Homework Assignment No. 7 - Solutions

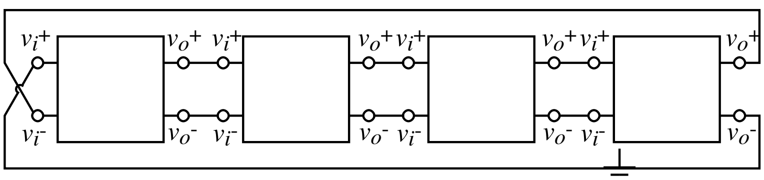
Problem 1 - (10 points)

A four-stage ring oscillator used as the VCO in a PLL is shown. Assume that M1 and M2 are matched and M3 and M4 are matched. Also assume that

$$g_m = \sqrt{2K'\frac{W}{L}I_D} \qquad \text{where } K'_N = 100\mu A/V^2 \text{ and } K'_P = 50\mu A/V^2$$

and that $r_{ds} = \infty$. The parasitic capacitors to ground at the outputs are 0.1pF each.

(a.) If I =2mA, find the frequency of oscillation in Hertz. (b.) Find the W/L ratio of M1 (M2) necessary for oscillation when I =2mA. (c.) If the current I is used to vary the frequency, express the relationship between ω_{osc} and I. In otherwords, find $\omega_{osc} = f(I)$.



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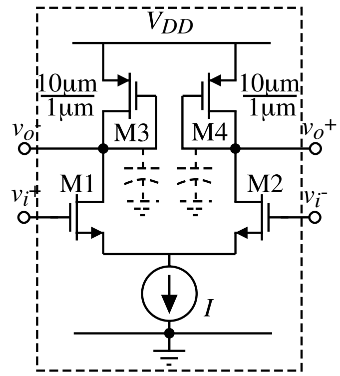


Solution

(a.) The small-signal transfer function of the stages can be written as,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_{m1}/g_{m3}}{s\frac{C}{g_{m3}}+1} \quad \rightarrow \quad \text{Arg}\left[\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right] = -\tan^{-1}\left(\frac{\omega C}{g_{m3}}\right)$$

From the above, we see that each stage must contribute –45° of phase to oscillate. Therefore,

$$\omega_{osc} = \frac{g_{m3}}{C} = \frac{\sqrt{2K'\,10\cdot 0.5I}}{C} = \frac{\sqrt{2\cdot 50\text{x}10^{-6}10\cdot 10^{-3}}}{10^{-13}} = 10^{10}\text{ rads/s} \rightarrow \boxed{f_{osc} = 1.59\text{GHz}}$$

(b.) The gain of the 4-stage ring oscillator at ω_{osc} should be equal to 1 so we can write,

$$1 = \left(\frac{g_{m1}/g_{m3}}{\sqrt{1+1}}\right)^4 = \frac{(g_{m1}/g_{m3})^4}{4} \rightarrow \qquad g_{m1} = 4^{0.25}\, g_{m3} = \sqrt{2}\, g_{m3} = \sqrt{2}\text{ mS}$$

$$\sqrt{2}\text{ mS} = \sqrt{2K'_N\,(W/L)\cdot 1\text{mA}} = \sqrt{2\cdot 100\text{x}10^{-6}\,(W/L)\cdot 1\text{mA}}$$

$$\therefore\ (W/L)_1 = \frac{2\text{mS}}{0.2\text{mS}} = 10 \quad \rightarrow \quad \boxed{(W/L)_1 = 10}$$

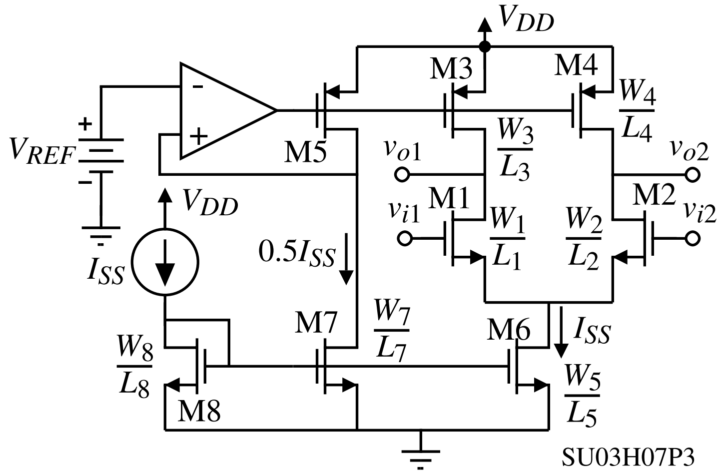
(c.) From part (a.) we get,

$$\omega_{osc} = \frac{g_{m3}}{C} = \frac{\sqrt{2K'\,10\cdot 0.5I}}{C} = \frac{\sqrt{2\cdot 50\text{x}10^{-6}10\cdot 0.5I}}{10^{-13}} = 2.36\text{x}10^{11}\sqrt{I}$$

$$\boxed{\omega_{osc} = 2.36\text{x}10^{11}\sqrt{I}}$$

Problem 2 – (10 points)

How does the oscillation frequency depend on I_{SS} for a ring oscillator using the stage shown? Express your answer in terms of V_{DD}, V_{REF}, I_{SS}, the simple large signal model parameters of the MOSFETs (K', V_T, λ) and the W/L values of the MOSFETs.



Solution

This topology uses a replica biasing circuit to define the on-resistance of M3 and M4 based on the on-resistance of M5. The on-resistance of M5 is

$$R_{on5} = \frac{V_{DD} - V_{REF}}{0.5 I_{SS}}$$

We can either assume that the W/Ls of M3, M4 and M5 are equal or since we know that R_{on} is inversely proportional to the W/L ratio, we can write that,

$$R_{on3} = R_{on4} = \frac{W_5/L_5}{W_3/L_3} R_{on5}$$

where $W_3/L_3 = W_4/L_4$.

Assuming a capacitance at each output of C_L, allows us to write the transfer function of the ring oscillator stage as,

$$\frac{V_{o2} - V_{o1}}{V_{i1} - V_{i2}} = \frac{g_{m1} R_{on3}}{s R_{on3} C_L + 1}$$

The phase shift due to a stage can be written as,

$$\theta i(j\omega) = -\tan^{-1}(\omega R_{on3} C_L)$$

To oscillate, this phase shift needs to be equal to some value, say k (in degrees). Therefore we can write that,

$$\omega_{osc} = \frac{k}{R_{on3} C_L} = \frac{0.5 I_{SS}\ k}{\frac{W_5/L_5}{W_3/L_3}(V_{DD} - V_{REF}) C_L}$$

Therefore, the oscillation frequency varies linearly with I_{SS}.

Problem 3 – (10 points)

In every practical oscillator, the LC tank is not the only source of phase shift. Hence, the actual oscillation frequency may differ somewhat from the resonant frequency of the tank. Using the time-varying model, explain why the oscillators's phase noise can degrade if such off-frequency oscillations occur.

Solution

If there is any off-frequency oscillations that are close to the actual oscillation frequency or harmonics of it, we know from the LTV theory that these frequencies and their associated noise will "fold" into the noise spectrum around the actual frequency and degrade the oscillator's phase noise. The following diagram illustrates the process.

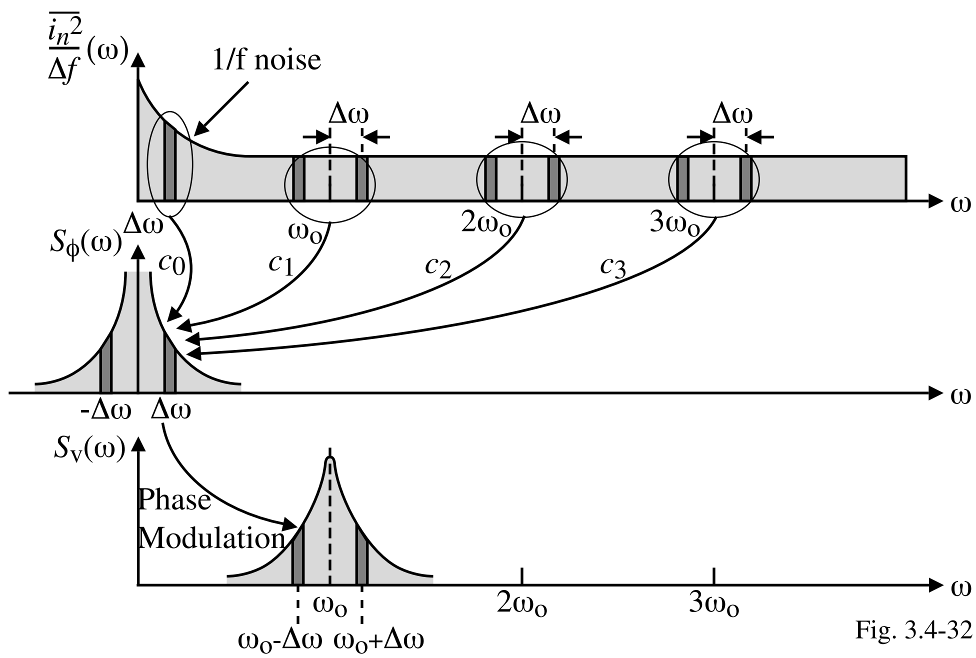
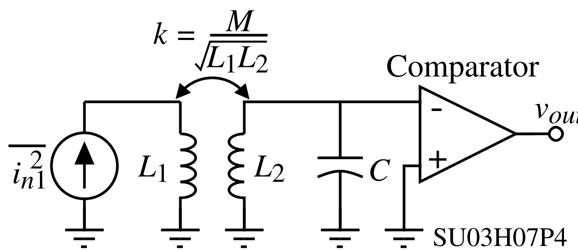


Fig. 3.4-32

Problem 4 – (10 points)

Assume that the steady-state output amplitude of the following oscillator is 1V. Calculate the phase noise in dBc/Hz at an offset of 100kHz from the carrier from the signal coming out of the ideal comparator. Assume that L_1 = 25nH, L_2 = 100nH, M = 10nH, and C = 100pF. Further assume that the noise current is



$$\overline{i_{n1}^2} = 4kTG_{eff}\Delta f$$

where $1/G_{eff} = 50\Omega$. The temperature of the circuit is 300°K.

Solution

First of all, several assumptions must be made to work this problem. They are:

1.) The load on the secondary of the transformer approximates a short.

2.) The output of the comparator is a square wave of amplitude 0.5V.

Our objective is to find the value of

$$\mathcal{L}\{f_m\} = 10\log_{10}\left[\frac{\overline{i_{n2}^2}/\Delta f\,\Gamma_{rms}{}^2}{2q_{max}{}^2(f_m)^2}\right]$$

First, the influence of the transformer. The equations of a general transformer are,

$$V_1 = sL_1I_1 + sMI_2 \qquad \text{and} \qquad V_2 = sMI_1 + sL_2I_2$$

If we assume that $V_2 \approx 0$, then $I_2 \approx \frac{M}{L_2}I_1 = 0.1I_1$. Since we are looking at the square of the current, we can write that the noise injected into the tank is

$$\frac{\overline{i_{n2}^2}}{\Delta f} = 0.01\frac{\overline{i_{n2}^2}}{\Delta f} = 0.04kTG_{eff} = \frac{0.04(1.381\text{x}10^{-23})300}{50} = 3.314\text{x}10^{-24}\ \text{A}^2/\text{Hz}$$

Next, we will evaluate $\Gamma_{rms}{}^2$. From the notes (page 160-21), we see that

$$\Gamma_{rms}{}^2 = \frac{1}{2}\sum_{n=0}^{\infty}c_n{}^2$$ where c_n are the coefficients of the ISF represented by a Fourier series.

What are the c_n? We shall assume that the ISF of the LC tank is a sinusoid of the same period. Therefore, only the c_1 coefficient is important. If the peak value of the ISF is 1V (a questionable assumption) then the *rms* value is 0.707. Thus $\Gamma_{rms}{}^2 \approx 0.25$.

$$q_{max} = Cv_{max} = 100\text{pF}(1\text{V}) = 10^{-10}\ \text{coulombs.}$$

$$\therefore \quad \mathcal{L}\{f_m\} = 10\log_{10}\left[\frac{3.314\text{x}10^{-24}(0.25)}{2\cdot10^{-20}(10^5)^2}\right] = 10\log_{10}(4.14\text{x}10^{-13}) = -123.82\ \text{dBc/Hz}$$

$$\mathcal{L}\{f_m\} = \underline{\underline{-123.82\ \text{dBc/Hz}}}$$

Problem 5 – (10 points)

A crystal reference oscillator and its associated transistor have the following specifications at 290°K.

Output frequency:	6.4MHz
Power output:	+10 dBm
Noise figure:	2.0 dB
Flicker corner:	15 kHz
Loaded Q:	$12\text{x}10^3$

(a.) Determine and plot the SSB phase noise in dBc as a function of the frequency offset from the carrier. Include the frequency range from 10Hz to 10MHz.

(b.) Suppose that this reference oscillator is used with a frequency synthesizer whose transfer function from the reference to the output is

$$\frac{\theta_{n,o}(s)}{\theta_{n,ref}(s)} = \frac{N}{Nref}\frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where N = 19,000, N_{ref} = 256, ζ = 0.7, and ω_n = 908 sec.$^{-1}$. Make a plot of the SSB reference noise in the output of the synthesizer.

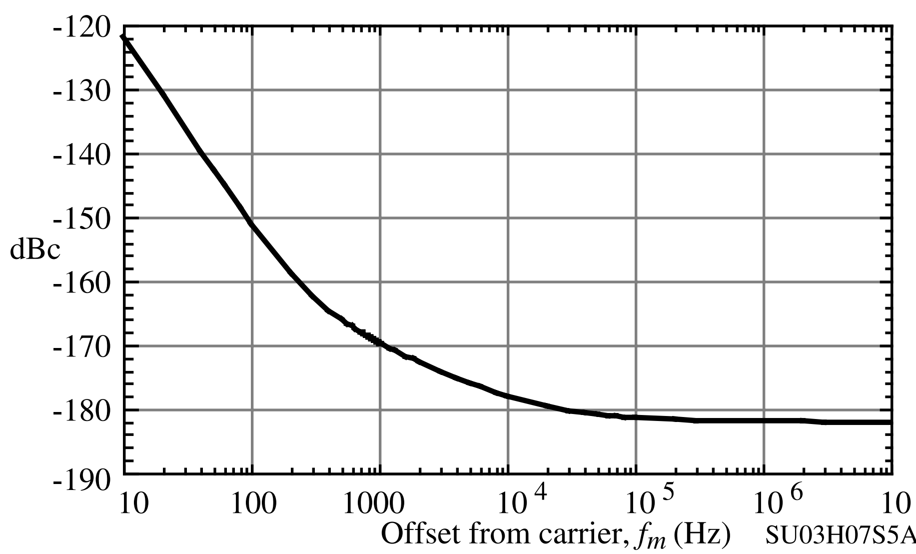
Solution

(a.) NF = 2.0dB, $F = 10^{2.0/10} = 1.585$, and P_o = 10 dBm = 0.01W

$$\mathcal{L}\{f_m\} = 10\log\left[\frac{FkT}{P_s}\left(1 + \frac{1}{4Q^2}\left(\frac{f_o}{f_m}\right)^2\right)\left(1 + \frac{f_c}{f_m}\right)\right]$$

$$= 10\log\left[\frac{1.585\cdot 1.38\text{x}10^{-23}\cdot 290}{0.01}\left(1 + \frac{1}{4(12\text{x}10^3)^2}\left(\frac{6.4\text{x}10^6}{f_m}\right)^2\right)\left(1 + \frac{15\text{kHz}}{f_m}\right)\right]$$

$$\mathcal{L}\{f_m\} = 10\log\left[6.348\text{x}10^{-19}\left(1 + \frac{71.11\text{x}10^3}{f_m^2}\right)\left(1 + \frac{1.5\text{x}10^4}{f_m}\right)\right]$$



Problem 5 – Continued

(b.) The VCO phase noise transfer function is

$$\frac{\theta_{n,o}(s)}{\theta_{n,ref}(s)} = \frac{N}{Nref}\frac{2\zeta\omega_n s + \omega_n{}^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2} = 74.219\,\frac{1271.2s^2 + 8.245\text{x}10^5}{s^2 + 635.6s + 8.245\text{x}10^5}$$

$$\theta_{n,ref}(\text{dBc}) = 10\log\left[\left|\frac{\theta_{n,o}(j\omega)}{\theta_{n,ref}(j\omega)}\right|^2 \mathcal{L}\{f_m\}\right]$$

Below is a plot of the above equation as well as the transfer function, $\theta_{n\backslash,o}(s)/\theta_{n\backslash,ref}(s)$, and the input reference noise.