FINAL EXAMINATION - SOLUTIONS

(Average Score = 77/100)

Problem 1 - (20 points – This problem must be attempted)

A DPLL frequency synthesizer has the following parameters:

$F(s) = \dfrac{1+0.01s}{s}$ $\qquad K_o = 2\text{x}10^6$ (rads/V) $\qquad K_d = 0.8$V/rad

$\beta = 2\pi$ $\qquad N = 150$ $\qquad f_{ref} = 120$ kHz

The temperature is 290°K and all circuits operate from a ±5V power supply.

(a.) What is the output frequency hold range in Hz?

(b.) What is the output frequency lock (capture) range in Hz? What is the lock (capture) time in seconds?

(c.) Assume that you wish to phase modulate the output of the synthesizer, where would you introduce the modulating voltage? What is the peak amplitude of the 1 kHz ac signal needed to produce an output peak phase deviation of 0.5 radians?

Solution

(a.) Since the filter is active PI, the hold range is limited by the loop components and is

$$\Delta\omega_{max} = \Delta\omega_H = \pm 5\text{V}\cdot K_o = \pm 5\text{V}(2\text{x}10^6 \text{ (rads/V)}) = \pm 10\text{Mrads/sec.}$$

$$\therefore \quad \Delta f_H = \frac{\pm 10\text{Mrads/sec}}{2\pi} = \pm 1.592 \text{ MHz} \quad \rightarrow \quad \boxed{\Delta f_H = \pm 1.592 \text{ MHz}}$$

(b.) First we find K (Lecture 90-02).

$$K = \frac{K_d K_o}{N} = \frac{0.8\cdot 2\text{x}10^6}{150} = 10{,}667$$

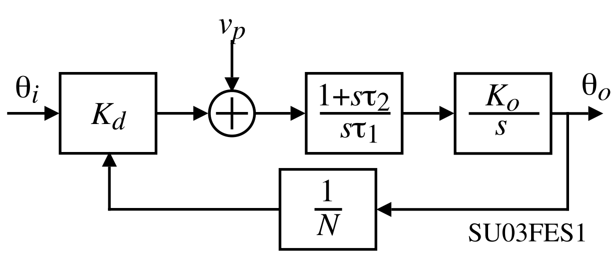
From Lecture 90-09,

$$\omega_n = \sqrt{\frac{K}{\tau_1}} = \sqrt{\frac{10{,}667}{1}} = 103.3 \text{ Rads/sec} \quad \text{and} \quad \zeta = \frac{\tau_2\omega_n}{2} = \frac{0.01\cdot 103.3}{2} = 0.516$$

$$\therefore \quad \Delta f_L = \frac{2\beta N\zeta\omega_n}{2\pi} = \frac{2(2\pi)150\cdot 0.516\cdot 103.3}{2\pi} = 15{,}991 \text{ Hz} \rightarrow \boxed{\Delta f_L = \pm 15{,}991\text{Hz}}$$

$$t_L = \frac{2\pi}{\omega_n} = \frac{2\pi}{103.3} = 60.8 \text{ msec} \qquad \boxed{t_L = 60.8 \text{ msec}}$$

(c.) See the following block diagram.



What is the BW? From Lect. 90-09,

$$BW = 103.3\sqrt{2(0.516^2)+1+\sqrt{2(0.516^2)+1}}$$

$$= 103.3(1.546) = 159.75 \text{ rads/sec.}$$

or $BW = 25.42$ Hz

Since, 1kHz >> BW, we can write,

$$\frac{\theta_o}{v_p} \approx \frac{\tau_2 K_o}{\tau_1\omega} \quad \rightarrow \quad v_p = \theta_o\frac{\tau_1\omega}{\tau_2 K_o} = 0.5\left(\frac{1\cdot 2000\pi}{0.01\cdot 2\text{x}10^6}\right) = 0.157\text{V} \qquad \boxed{v_p = 0.157\text{V}}$$

<u>Problem 2 - (20 points – This problem must be attempted)</u>

(a.) What is non-return-to-zero data and why is it preferable over return-to-zero data?

NRZ is digital data that takes the value of the data over the entire bit period whereas RZ data returns to zero before the bit period is complete. The bandwidth of RZ data is greater than NRZ data by at least a factor of two.

(b.) What is the differences between a linear and bang-bang phase detector for clock and data recovery (CDR) applications?

Linear PD: Has linear gain characteristics, gain is sensitive to data density, has small jitter generation, suffers from bandwidth limitations, has static phase offset due to mismatch.

Bang-bang PD: High phase detector gain, higher output jitter, state phase offset set by sampling aperature errors, widely used in digital PLL and DLL's

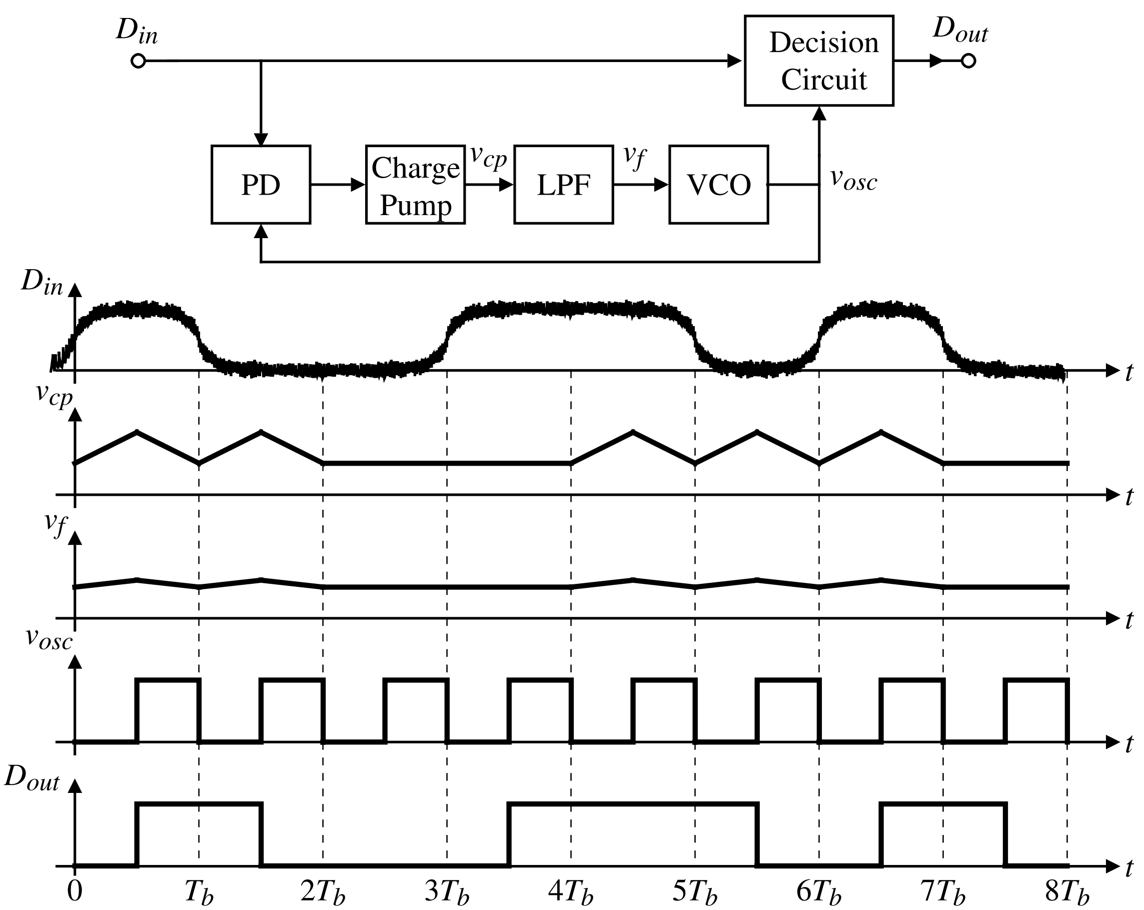
(c.) What are the sources of jitter in a CDR?

1.) Jitter on the input data. 2.) Jitter on the VCO due to device noise. 3.) Ripple on the VCO control line. 4.) Injection pulling of the VCO by the input data. 5.) Substrate and supply noise.

(d.) What is a half-rate phase detector? How does it work?

A half-rate phase detector is a phase detector that senses the input random data at full rate but employs a VCO running at half the input rate. This is done by sensing the data at both the rising and falling edges of the clock or using a quadrature clock.

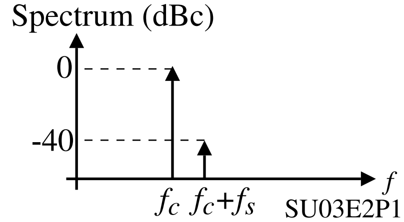
(e.) Sketch the approximate waveforms for optimum performance for the CDR circuit shown on the plots given.



SU03FES2

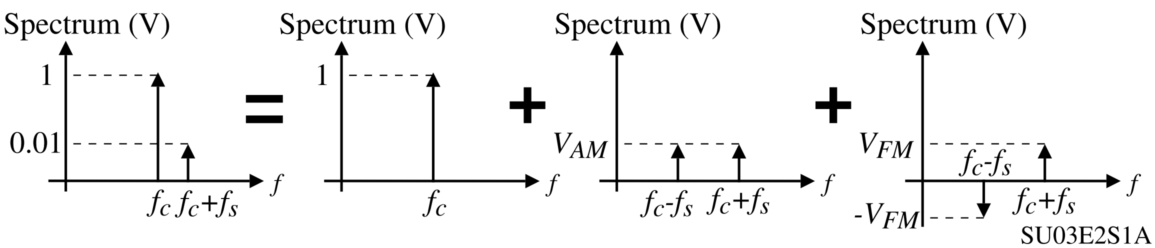
Problem 3 - (20 points – This problem is optional)

A carrier together with a single –40dBc spur 1kHz above the carrier are applied to an ideal limiter. Sketch the spectrum of the output of the limiter considering frequencies through the 5th harmonic of the carrier. Assume the limiter acts like a square wave modulator resulting in odd harmonics only whose amplitudes are given as $a_n = (4/n\pi)$ where n = the harmonic.



Solution

The amplitude limiter will only influence the amplitude modulation so that we must resolve this spectrum into a pair of AM and FM sidebands. This will be done as follows where f_s = 1 kHz.



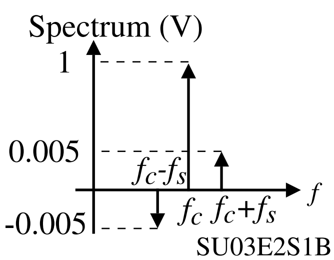
The amplitude of the single spur is found as

$$V_{spur} = 10^{-40/20} = 0.01\text{V}$$

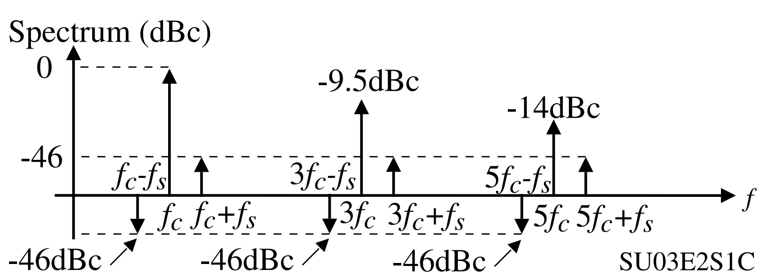
We know that $V_{AM} = V_{PM}$ and that $V_{spur} = 0.01\text{V} = 2V_{AM} = 2V_{PM}$. Therefore,

$$V_{AM} = V_{PM} = 0.005 \quad \rightarrow \quad V_{PM}(\text{dBc}) = 20\log_{10}(0.005) = -46 \text{ dBc}$$

The limiter removes the AM and the spectrum looks like the following



The key to answering this question is to realize that the carrier will be reduced by n the harmonic number because of the mixing action of the limiter. The FM spurs will also be reduced by the same amount, but because of the multiplication, the spurs will be increased by the same amount. Thus, the spurs do not change for the various harmonics. The resulting spectrum out to the fifth harmonic is given below.

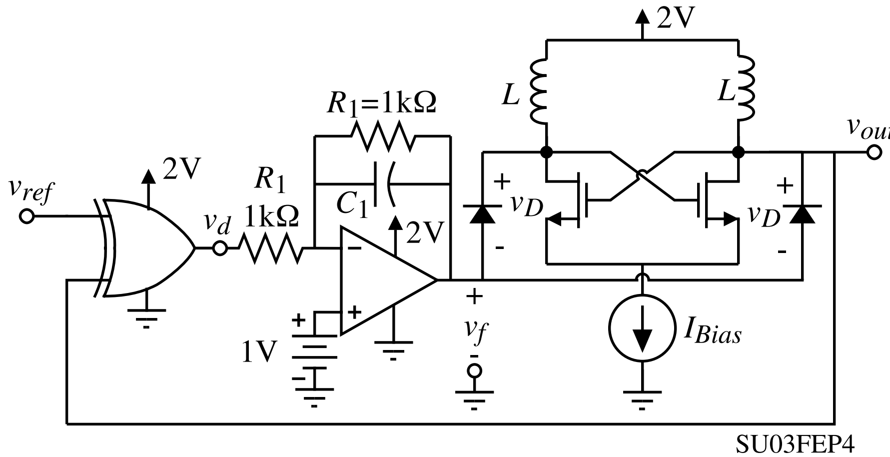


Problem 4 - (20 points – This problem is optional)

A simple implementation of a CMOS PLL is shown. The phase detector output is fed to a first-order lowpass filter whose output changes the varactor capacitance to change the VCO frequency. If the VCO frequency is 500 MHz and the varactor capacitance varies as

$$C = \frac{C_o}{\sqrt{1+v_D}}$$

what value of C_1 will give a phase margin of 45° if $v_f \approx$ 1V? If you need to make any assumptions in working this problem, make sure they are clearly stated.



Solution

To find the phase margin, we must find the open-loop gain which is given in general as,

$$LG = \frac{K_d K_o F(s)}{s}$$

Assuming the output swing of the EXOR gate is 2V, then $K_d = \frac{2}{\pi}$

The frequency of the LC oscillator can be written as,

$$\omega_{osc} = \frac{1}{\sqrt{2L\,\frac{C}{2}}} = \frac{1}{\sqrt{LC}} \quad \rightarrow \quad \omega_{osc}^2 = \frac{1}{LC} = \frac{\sqrt{1+v_D}}{LC_o} = \omega_o^2\sqrt{1+v_D}$$

But, $v_D = 2 - v_f$, so that $\omega_{osc}^2 = \omega_o^2\sqrt{3-v_f}$. Now differentiate ω_{osc} with respect to v_f.

$$2\,\omega_{osc}\frac{d\omega_{osc}}{dv_f} = \frac{\omega_o^2}{2\sqrt{3-v_f}} \quad \rightarrow \quad \frac{d\omega_{osc}}{dv_f} = -\frac{\omega_o^2}{4\omega_{osc}\sqrt{3-v_f}} \approx -\frac{\omega_o}{4\sqrt{3-v_f}} \text{ if } \omega_{osc} \approx \omega_o$$

Now, assuming $v_f \approx 1$ gives,

$$K_o = \frac{d\omega_{osc}}{dv_f} = -\frac{\omega_o}{4\sqrt{3-v_f}} = -\frac{1000\pi\text{x}10^6}{4\sqrt{2}} = -0.555\text{x}10^9 \text{ rads/V}$$

$$LG(s) = \frac{2}{\pi}\,\frac{(-1000\pi\text{x}10^6)}{4\sqrt{2}\;s}\,\frac{-\omega_1}{s+\omega_1} = 353\text{x}10^6\,\frac{\omega_1}{s(s+\omega_1)}$$

We know that the filter has to provide 45° phase shift to get a PM of 45°. Therefore, the magnitude of the filter is $1/\sqrt{2}$. Therefore, the unity gain frequency can be found as,

$$LG(j\omega_{0\text{dB}}) = \frac{353\text{x}10^6}{\sqrt{2}\omega_{0\text{dB}}} = 1 \quad \rightarrow \quad \omega_{0\text{dB}} = \frac{353\text{x}10^6}{\sqrt{2}} = 250\text{x}10^6 \text{ rads/sec.}$$

$$45° = \tan^{-1}\left(\frac{\omega_{0\text{dB}}}{\omega_1}\right) \quad \rightarrow \quad \omega_{0\text{dB}} = \omega_1 \quad \rightarrow \quad \frac{1}{R_1C_1} = 250\text{x}10^6 \text{ rads/sec.}$$

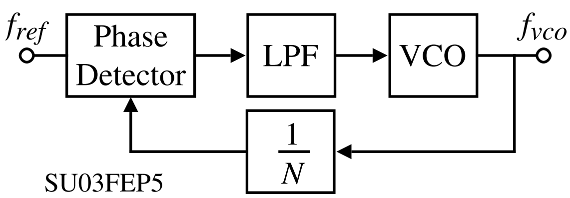
$$\therefore \quad C_1 = \frac{1}{R_1 250\text{x}10^6} = \frac{1}{10^3\cdot 250\text{x}10^6} = 4\text{pF} \qquad \boxed{C_1 = 4\text{pF}}$$

Problem 5 – (20 points – This problem is optional)

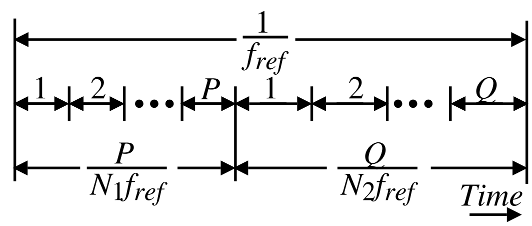
The block diagram for a fractional-N frequency synthesizer is shown. The effective N is normally given as shown below (see page 100-30 of the lecture notes) where the divider divides by N_1 for P cycles (periods) and N_2 for Q cycles (periods).

$$N_{eff} = \frac{PN_1+QN_2}{P+Q}$$

The above relationship is only good when N_{eff} is large. (a.) Derive a better expression for N_{eff} based on the diagram shown which takes into account that f_{ref} always remains constant but that the output frequency of the VCO, f_{VCO}, changes as N changes. (b.) If $P = 1$, $Q = M$-1, $N_1 = N$+1 and $N_2 = N$, find N_{eff} for both the above expression and the one derived in (a.). (c.) If $M = 100$ and $N = 1000$, what is the effective N for both expressions?



SU03FEP5



Solution

(a.) We know that f_{ref} never changes so $f_{vco1}=N_1f_{ref}$ and $f_{vco2}=N_2f_{ref}$ also $T_{vco1} = 1/f_{vco1}$ and $T_{vco2} = 1/f_{vco2}$. We can express the average clock period, T_{vco}(aver) as

$$T_{vco}(\text{aver}) = \frac{T_{vco1}P+T_{vco2}Q}{P+Q} = \frac{\frac{P}{f_{vco1}} + \frac{Q}{f_{vco2}}}{P+Q} = \frac{\frac{P}{N_1} + \frac{Q}{N_2}}{(P+Q)f_{ref}}$$

$$\therefore\ f_{vco}(\text{aver}) = \frac{1}{T_{vco}(\text{aver})} = \frac{(P+Q)f_{ref}}{\frac{P}{N_1} + \frac{Q}{N_2}} = N_{eff}f_{ref} \rightarrow \boxed{N_{eff} = \frac{(P+Q)}{\frac{P}{N_1} + \frac{Q}{N_2}} = \frac{N_1N_2(P+Q)}{N_2P+N_1Q}}$$

(b.) $$N_{eff1} = \frac{PN_1+QN_2}{P+Q} = \frac{(N+1)+N(M\text{-}1)}{M} = N + \frac{1}{M}$$

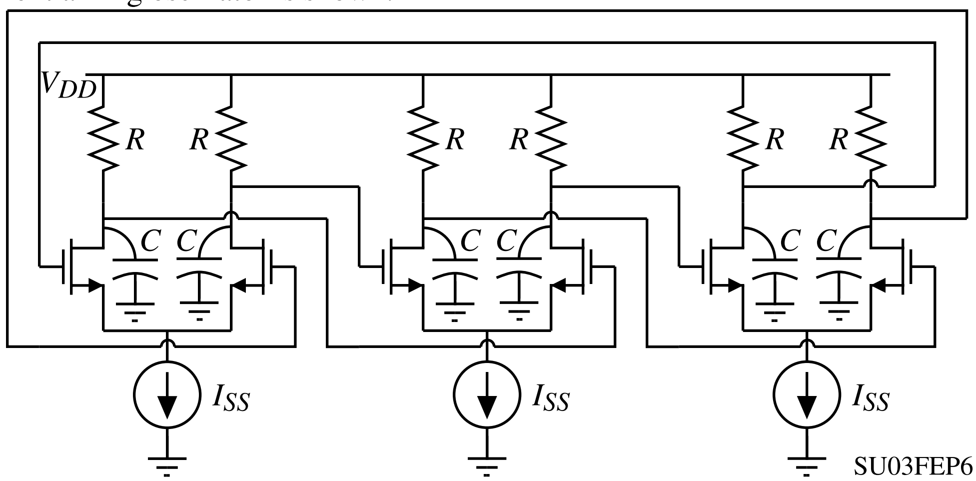
$$N_{eff2} = \frac{N_1N_2(P+Q)}{N_2P+N_1Q} = \frac{N(N+1)M}{N+(N+1)(M\text{-}1)} = \frac{NM(N+1)}{M(N+1)\text{-}1} = \frac{N}{1 - \frac{1}{M(N+1)}} \approx N + \frac{N}{M(N+1)}$$

$$\therefore\quad \boxed{N_{eff1} = N + \frac{1}{M} \quad \text{and } N_{eff2} = \frac{N}{1 - \frac{1}{M(N+1)}} \approx N + \frac{N}{M(N+1)}}$$

(c.) $\boxed{N_{eff1} = 1000.01 \qquad \text{and } N_{eff2} = 1000.00999}$

Problem 6 – (20 points – This problem is optional)

A differential ring oscillator is shown.



(a.) Find the frequency of oscillation in Hz if R = 1kΩ and C = 1pF.

(b.) What value of g_m is required for oscillation assuming all stages are identical?

(c.) What is the maximum positive and maximum negative voltage swing at the drains if I_{SS} = 1mA and V_{DD} = 2V?

Solution

(a.) The voltage transfer function of a single stage is,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m R}{sRC+1} = \frac{g_m R}{s\tau_1+1} \qquad \text{where } \tau_1 = RC = 10^{-9} \text{ secs.}$$

When the phase shift of each stage is equal to –60°, the phase shift around the loop will be 360° or 0°. Therefore, the oscillation frequency can be found as,

$$-\tan^{-1}(\omega_{osc}\tau_1) = -60° \quad \rightarrow \quad f_{osc} = \frac{1.732}{2\pi\cdot RC} = \frac{1.732}{2\pi\cdot 10^{-9}} = 275.67 \text{ MHz}$$

$$\boxed{f_{osc} = 275.67 \text{ MHz}}$$

(b.) The magnitude of the loop gain at the oscillation frequency is given as

$$\left(\frac{g_m R}{\sqrt{1+(\omega_{osc}RC)^2}}\right)^3 = 1 \quad \rightarrow \quad g_m R = \sqrt{1+1.732^2} = \sqrt{1+3} = 2$$

$$\therefore \quad g_m = \frac{2}{R} = 2\text{mS} \qquad \boxed{g_m = 2\text{mS}}$$

(c.) $v_{max} = V_{DD} = 2\text{V}$ and $v_{min} = V_{DD} - I_{SS}R = 2 - 1 = 1\text{V}$

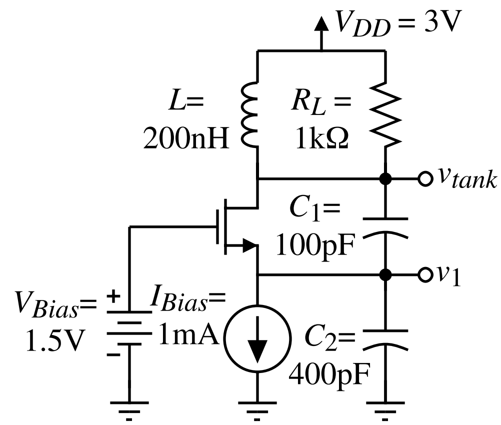
$$\boxed{v_{max} = 2\text{V and } v_{min} = 1\text{V}}$$

Problem 7 – (20 points – This problem is optional)

An LC oscillator is shown (a.) What is the frequency of oscillation of this oscillator? (b.) At room temperature, assume that the noise of R_L is dominant over all other noise sources and calculate the single sideband phase noise resulting from the resistor's noise using the linear time varying theory if $\Gamma_{rms}{}^2 = 0.5$. Note that the tank voltage, v_{tank} can be approximated as

$$v_{tank} \approx 2I_{Bias}R_L\left(1 - \frac{C_1}{C_1+C_2}\right)$$

(c.) At an offset of 200kHz, what is the phase noise in dBc?



SU03FEP7

Solution

(a.) The frequency of oscillation is

$$\omega_{osc} = \frac{1}{\sqrt{L\frac{C_1C_2}{C_1+C_2}}} = \frac{1}{\sqrt{200\text{nH}\cdot 80\text{pF}}} = 250 \text{ Mrads/sec.} \quad \rightarrow \quad f_{osc} = 39.79 \text{ MHz}$$

$$\boxed{\omega_{osc} = 250 \text{ Mrads/sec and } f_{osc} = 39.79\text{MHz}}$$

(b.) The LTV noise theory gives the SSB noise as,

$$\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{\frac{\overline{i_n{}^2}}{\Delta f}\,\Gamma_{rms}{}^2}{2q_{max}{}^2\Delta\omega^2}\right]$$

The noise due to R_L is found as

$$\frac{\overline{i_n{}^2}}{\Delta f} = \frac{4kT}{R_L} = \frac{4\cdot 1.381\text{x}10^{-23}\cdot 300}{10^3} = 1.6572\text{x}10^{-23} \text{ A}^2/\text{Hz}$$

q_{max} is the maximum charge across the equivalent tank capacitance (80pF) given as

$$q_{max} = C_{eq}\, v_{max}$$

Let us assume that v_{max} is equal to v_{tank} which is given from the above expression as

$$v_{tank} \approx 2I_{Bias}R_L\left(1 - \frac{C_1}{C_1+C_2}\right) = 2\cdot 1\text{mA}\cdot 1\text{k}\Omega\left(1 - \frac{100}{500}\right) = 1.6\text{V}$$

$\therefore \quad q_{max} = 80\text{pF}\cdot 1.6\text{V} = 128 \text{ pC}$

$$\boxed{\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{0.000253}{(\Delta\omega)^2}\right]}$$

(c.) $$\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{1.6572\text{x}10^{-23}}{4(128\text{x}10^{-12})^2(0.4\pi\text{x}10^6)^2}\right] = 10\log_{10}(6.405\text{x}10^{-16}) = -151.9\text{dBc}$$

$$\boxed{\mathcal{L}\{\Delta\omega\} = -151.9\text{dBc at an offset of 200kHz}}$$