Homework Assignment No. 2 - Solution

Problem 1 - (10 points)

A phase-locked loop has a center frequency of 10^5 rads/s, a $K_o$ of 10^3 rad/V-s, and a $K_d$ of 1 V/rad. Assume there is no other gain in the loop. Determine the loop bandwidth in the first-order loop configuration. Determine the single-pole, loop-filter pole location to give the closed-loop poles located on 45° radials from the origin of the complex frequency plane.

*Solution*

The loop bandwidth = $K_v = K_o K_p = \frac{10^3}{s}$

In order to produce poles at 45° to the axis, we add a loop pole at $\omega_1$ where

$$\omega_1 = 2K_v = 2000 \text{ rads/sec}.$$ 

The filter transfer function becomes,

$$F(s) = \frac{\omega_1}{s + \omega_1} = \frac{2000}{s + 2000}$$

Problem 2 - (10 points)

For the same PLL of the previous problem, design a loop filter with a zero that gives a crossover frequency for the loop gain of 100 rads/sec. The loop phase shift at the loop crossover frequency should be –135°.

*Solution*

A plot of the desired loop gain is shown below.

![Plot of desired loop gain](SU03H02S2)

If $\omega_2$ (the zero frequency) is at the unity gain point, then the loop phase shift will be –135° at this point. Therefore, we require that $\omega_2 = 100$ radians/sec. If $\omega_1 = 10$ radians/sec, the requirement will be satisfied as shown in the above plot.

The design of the filter becomes,

$$\omega_2 = \frac{1}{R_2C} \quad \text{and} \quad \omega_1 = \frac{1}{(R_1+R_2)C}$$

$$\therefore \frac{\omega_2}{\omega_1} = 1 + \frac{R_1}{R_2} = 10 \quad \rightarrow \quad R_1 = 9R_2$$

Now appropriate values of $R_1, R_2, and C$ can be chosen.
Problem 3 – (10 points)
Estimate the capture range of the PLL of the previous problem assuming that it is not artificially limited by the VCO frequency range.

Solution
For capture we need

$$|\omega_i - \omega_o| < \frac{\pi}{2} K_v |F(j(\omega_i - \omega_o))|$$

If we assume that

$$|\omega_i - \omega_o| = \frac{\pi}{2} K_v |F(j(\omega_i - \omega_o))|$$

then with $\omega_o = 10^5$ rads/sec and $K_v = 1000$ rads/sec. we get,

$$|\omega_i - \omega_o| = 1570|F(j(\omega_i - \omega_o))|$$

Now from the previous problem, we know that $|F(j\omega)|$ is given as

![Graph](image-url)

From this figure we can solve the above equation to find that

$$\omega_i - \omega_o = 157$$ rads/sec.

which is the capture range.
Problem 4 – (10 points)

A filter for a phase locked loop is specified as

$$|F(s)| = \frac{10\omega_1}{s+\omega_1} = \frac{1,000,000}{s+100,000}$$

and must be implemented on a CMOS chip using resistors no larger than 10kΩ and capacitors no larger than 10pF. Using the circuit shown, find the values of $R_1$ and $R_2$ that will satisfy the component value constraints.

**Solution**

Find the currents $i_1$ and $i_2$,

$$i_1 = \frac{v_{in}}{R} \frac{R}{R+R_1} \frac{R}{R+R_1} = \frac{R_1 v_{in}}{2RR_1 + R_2} = \frac{v_{in}}{R_T 1}$$

and

$$i_2 = \frac{v_{out}}{R} \frac{R}{R+R_1} + sCv_{out} \frac{R_2}{R+R_1} v_{out} = \frac{v_{out}}{R_T 2} + sCv_{out}$$

Solving for the sum of the currents flow toward the minus op amp input terminal gives,

$$\frac{v_{in}}{R_T 1} + \frac{v_{out}}{R_T 2} + sCv_{out} = 0 \rightarrow \frac{v_{out}}{v_{in}} = R_T 2 \frac{1}{sRCR_T 2 + 1} = -10 \frac{1}{s 10^5 + 1}$$

$\therefore \quad CR_T 2 = 10^{-5} \quad \rightarrow \quad R_T 2 = \frac{10^{-5}}{10^{-11}} = 10^6$

$$R_T 2 = \frac{2RR_2 + R_2}{R_2} = 2R + \frac{R_2^2}{R_2} = 20 \cdot 10^3 + \frac{100 \cdot 10^6}{R_2} = 10^6$$

$\therefore \quad R_2 = \frac{100 \cdot 10^6}{10^6 - 20 \cdot 10^3} = 100\Omega$

$$R_T 1 = \frac{R_T 2}{10^5} = 10^5 \quad \rightarrow \quad 2R + \frac{R_2^2}{R_1} = 20 \cdot 10^3 + \frac{100 \cdot 10^6}{R_1} = 10^5$$

$\therefore \quad R_1 = \frac{100 \cdot 10^6}{10^5 - 20 \cdot 10^3} = 10000\Omega$

This problem shows how a clever circuit technique can make a filter suitable for integrated circuit implementation.
Problem 5 – (20 points)

This homework is designed to provide practical inductor design experience for students. Use ASITIC for the design and analysis. However, other tools are acceptable if they give all the results including layout. (See lecture 045 on Monolithic Inductor Design in Silicon Technology)

A 5GHz LC tank will be designed as a part of LC oscillator. C value is given as 1pF.

(a) Find L value. (b) Design and simulate a spiral inductor with this L value (± 5% range). Optimize design parameters, W, S, D and N to get a high Q (Q_{\text{min}} = 5). Show L, Q, f_{SR} value obtained from simulation. (c) Show the layout. (d) Give a lumped circuit model with component values.

Solution

(a) LC tank oscillation frequency is given as 5GHz.

\[ L = \frac{1}{\omega_{\text{osc}}^2 C} = \frac{1}{(2\pi \times 5 \times 10^9)(1 \times 10^{-12})} = 1.01 \times 10^{-9} \]

(b) One possible solution is

- Parameters: W = 16um, S = 2um, D = 150um, N = 2.5
- Resulting inductor: L = 0.952nH, Q = 8.54, f_{SR} = 19.35GHz @ 5GHz

(c) Layout

(d) Pi model from ASITIC is shown below. This is the analysis result from ‘pix’ command.