LECTURE 050 – LINEAR PHASE LOCK LOOPS - I
(References [2])

INTRODUCTION TO PHASE LOCK TECHNIQUES

Introduction

Objective:

Understand the principles and applications of phase locked loops using integrated circuit technology with emphasis on CMOS technology.

Organization:

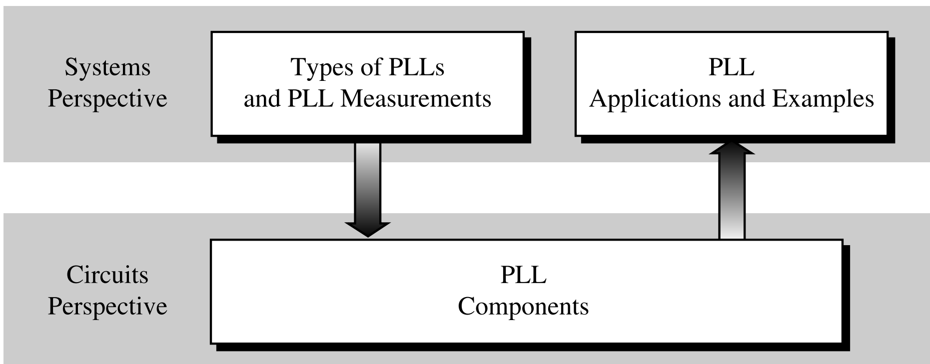


Fig. 050-01

Introduction - Continued

Outline:

- Operating Principles of PLLs
- Classification of PLL Types

Pertinent References:

1. F.M Gardner, *Phaselock Techniques*, 2nd edition, John-Wiley & Sons, Inc., New York, 1979.
2. B. Razavi (ed.), *Monolithic Phase-Locked Loops and Clock Recovery Circuits*, IEEE Press, 1997.
3. R.E. Best, *Phase-Locked Loops: Design, Simulation, and Applications*, 4th edition, McGraw-Hill, 1999.
4. Recent publications of the *IEEE Journal of Solid-State Circuits*.

OPERATING PRINCIPLES OF PLLs

What is a PLL?

A PLL contains three basic components as shown below:

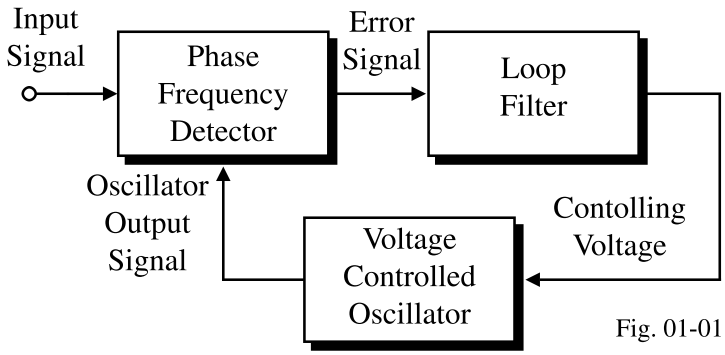


Fig. 01-01

- Phase/frequency detector determines the difference between the phase or frequency of two signals
- The loop filter removes the high-frequencies from the voltage-controlled oscillator (VCO) controlling voltage
- The VCO produces and output frequency controlled by a voltage

More Detailed PLL Block Diagram

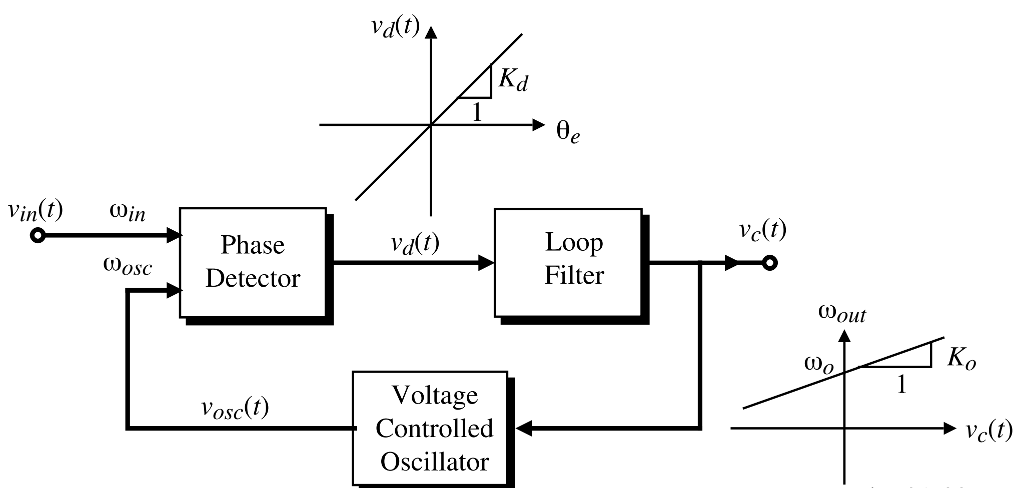


Fig. 01-02

$v_{in}(t)$ – The input or reference signal
ω_{in} – The radian frequency of the input signal
$v_{osc}(t)$ – The output of the VCO
ω_{osc} – The radian frequency of the VCO
$v_d(t)$ – The detector output voltage = $K_d\theta_e$
θ_e – Phase error between $v_{in}(t)$ and $v_{out}(t) = \theta_{in} - \theta_{osc}$
$v_c(t)$ – The output voltage of the loop filter and the control voltage for the VCO

The Phase Detector and VCO in more Detail

Phase Detector:

$$v_d(t) = K_d\theta_e = K_d(\theta_{in} - \theta_{osc})$$

where K_d is the gain of the phase detector.

The units of K_d are volts/radians or simply volts assuming all phase shifts are in radians and not degrees.

Voltage Controlled Oscillator:

$$\omega_{osc} = \omega_o + K_o\, v_c(t)$$

where K_o is the VCO gain and ω_o is the free-running radian frequency.

The units of K_o are rads/sec·V or simply $(\text{sec}\cdot\text{V})^{-1}$ assuming all phase shifts are in radians and not degrees.

PLL Operation

Locked Operation:

- The loop is *locked* when the frequency of the VCO is exactly equal to the average frequency of the input signal.
- If the input signal has noise, the phase locked loop will remove much of the noise on the input signal.
- To maintain the control voltage needed for locked conditions, it is generally necessary for the output of the phase/frequency detector to be nonzero.

Unlocked Operation:

- The VCO runs at a frequency called the *free running frequency*, ω_o, which corresponds to no control voltage.
- The capture process is the means by which the loop goes from unlocked, free-running state to that of the locked state.

Transient Response of the PLL

Assume the input frequency is increased by an amount $\Delta\omega$.

1.) ω_{in} increases by $\Delta\omega$ at t_o.

2.) The input signal leads the VCO and v_d begins to increase.

3.) After a delay due to the loop filter, the VCO increases ω_{osc}.

4.) As ω_{osc} increases, the phase error reduces.

5.) Depending on the loop filter, the final phase error will be reduced to zero or to a finite value.

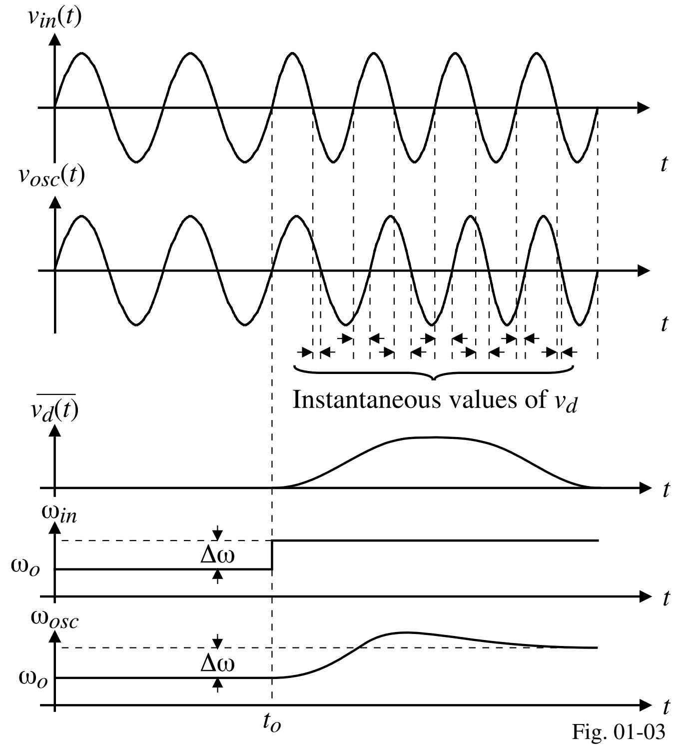


Fig. 01-03

CLASSIFICATION OF PLL TYPES

Types of PLLs

PLL Type	Phase Detector	Loop Filter	Controlled Oscillator
Linear PLL (LPLL)	Analog multiplier	RC passive or active	Voltage
Digital PLL (DPLL)	Digital detector	RC passive or active	Voltage
All digital PLL (ADPLL)	Digital detector	Digital filter	Digitally controlled
Software PLL (SPLL)	Software multiplier	Software filter	Software oscillator

The digital PLL (DPLL) has been the mainstay of most PLLs and is called the “classical” digital PLL.

The Linear PLL (LPLL)

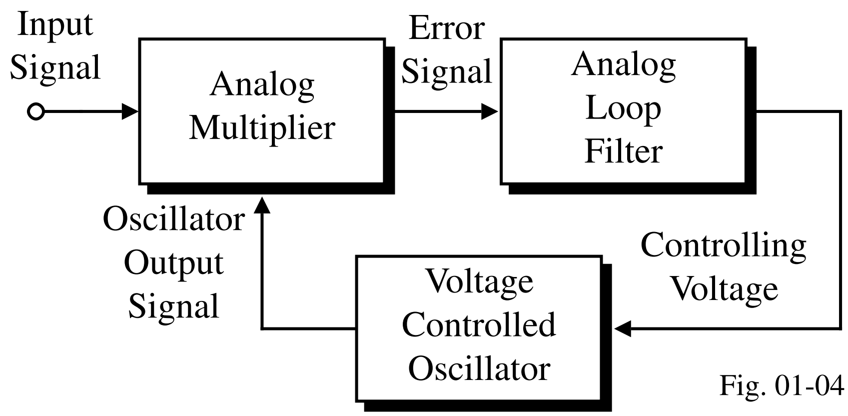


Fig. 01-04

The Digital PLL (DPLL)

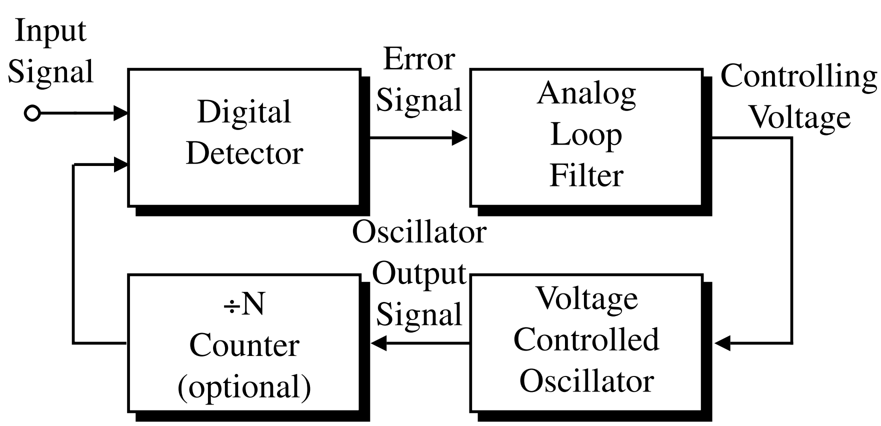


Fig. 01-05

The All-Digital PLL (ADPLL)

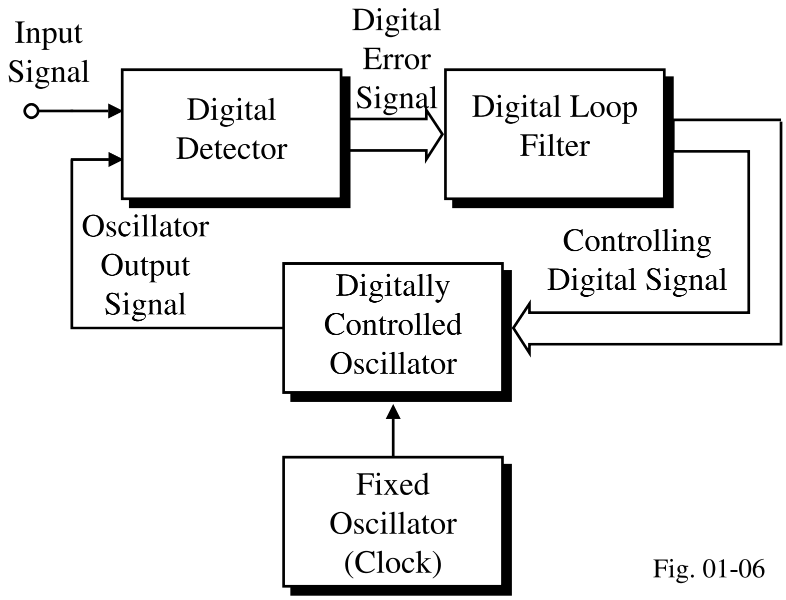


Fig. 01-06

The Software PLL (SPLL)

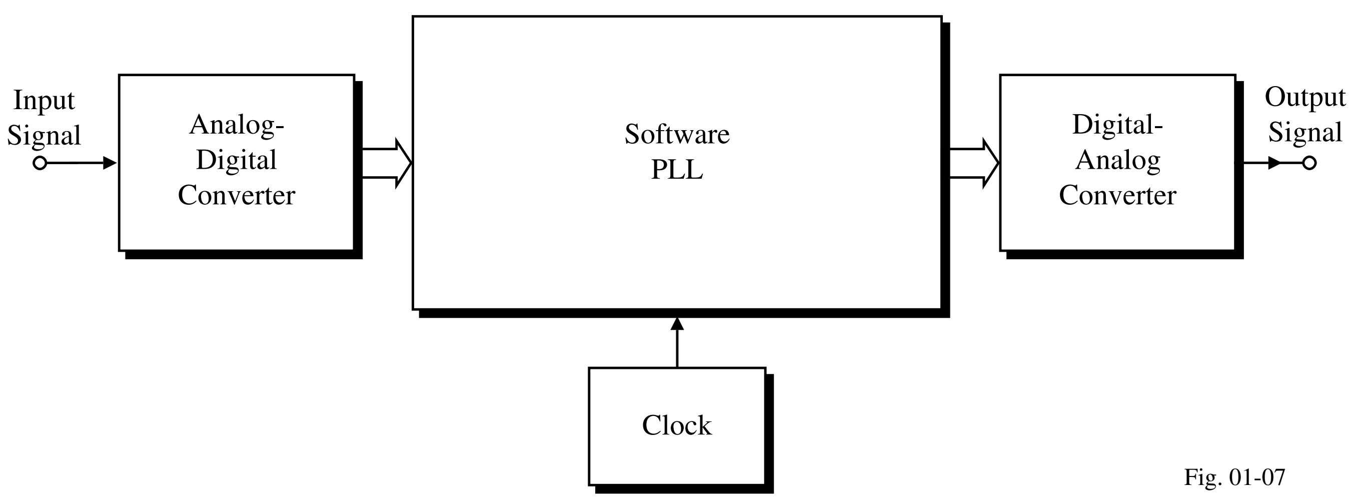


Fig. 01-07

SYSTEMS PERSPECTIVE OF PLLs

Outline

- Linear PLL
- Classical Digital PLL
- All-Digital PLL
- Measurement of PLLs

Roadmap

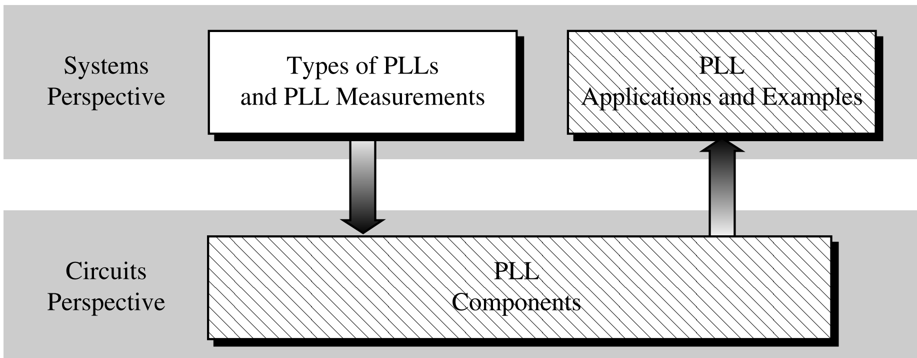


Fig. 050-02

LINEAR PHASE LOCKED LOOPS

Outline

- PLL Components
- Locked State
- Order of the LPLL System
- The Acquisition Process - Unlocked State
- Noise in the LPLL
- LPLL System Design
- Simulation of LPLLs

PLL COMPONENTS

Building Block of the LPLL

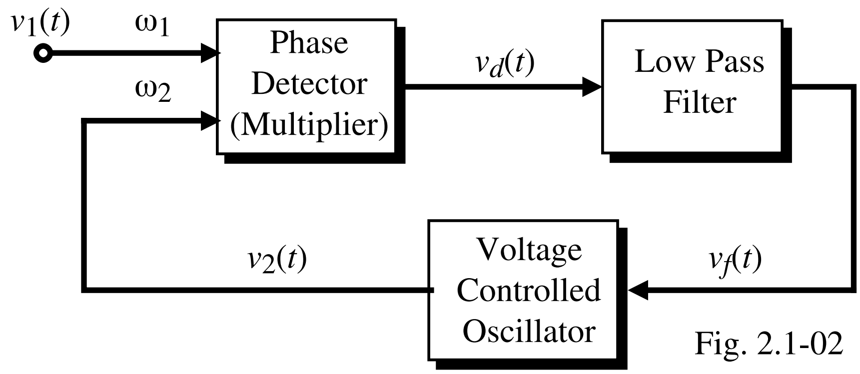


Fig. 2.1-02

$v_1(t)$ = Input signal, generally sinusoidal

$v_2(t)$ = VCO output signal, may be sinusoidal or square wave

$v_d(t)$ = Phase detector output signal

$v_f(t)$ = Loop filter output signal and controlling signal to the VCO

ω_1 = Frequency of the input signal

ω_2 = Frequency of the VCO

Loop Filters

In the PLL, there are many high frequencies including noise that must be removed by the use of a low pass filter in order to achieve optimum performance.

Types of Loop Filters:

1.) Passive lag filter (*lag-lead*)

$$F(s) = \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)} \qquad \text{where} \qquad \tau_1 = R_1C \text{ and } \tau_2 = R_2C$$

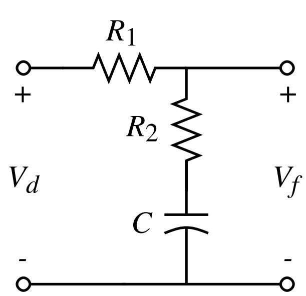
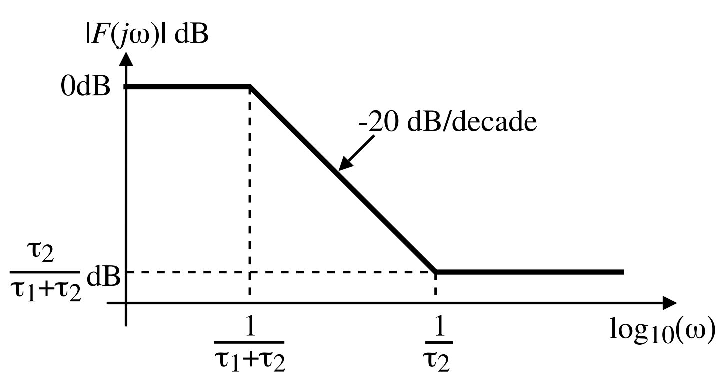


Fig. 2.1-03



Pole is at $1/(\tau_1+\tau_2)$ and the zero at $1/\tau_2$.

- Since the pole is smaller than the zero, the filter is lag-lead
- Passive filters should have no amplitude nonlinearity

Loop Filters - Continued

2.) Active Lag filter

$$F(s) = K_a \frac{1+s\tau_2}{1+s\tau_1} \quad \text{where} \quad \tau_1 = R_1C_1,\ \tau_2 = R_2C_2 \text{ and } K_a = -\frac{C_1}{C_2}$$

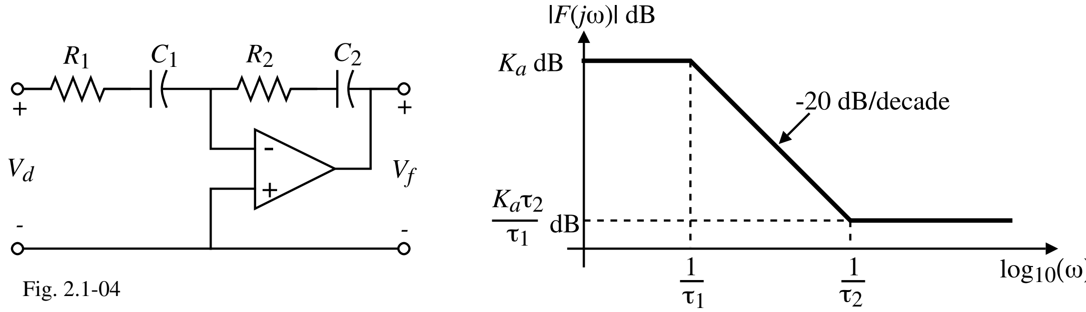


Fig. 2.1-04

- Easier to make lead-lag
- Can have gain (not necessarily desirable)
- Limited by the linearity and noise of the op amp

Loop Filters - Continued

3.) Active Proportional-Integral (PI) Filter

$$F(s) = \frac{1+s\tau_2}{s\tau_1} \quad \text{where} \quad \tau_1 = R_1C \text{ and } \tau_2 = R_2C$$

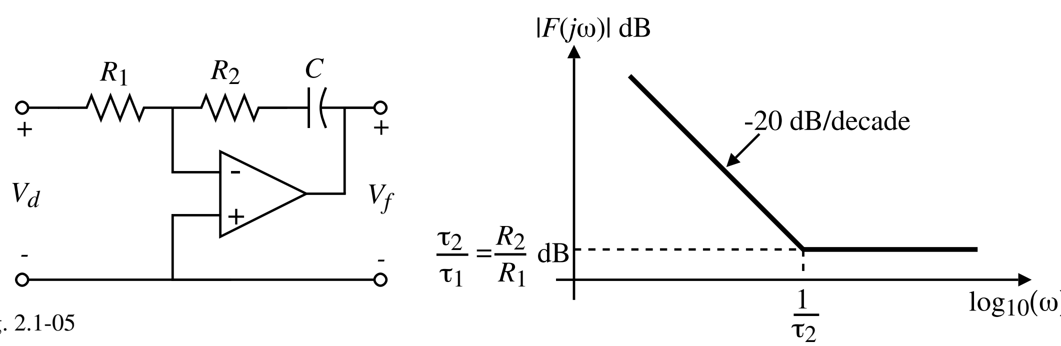


Fig. 2.1-05

- Has large open loop gain at low frequencies ⇒ Large hold range
- Limited by the linearity and noise of the op amp
- Gain limits at the op amp open loop gain

Stability:

To keep the loop stable, it is important to pick the loop filter so that it does not introduce more than a 90° phase shift in the loop.

Phase Signals

It is important to remember that frequency and phase are related as

$$\frac{d\theta}{dt} = \omega \qquad \rightarrow \qquad \theta = \int \omega \cdot dt$$

Transfer functions:

$$H(s) = \frac{V_2(s)}{V_1(s)}$$

where $V_2(s)$ and $V_1(s)$ are the Laplace transforms of $v_2(t)$ and $v_1(t)$.

To examine phase signals, let us assume that,

$v_1(t) = V_{10}\, sin[\omega_1 t + \theta_1(t)]$ and $v_2(t) = V_{20}\, sin[\omega_2 t + \theta_2(t)]$

For phase signals, the information is carried only in $\theta(t)$.

Next, we consider some simple phase signals that are used to excite a PLL.

Phase Signals – Continued

1.) A step phase shift which is an example of phase modulation.

$$\theta_1(t) = \Delta\Phi \;\; u(t)$$

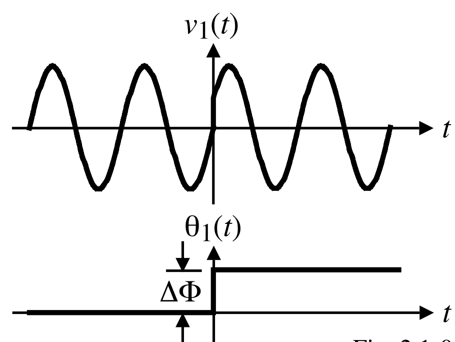


Fig. 2.1-06

2.) A step frequency change assuming that $\omega_1(t) = \omega_o$ for $t < 0$. We may express $v_1(t)$ as,

$$v_1(t) = V_{10}\, sin[\omega_o t + \Delta\omega \cdot t]$$

$$= V_{10}\, sin[\omega_o t + \theta_1(t)]$$

$$\therefore \quad \theta_1(t) = \Delta\omega \cdot t$$

(the phase becomes a ramp signal)

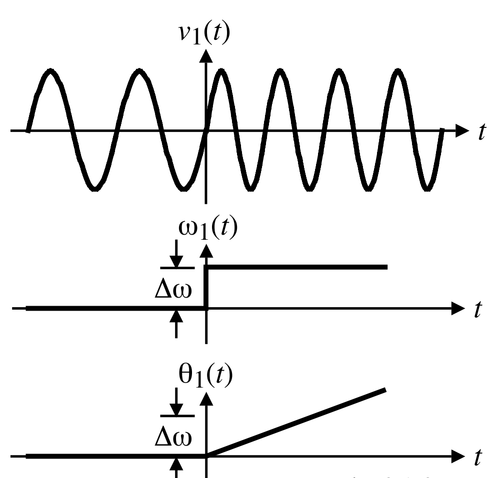


Fig. 2.1-07

Phase Signals – Continued

3.) Frequency ramp

$$\omega_1(t) = \omega_o + \Delta\dot{\omega} \cdot t$$

where $\Delta\dot{\omega}$ is the rate of change of the angular frequency.

$$\therefore \quad v_1(t) = V_{10}\, sin\left[\int_0^t (\omega_o + \Delta\dot{\omega}\tau)d\tau\right]$$

$$= V_{10}\, sin\left(\omega_o t + \Delta\dot{\omega}\frac{t^2}{2}\right)$$

$$\theta_1(t) = \Delta\dot{\omega}\frac{t^2}{2}$$

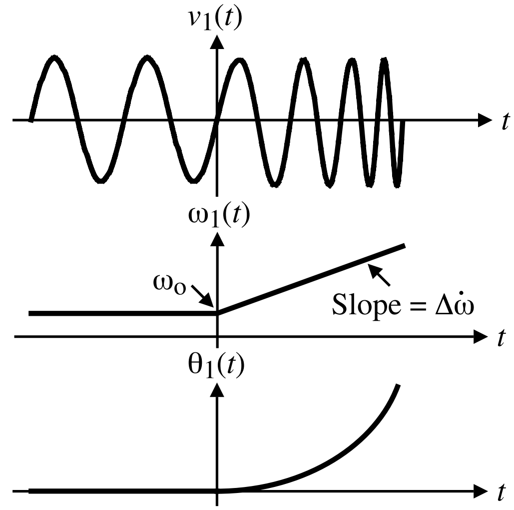


Fig. 2.1-08

LOCKED STATE OF THE LPLL

Transfer Function of the Phase Detector

Input sinusoidal and VCO sinusoidal:

$$v_1(t) = V_{10}\, sin[\omega_1 t + \theta_1(t)] \qquad \text{and} \qquad v_2(t) = V_{20}\, cos[\omega_2 t + \theta_2(t)]$$

$$\therefore\ v_d(t) = v_1(t)\cdot v_2(t) = V_{10}V_{20}\, sin[\omega_1 t + \theta_1(t)]cos[\omega_2 t + \theta_2(t)]$$

$$= \frac{V_{10}V_{20}}{2} sin[\omega_1 t + \theta_1(t) - \omega_2 t - \theta_2(t)] - \frac{V_{10}V_{20}}{2} sin[\omega_1 t + \theta_1(t) + \omega_2 t + \theta_2(t)]$$

If the loop is locked, then $\omega_1 = \omega_2$ and

$$v_d(t) = \frac{V_{10}V_{20}}{2} sin[\theta_1(t) - \theta_2(t)] - \frac{V_{10}V_{20}}{2} sin[2\omega_1 t + \theta_1(t) + \theta_2(t)]$$

Ignoring the high-frequency terms gives,

$$v_d(t) \approx \frac{V_{10}V_{20}}{2} sin[\theta_1(t) - \theta_2(t)] = \frac{V_{10}V_{20}}{2} sin\theta_e(t) = K_d\, sin\theta_e(t) \approx K_d\, \theta_e(t)$$

if $\theta_e(t)$ is small.

$$K_d = \text{detector gain} = \frac{V_{10}V_{20}}{2}$$

$$\therefore\ v_d(t) \approx K_d\, \theta_e(t) \qquad \Rightarrow \qquad V_d(s) \approx K_d\, \Theta_e(s)$$

Transfer Function of the Phase Detector – Continued

Input signals when VCO output is a square wave:

$$v_1(t) = V_{10}\, sin[\omega_1 t + \theta_1(t)]$$

$$v_2(t) = V_{20}\, sgn[\omega_2 t + \theta_2(t)] = V_{20}\left[\frac{4}{\pi}cos[\omega_2 t + \theta_2(t)] + \frac{4}{3\pi}cos[3\omega_2 t + \theta_2(t)] + \cdots\right]$$

$$\therefore\ v_d(t) = v_1(t)\cdot v_2(t)$$

$$= V_{10}V_{20}\, sin[\omega_1 t + \theta_1(t)]\left[\frac{4}{\pi}cos[\omega_2 t + \theta_2(t)] + \frac{4}{3\pi}cos[3\omega_2 t + \theta_2(t)] + \cdots\right]$$

$$= \frac{4V_{10}V_{20}}{\pi}\left[sin[\omega_1 t + \theta_1(t)]cos[\omega_2 t + \theta_2(t)] + \frac{1}{3}\, cos[\omega_2 t + \theta_2(t)]cos[3\omega_2 t + \theta_2(t)] + \cdots\right]$$

When the loop is locked,

$$v_d(t) = \frac{2V_{10}V_{20}}{\pi}\left[sin[\theta_1(t) - \theta_2(t)] + sin[2\omega_1 t + \theta_1(t) + \theta_2(t)] + \cdots\right]$$

$$\approx \frac{2V_{10}V_{20}}{\pi}\, sin\theta_e(t) = K_d\, sin\theta_e(t) \quad\rightarrow\quad v_d(t) \approx K_d\, \theta_e(t)$$

where the detector gain is $K_d = \frac{2V_{10}V_{20}}{\pi}$ (a little better than sinusoidal inputs only)

The transfer function is $V_d(s) \approx K_d\, \Theta_e(s)$ or $\frac{V_d(s)}{\Theta_e(s)} = K_d$

VCO Transfer Function

The angular frequency of the VCO was expressed as,

$$\omega_2(t) = \omega_o + \Delta\omega_2(t) = \omega_o + K_o\, v_f(t)$$

where K_o is the VCO gain in units of radians/sec or simply sec^{-1}.

However, what we want is the phase of the VCO output.

$$\therefore\quad \theta_2(t) = \int\Delta\omega_2\, dt = K_o\int v_f(t)dt$$

Taking the Laplace transform gives,

$$\Theta_2(s) = \mathscr{L}[\theta_2(t)] = \frac{K_o}{s}\, V_f(s) \quad\rightarrow\quad \frac{\Theta_2(s)}{V_f(s)} = \frac{K_o}{s}$$

Linear Model of the LPLL

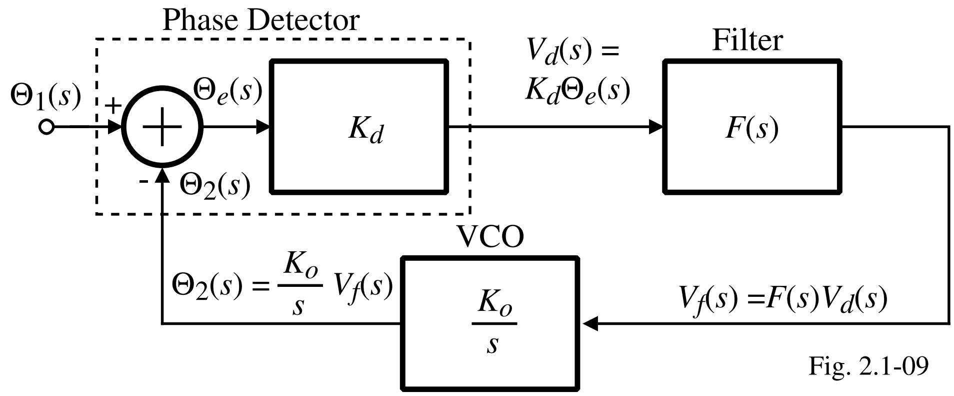


Fig. 2.1-09

Phase transfer function:

$$H(s) = \frac{\Theta_2(s)}{\Theta_1(s)} = ?$$

$$\Theta_2(s) = \frac{K_o}{s} V_f(s) = \frac{K_o}{s} F(s)V_d(s) = \frac{K_oK_d}{s} F(s)\Theta_e(s) = \frac{K_oK_d}{s} F(s)[\Theta_1(s) - \Theta_2(s)]$$

$$s\Theta_2(s) = K_oK_dF(s)[\Theta_1(s) - \Theta_2(s)] \quad \rightarrow \quad \Theta_2(s)[s + K_oK_dF(s)] = K_oK_dF(s)\Theta_1(s)$$

$$\therefore \quad H(s) = \frac{\Theta_2(s)}{\Theta_1(s)} = \frac{K_oK_dF(s)}{s + K_oK_dF(s)} \qquad \text{Also, } H_e(s) = 1 - H(s) = \frac{s}{s + K_oK_dF(s)}$$

LPLL Transfer Function for Various Loop Filters

1.) Passive lag filter.

$$F(s) = \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)} \quad \rightarrow \quad H(s) = \frac{K_oK_d\left(\frac{1+s\tau_2}{\tau_1+\tau_2}\right)}{s^2 + s\left(\frac{1+K_oK_d\tau_2}{\tau_1+\tau_2}\right) + \frac{K_oK_d}{\tau_1+\tau_2}}$$

2.) The active lag filter.

$$F(s) = K_a\frac{1 + s\tau_2}{1 + s\tau_1} \quad \rightarrow \quad H(s) = \frac{K_oK_dK_a\left(\frac{1+s\tau_2}{\tau_1}\right)}{s^2 + s\left(\frac{1+K_oK_dK_a\tau_2}{\tau_1}\right) + \frac{K_oK_dK_a}{\tau_1}}$$

3.) The active PI filter.

$$F(s) = \frac{1 + s\tau_2}{s\tau_1} \quad \rightarrow \quad H(s) = \frac{K_oK_d\left(\frac{1+s\tau_2}{\tau_1}\right)}{s^2 + s\left(\frac{K_oK_d\tau_2}{\tau_1}\right) + \frac{K_oK_d}{\tau_1+\tau_2}}$$

Normalized Form of the Transfer Functions

The normalized form of the denominator of a second-order transfer function is

$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

where ω_n is the natural frequency and ζ is the damping factor.

1.) Passive lag filter.

$$\omega_n = \sqrt{\frac{K_oK_d}{\tau_1+\tau_2}} \qquad \text{and} \qquad \zeta = \frac{\omega_n}{2}\left(\tau_2 + \frac{1}{K_oK_d}\right)$$

2.) Active lag filter.

$$\omega_n = \sqrt{\frac{K_oK_dK_a}{\tau_1}} \qquad \text{and} \qquad \zeta = \frac{\omega_n}{2}\left(\tau_2 + \frac{1}{K_oK_dK_a}\right)$$

3.) Active PI filter.

$$\omega_n = \sqrt{\frac{K_oK_d}{\tau_1}} \qquad \text{and} \qquad \zeta = \frac{\omega_n\tau_2}{2}$$

Normalized Phase Functions

1.) Passive lag filter.

$$H(s) = \frac{s\omega_n\left(2\zeta - \frac{\omega_n}{K_oK_d}\right) + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2.) Active lag filter.

$$H(s) = \frac{s\omega_n\left(2\zeta - \frac{\omega_n}{K_oK_dK_a}\right) + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

3.) Active PI filter.

$$H(s) = \frac{2s\zeta\omega_n + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

If $K_dK_o >> \omega_n$ or $K_dK_oK_a >> \omega_n$, then all of the above transfer functions simplify to,

$$H(s) = \frac{2s\zeta\omega_n + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \text{and} \qquad H_e(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Frequency Response of *H*(*s*)

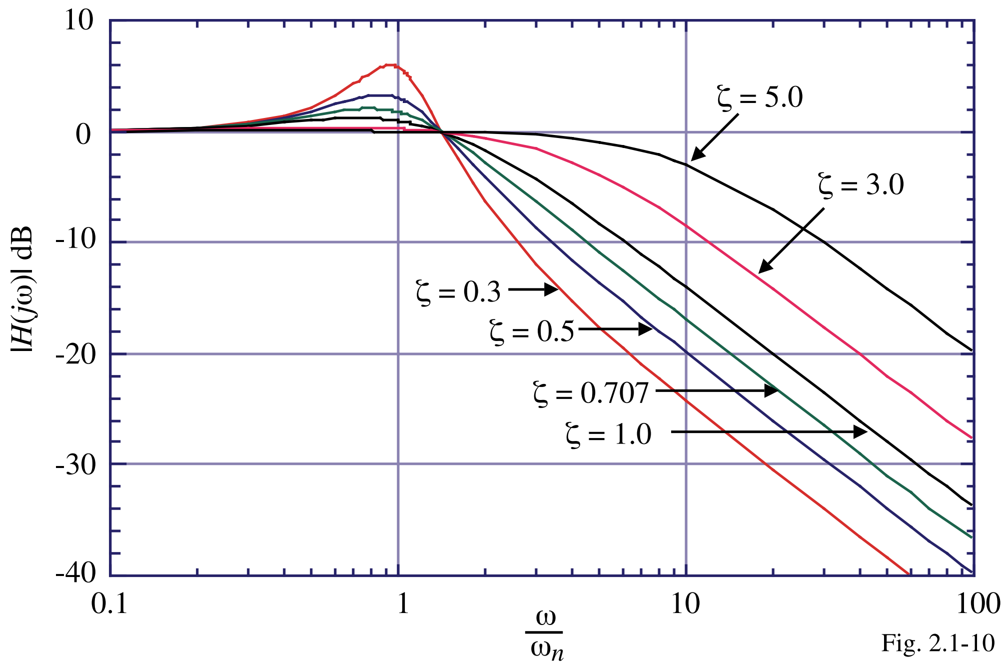


Fig. 2.1-10

Phase Step Response

Assume that $\theta_1(t) = \Delta\Phi \cdot u(t)$ → $\Theta_1(s) = \frac{\Delta\Phi}{s}$

Phase error:

$$\Theta_e(s) = H_e(s)\frac{\Delta\Phi}{s} = \frac{\Delta\Phi s^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\theta_e(t) = \mathcal{L}^{-1}[\Theta_e(s)] = \Delta\Phi\left(\cos\sqrt{1-\zeta^2}\,\omega_n t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right)e^{-\zeta\omega_n t}, \quad \zeta < 1$$

$$= \Delta\Phi(1 - \omega_n t)e^{-\zeta\omega_n t}, \quad \zeta = 1$$

$$= \Delta\Phi\left(\cosh\sqrt{\zeta^2-1}\,\omega_n t - \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\sqrt{1-\zeta^2-1}\,\omega_n t\right)e^{-\zeta\omega_n t}, \quad \zeta > 1$$

Steady state error:

$$\theta_e(\infty) = \lim_{s\to 0} s\Theta_e(s) = 0$$

Frequency Step Response

Assume that $\omega_1(t) = \omega_o + \Delta\omega \cdot u(t)$

However, $\theta_1(t) = \Delta\omega \cdot t$ $\rightarrow$ $\Theta_1(s) = \dfrac{\Delta\omega}{s^2}$

Phase error:

$$\Theta_e(s) = H_e(s)\frac{\Delta\omega}{s^2} = \frac{\Delta\omega s^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{\Delta\omega}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\theta_e(t) = \mathcal{L}^{-1}[\Theta_e(s)] = \frac{\Delta\omega}{\omega_n}\left(\frac{1}{\sqrt{1 - \zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right)e^{-\zeta\omega_n t},\ \ \zeta < 1$$

$$= \frac{\Delta\omega}{\omega_n}(\omega_n t)e^{-\zeta\omega_n t},\ \ \zeta = 1$$

$$= \frac{\Delta\omega}{\omega_n}\left(\frac{1}{\sqrt{\zeta^2-1}}\sinh\sqrt{1-\zeta^2-1}\,\omega_n t\right)e^{-\zeta\omega_n t},\ \ \zeta > 1$$

Steady state error:

$$\theta_e(\infty) = \lim_{s\to 0} s\Theta_e(s) = 0 \text{ (high gain loops)} \qquad \theta_e(\infty) = \frac{\Delta\omega}{K_d K_o F(0)} \text{ (low gain loops)}$$

Frequency Ramp Response

Assume that $\omega_1(t) = \omega_o + \Delta\dot{\omega} \cdot t$

However, $\theta_1(t) = \Delta\dot{\omega}\dfrac{t^2}{2}$ $\rightarrow$ $\Theta_1(s) = \dfrac{\Delta\dot{\omega}}{s^3}$

Phase error:

$$\Theta_e(s) = H_e(s)\frac{\Delta\dot{\omega}}{s^3} = \frac{\Delta\dot{\omega}\, s^2}{s^3(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{\Delta\dot{\omega}}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\theta_e(t) = \mathcal{L}^{-1}[\Theta_e(s)] = \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}\left(\cos\sqrt{1-\zeta^2}\,\omega_n t + \frac{\zeta}{\sqrt{1+\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t\right)e^{-\zeta\omega_n t},\ \ \zeta < 1$$

$$= \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}(1 + \omega_n t)e^{-\zeta\omega_n t},\ \ \zeta = 1$$

$$= \frac{\Delta\dot{\omega}}{\omega_n^2} - \frac{\Delta\dot{\omega}}{\omega_n^2}\left(\cosh\sqrt{\zeta^2-1}\,\omega_n t + \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\sqrt{1-\zeta^2-1}\,\omega_n t\right)e^{-\zeta\omega_n t},\ \ \zeta > 1$$

Steady state error:

$$\theta_e(\infty) = \lim_{s\to 0} s\Theta_e(s) = \frac{\Delta\dot{\omega}}{\omega_n^2} \text{ (High loop gain)} \quad \theta_e(\infty) = \frac{\Delta\dot{\omega}\, t}{K_d K_o F(0)^2} + \frac{\Delta\dot{\omega}}{\omega_n^2} \text{ (Low loop gain)}$$

THE ORDER OF A LPLL SYSTEM

Definition of Order

The number of roots in the denominator (poles) of the PLL transfer function determines the order.

Generally, the order of a PLL is one greater than the order of $F(s)$.

Implication of the order:

- Greater than 2 will be unstable unless corrected by zeros
- Less than 2 will have poor noise suppression.

First-Order PLL

A first–order PLL occurs when $F(s) = 1$. From previous results we have,

$$H(s) = \frac{\Theta_2(s)}{\Theta_1(s)} = \frac{K_oK_d}{s + K_oK_d} \qquad \text{Also, } H_e(s) = 1 - H(s) = \frac{s}{s + K_oK_d}$$

The –3dB bandwidth of $H(s)$ is K_oK_d.

Comments:

- $F(s)$ causes the –3dB bandwidth to be reduced in higher-order systems which means that the first-order PLL has a wider bandwidth
- The hold range of the first-order PLL will be larger than for higher-order PLLs
- The first-order PLL will track the signal variations more quickly than higher-order PLLs
- The first-order PLL does not suppress noise superimposed on the input signal to the extent of higher-order PLLs.

Higher-Order PLLs

Comments:

- Generally $F(s)$ has a pole and a zero in order to get better noise rejection without sacrificing speed.

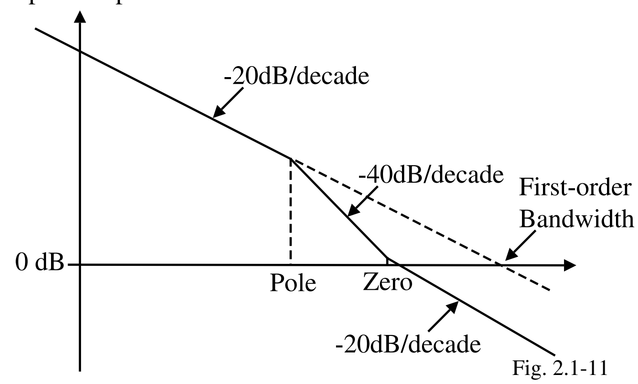


Fig. 2.1-11

- If the phase shift of the open loop system is more than 90°, the stability of the loop may be poor (ζ is small).

(To be continued in Lecture 060)

