.

The open-loop gain, V_{gs}'/V_{gs} can be found as,

$$\frac{V_{gs}(s)}{V_{gs}(s)} = -\left(\frac{1}{sC_1}\right)\left[\frac{g_m\,[r_{ds}||(1/sC_2)]}{sL + (1/sC_1) + r_{ds}||(1/sC_2)}\right] = -\left(\frac{1}{sC_1}\right)\left[\frac{g_m\left(\frac{r_{ds}}{sC_2 r_{ds}+1}\right)}{\frac{s^2LC_1+1}{sC_1} + \frac{r_{ds}}{sC_2 r_{ds}+1}}\right]$$

$$LG(s) = \frac{V_{gs}(s)}{V_{gs}(s)} = -\left[\frac{g_m r_{ds}}{(s^2LC_1+1)(sC_2 r_{ds}+1) + sC_1 r_{ds}}\right]$$

$$= -\left[\frac{g_m r_{ds}}{s^3LC_1C_2 r_{ds} + s^2LC_1 + s r_{ds}\,(C_1+C_2) + 1}\right]$$

$$LG(j\omega) = \left[\frac{-g_m r_{ds}}{1 - \omega^2LC_1 + j\omega[\,r_{ds}(C_1+C_2) - \omega^2LC_1C_2 r_{ds}\,]}\right] = 1 + j0$$

$$\therefore \quad \omega_{osc}{}^2 = \frac{C_1+C_2}{LC_1C_2} \quad \rightarrow \quad \boxed{\omega_{osc} = \frac{1}{\sqrt{\frac{LC_1C_2}{C_1+C_2}}}}$$

At the oscillation frequency, we can write that,

$$-g_m r_{ds} = 1 - \omega_{osc}{}^2 LC_1 = 1 - \frac{C_1+C_2}{C_2} = 1 - 1 - \frac{C_1}{C_2} = -\frac{C_1}{C_2}$$

$$\therefore \quad \boxed{g_m r_{ds} = \frac{C_1}{C_2}}$$

Problem 4 – (25 points)

A model for single sideband noise using the time-invarient theory is given by

$$\mathcal{L}\{f_m\} = 10 \log \left\{ \frac{2FkT}{P_s} \left[1 + \frac{1}{4Q^2}\left(\frac{f_o}{f_m}\right)^2 \right] \left(1 + \frac{f_c}{f_m}\right) \right\}$$

(a.) Describe each term in this equation and give the units of the term.

(b.) If F = 2dB, what is the noise floor if the carrier power is 10 dBm at room temperature (27°C) and $k = 1.381\text{x}10^{-23}$ Joules/K°?

(c.) Make an approximate sketch of $\mathcal{L}\{f_m\}$ in dBc as a function of log10(f_m) and identify the various regions.

Solution

(a.)

F = the noise figure or factor depending upon terminology. It is unitless.
k = Boltsmann's constant and is equal to $1.381\text{x}10^{-23}$ Joules/K°.
T = temperature in °K
P_s = power in the carrier in watts.
Q = open-loop Q of the oscillator. It is unitless.
f_o = carrier frequency in Hz.
f_m = deviation frequency from the carrier in Hz.
f_c = corner frequency in Hz associate where the $1/f$ noise is no longer significant.

(b.) The noise floor is 10 log $\left(\frac{2FkT}{P_s}\right)$. We need to perform some "preprocessing" first before using the equation.

$$F = 2\text{dB} \rightarrow F = 10^{2/10} = 1.585 \quad \text{and} \quad P_s = 10\text{dBm} \quad \rightarrow \quad P_s = 10^{10/10} = 10\text{mW}$$

$$\mathcal{L}\{f_m\} = 10 \log\left(\frac{2\cdot 1.585\cdot 1.381\text{x}10^{-23}\cdot 300}{10\text{x}10^{-3}}\right) = -178.8 \text{ dBc}$$

(c.)