**FINAL EXAMINATION - SOLUTIONS**

(Average Score = 77/100)

**Problem 1 - (20 points – This problem must be attempted)**

A DPLL frequency synthesizer has the following parameters:

\[ F(s) = \frac{1 + 0.01s}{s} \quad K_o = 2 \times 10^6 \text{ (rads/V)} \quad K_d = 0.8 \text{V/rad} \]

\[ \beta = 2\pi \quad N = 150 \quad f_{ref} = 120 \text{ kHz} \]

The temperature is 290°K and all circuits operate from a ±5V power supply.

(a.) What is the output frequency hold range in Hz?

(b.) What is the output frequency lock (capture) range in Hz? What is the lock (capture) time in seconds?

(c.) Assume that you wish to phase modulate the output of the synthesizer, where would you introduce the modulating voltage? What is the peak amplitude of the 1 kHz ac signal needed to produce an output peak phase deviation of 0.5 radians?

**Solution**

(a.) Since the filter is active PI, the hold range is limited by the loop components and is

\[ \Delta \omega_{max} = \Delta \omega_H = \pm 5V \cdot K_o = \pm 5V(2\times10^6 \text{ (rads/V)}) = \pm 10\text{Mrads/sec}. \]

\[ \therefore \Delta f_H = \frac{\pm 10\text{Mrads/sec}}{2\pi} = \pm 1.592 \text{ MHz} \]

(b.) First we find \( K \) (Lecture 90-02).

\[ K = \frac{K_d K_o}{N} = \frac{0.8\cdot2 \times 10^6}{150} = 10,667 \]

From Lecture 90-09,

\[ \omega_n = \sqrt{\frac{K}{\tau_1}} = \sqrt{\frac{10,667}{1}} = 103.3 \text{ Rads/sec} \quad \text{and} \quad \zeta = \frac{\tau_2 \omega_n}{2} = \frac{0.01\cdot103.3}{2} = 0.516 \]

\[ \therefore \Delta f_L = \frac{2\beta N \zeta \omega_n}{2\pi} = \frac{2(2\pi)150\cdot0.516\cdot103.3}{2\pi} = 15,991 \text{ Hz} \rightarrow \Delta f_L = \pm 15,991 \text{Hz} \]

\[ t_L = \frac{2\pi}{\omega_n} = \frac{2\pi}{103.3} = 60.8 \text{ msec} \]

(c.) See the following block diagram.

\[ v_p = \frac{\theta_o}{\tau_2 K_o} \rightarrow \frac{\theta_o}{\tau_1 \omega} = \frac{\tau_2 K_o}{\tau_1 \omega} = 0.5 \left( \frac{1.2000\pi}{0.01\cdot2\times10^6} \right) = 0.157 \text{V} \]

\[ v_p = 0.157 \text{V} \]

What is the BW? From Lect. 90-09,

\[ BW = 103.3\sqrt{2(0.51\omega)^2 + 1} + \sqrt{2(0.51\omega)^2 + 1} = 103.3(1.546) = 159.75 \text{ rads/sec.} \]

or \( BW = 25.42 \text{ Hz} \)

Since, 1kHz >> BW, we can write,
Problem 2 - (20 points – This problem must be attempted)

(a.) What is non-return-to-zero data and why is it preferable over return-to-zero data?

NRZ is digital data that takes the value of the data over the entire bit period whereas RZ data returns to zero before the bit period is complete. The bandwidth of RZ data is greater than NRZ data by at least a factor of two.

(b.) What is the differences between a linear and bang-bang phase detector for clock and data recovery (CDR) applications?

Linear PD: Has linear gain characteristics, gain is sensitive to data density, has small jitter generation, suffers from bandwidth limitations, has static phase offset due to mismatch.

Bang-bang PD: High phase detector gain, higher output jitter, state phase offset set by sampling aperture errors, widely used in digital PLL and DLL’s

(c.) What are the sources of jitter in a CDR?

1.) Jitter on the input data. 2.) Jitter on the VCO due to device noise. 3.) Ripple on the VCO control line. 4.) Injection pulling of the VCO by the input data. 5.) Substrate and supply noise.

(d.) What is a half-rate phase detector? How does it work?

A half-rate phase detector is a phase detector that senses the input random data at full rate but employs a VCO running at half the input rate. This is done by sensing the data at both the rising and falling edges of the clock or using a quadrature clock.

(e.) Sketch the approximate waveforms for optimum performance for the CDR circuit shown on the plots given.
Problem 3 - (20 points – This problem is optional)

A carrier together with a single −40dBc spur 1kHz above the carrier are applied to an ideal limiter. Sketch the spectrum of the output of the limiter considering frequencies through the 5th harmonic of the carrier. Assume the limiter acts like a square wave modulator resulting in odd harmonics only whose amplitudes are given as $a_n = (4/n\pi)$ where $n =$ the harmonic.

Solution

The amplitude limiter will only influence the amplitude modulation so that we must resolve this spectrum into a pair of AM and FM sidebands. This will be done as follows where $f_s = 1$ kHz.

The amplitude of the single spur is found as

$$V_{spur} = 10^{-40/20} = 0.01V$$

We know that $V_{AM} = V_{PM}$ and that $V_{spur} = 0.01V = 2V_{AM} = 2V_{PM}$. Therefore,

$$V_{AM} = V_{PM} = 0.005 \rightarrow V_{PM}(dBc) = 20 \log_{10}(0.005) = -46 \text{ dBC}$$

The limiter removes the AM and the spectrum looks like the following

The key to answering this question is to realize that the carrier will be reduced by $n$ the harmonic number because of the mixing action of the limiter. The FM spurs will also be reduced by the same amount, but because of the multiplication, the spurs will be increased by the same amount. Thus, the spurs do not change for the various harmonics. The resulting spectrum out to the fifth harmonic is given below.
**Problem 4 - (20 points – This problem is optional)**

A simple implementation of a CMOS PLL is shown. The phase detector output is fed to a first-order lowpass filter whose output changes the varactor capacitance to change the VCO frequency. If the VCO frequency is 500 MHz and the varactor capacitance varies as

\[ C = \frac{C_o}{\sqrt{1 + v_D}} \]

what value of \( C_1 \) will give a phase margin of 45° if \( v_f \approx 1 \) V? If you need to make any assumptions in working this problem, make sure they are clearly stated.

**Solution**

To find the phase margin, we must find the open-loop gain which is given in general as,

\[ LG(s) = \frac{K_d K_o F(s)}{s} \]

Assuming the output swing of the EXOR gate is 2V, then \( K_d = \frac{2}{\pi} \)

The frequency of the LC oscillator can be written as,

\[ \omega_{osc} = \frac{1}{\sqrt{2LC}} = \frac{1}{\sqrt{LC}} \rightarrow \omega_{osc}^2 = \frac{1}{LC} = \frac{1}{LC_o} = \omega_o^2 \sqrt{1 + v_D} \]

But, \( v_D = 2 - v_f \), so that \( \omega_{osc}^2 = \omega_o^2 \sqrt{3 - v_f} \). Now differentiate \( \omega_{osc} \) with respect to \( v_f \).

\[
2 \frac{d \omega_{osc}}{d v_f} = \frac{\omega_o^2}{2 \sqrt{3 - v_f}} \rightarrow \frac{d \omega_{osc}}{d v_f} = -\frac{\omega_o^2}{4 \omega_{osc} \sqrt{3 - v_f}} = -\frac{\omega_o}{4 \sqrt{3 - v_f}} \text{ if } \omega_{osc} = \omega_o
\]

Now, assuming \( v_f \approx 1 \) gives,

\[
K_o = \frac{d \omega_{osc}}{d v_f} = -\frac{\omega_o}{4 \sqrt{3 - v_f}} = -\frac{1000 \pi \times 10^6}{4 \sqrt{2}} = -0.555 \times 10^9 \text{ rads/V}
\]

\[
LG(s) = \frac{2}{\pi} \frac{(-1000 \pi \times 10^6)}{4 \sqrt{2}} \left( \frac{1}{s + \omega_1} \right) = 353 \times 10^6 \left( \frac{1}{s(s + \omega_1)} \right)
\]

We know that the filter has to provide 45° phase shift to get a PM of 45°. Therefore, the magnitude of the filter is \( 1/\sqrt{2} \). Therefore, the unity gain frequency can be found as,

\[
LG(j\omega_{0dB}) = \frac{353 \times 10^6}{\sqrt{2} \omega_{0dB}} = 1 \rightarrow \omega_{0dB} = \frac{353 \times 10^6}{\sqrt{2}} = 250 \times 10^6 \text{ rads/sec.}
\]

\[
45° = \tan^{-1} \left( \frac{\omega_{0dB}}{\omega_1} \right) \rightarrow \omega_{0dB} = \omega_1 \rightarrow \frac{1}{R_1C_1} = 250 \times 10^6 \text{ rads/sec.}
\]

\[
C_1 = \frac{1}{R_1 250 \times 10^6} = \frac{1}{10^3 \cdot 250 \times 10^6} = 4 \text{ pF}
\]

\[ \therefore C_1 = 4 \text{ pF} \]
Problem 5 – (20 points – This problem is optional)

The block diagram for a fractional-N frequency synthesizer is shown. The effective \( N \) is normally given as shown below (see page 100-30 of the lecture notes) where the divider divides by \( N_1 \) for \( P \) cycles (periods) and \( N_2 \) for \( Q \) cycles (periods).

\[
N_{\text{eff}} = \frac{PN_1 + QN_2}{P+Q}
\]

The above relationship is only good when \( N_{\text{eff}} \) is large. (a.) Derive a better expression for \( N_{\text{eff}} \) based on the diagram shown which takes into account that \( f_{\text{ref}} \) always remains constant but that the output frequency of the VCO, \( f_{\text{VCO}} \), changes as \( N \) changes. (b.) If \( P = 1 \), \( Q = M-1 \), \( N_1 = N+1 \) and \( N_2 = N \), find \( N_{\text{eff}} \) for both the above expression and the one derived in (a.). (c.) If \( M = 100 \) and \( N = 1000 \), what is the effective \( N \) for both expressions?

\[
\begin{align*}
\text{Solution} \\
\text{(a.) } \quad &\text{We know that } f_{\text{ref}} \text{ never changes so } f_{\text{VCO}_1} = N_1 f_{\text{ref}} \text{ and } f_{\text{VCO}_2} = N_2 f_{\text{ref}} \text{ also } T_{\text{VCO}_1} = 1/f_{\text{VCO}_1} \\
&\text{and } T_{\text{VCO}_2} = 1/f_{\text{VCO}_2}. \text{ We can express the average clock period, } T_{\text{VCO}(\text{aver})} \text{ as} \\
&\quad T_{\text{VCO}(\text{aver})} = \frac{T_{\text{VCO}_1} + T_{\text{VCO}_2}}{P+Q} = \frac{P f_{\text{VCO}_1} + Q f_{\text{VCO}_2}}{P+Q} = \frac{P f_{\text{VCO}_1} + Q f_{\text{VCO}_2}}{(P+Q) f_{\text{ref}}} \\
&\therefore \quad f_{\text{VCO}(\text{aver})} = \frac{1}{T_{\text{VCO}(\text{aver})}} = \frac{P}{P+Q} f_{\text{ref}} = \frac{P}{P+Q} f_{\text{ref}} \\
&\text{N_{eff} = } \frac{PN_1 + QN_2}{P+Q} = \frac{(N+1)+N(M-1)}{M} = N + \frac{1}{M} \\
&\therefore \quad N_{\text{eff}}_1 = N + \frac{1}{M} \quad \text{and} \quad N_{\text{eff}}_2 = \frac{N_1 N_2 (P+Q)}{N_2 P + N_1 Q} = \frac{N(M+1)}{M(N+1)} \\
&\quad \therefore \quad N_{\text{eff}}_1 = N + \frac{1}{M} \quad \text{and} \quad N_{\text{eff}}_2 = \frac{N}{1 - \frac{M}{M(N+1)}} = N + \frac{N}{M(N+1)} \\
&\text{(c.) } \quad N_{\text{eff}}_1 = 1000.01 \quad \text{and} \quad N_{\text{eff}}_2 = 1000.009999
\end{align*}
\]
Problem 6 – (20 points – This problem is optional)

A differential ring oscillator is shown.

(a.) Find the frequency of oscillation in Hz if $R = 1\, \text{k}\Omega$ and $C = 1\, \text{pF}$.

(b.) What value of $g_m$ is required for oscillation assuming all stages are identical?

(c.) What is the maximum positive and maximum negative voltage swing at the drains if $I_{SS} = 1\, \text{mA}$ and $V_{DD} = 2\, \text{V}$?

Solution

(a.) The voltage transfer function of a single stage is,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m R}{s R C + 1} = \frac{g_m R}{s \tau_1 + 1}$$

where $\tau_1 = R C = 10^{-9} \, \text{sec}$.

When the phase shift of each stage is equal to $-60^\circ$, the phase shift around the loop will be $360^\circ$ or $0^\circ$. Therefore, the oscillation frequency can be found as,

$$-\tan^{-1}(\omega_{osc} \tau_1) = -60^\circ \quad \rightarrow \quad f_{osc} = \frac{1.732}{2\pi R C} = \frac{1.732}{2\pi \times 10^{-9}} = 275.67 \, \text{MHz}$$

(b.) The magnitude of the loop gain at the oscillation frequency is given as

$$\left(\frac{g_m R}{\sqrt{1 + (\omega_{osc} R C)^2}}\right)^3 = 1 \quad \rightarrow \quad g_m R = \sqrt{1 + 1.732^2} = \sqrt{1 + 3} = 2$$

∴ $g_m = \frac{2}{R} = 2\, \text{mS}$

(c.) $v_{max} = V_{DD} = 2\, \text{V}$ and $v_{min} = V_{DD} - I_{SS} R = 2 - 1 = 1\, \text{V}$

$$v_{max} = 2\, \text{V} \quad \text{and} \quad v_{min} = 1\, \text{V}$$
Problem 7 – (20 points – This problem is optional)

An LC oscillator is shown (a.) What is the frequency of oscillation of this oscillator? (b.) At room temperature, assume that the noise of $R_L$ is dominant over all other noise sources and calculate the single sideband phase noise resulting from the resistor’s noise using the linear time varying theory if $\Gamma_{rms}^2 = 0.5$. Note that the tank voltage, $v_{tank}$ can be approximated as

$$v_{tank} = 2I_{Bias}R_L \left( 1 - \frac{C_1}{C_1+C_2} \right)$$

(c.) At an offset of 200kHz, what is the phase noise in dBc?

Solution

(a.) The frequency of oscillation is

$$\omega_{osc} = \frac{1}{\sqrt{\frac{C_1C_2}{LC_1+C_2}}} = \frac{1}{\sqrt{200nH\cdot 80pF}} = 250 \text{ Mrads/sec.} \rightarrow f_{osc} = 39.79 \text{ MHz}$$

$$\omega_{osc} = 250 \text{ Mrads/sec and } f_{osc} = 39.79 \text{ MHz}$$

(b.) The LTV noise theory gives the SSB noise as,

$$\mathcal{L} \{\Delta \omega\} = 10\log_{10} \left[ \frac{i_n^2}{2q_{max}^2\Delta f^2} \right]$$

The noise due to $R_L$ is found as

$$\frac{i_n^2}{\Delta f} = \frac{4kT}{R_L} = \frac{4\cdot 1.381x10^{-23}\cdot 300}{10^3} = 1.6572\times 10^{-23} \text{ A}^2/\text{Hz}$$

$q_{max}$ is the maximum charge across the equivalent tank capacitance (80pF) given as

$$q_{max} = C_{eq} v_{max}$$

Let us assume that $v_{max}$ is equal to $v_{tank}$ which is given from the above expression as

$$v_{tank} = 2I_{Bias}R_L \left( 1 - \frac{C_1}{C_1+C_2} \right) = 2\cdot 1mA\cdot 1k\Omega \left( 1 - \frac{100}{500} \right) = 1.6V$$

$$q_{max} = 80pF\cdot 1.6V = 128 \text{ pC}$$

$$\mathcal{L} \{\Delta \omega\} = 10\log_{10} \left[ \frac{0.000253}{(\Delta \omega)^2} \right]$$

(c.) $\mathcal{L} \{\Delta \omega\} = 10\log_{10} \left[ \frac{1.6572\times 10^{-23}}{4(128\times 10^{-12})^2(0.4\pi x 10^6)^2} \right] = 10\log_{10}(6.405\times 10^{-16}) = -151.9 \text{ dBc}$

$$\mathcal{L} \{\Delta \omega\} = -151.9 \text{ dBc at an offset of 200kHz}$$