LECTURE 090 – PLL DESIGN EQUATIONS AND PLL MEASUREMENTS
(Reference [2, Previous ECE6440 Notes])

Objective
The objective of this presentation is
1.) To provide a summary of relationships and equations that can be used to design PLLs.
2.) Illustrate the design of a DPLL frequency synthesizer
3.) Show how to make measurements on PLLs

Outline
• PLL design equations
• PLL design example
• PLL measurements
• Summary

PLL DESIGN EQUATIONS†

Introduction
The following design equations are to be used in designing PLLs and apply both to LPLLs and DPLLs with the following definitions:

LPLLs: $N = 1$ and $\beta = 1$
where $N$ is the divider in the feedback loop and $\beta$ is the loop expansion factor determined by the type of PFD.

Loop gain $= K = \frac{K_d K_o F(0)}{N} = \frac{K_v F(0)}{N}$

Goal of these equations:
Permit the basic design of an LPLL or DPLL.

† These notes are taken from PLL Design Equations Notes by R.K Feeney, July 1998
Type – I, First-Order Loop ($F(0) = 1$)

Crossover frequency (frequency at which the loop gain is 1 or 0dB): 
\[ \omega_c = K \text{ (radians/sec.)} \]

-3dB Bandwidth (frequency at which the closed-loop gain is equal to –3dB):

Closed loop transfer function \( \frac{K}{s + K} \) \[ \rightarrow \omega_{-3dB} = K \text{ (radians/sec.)} \]

Noise Bandwidth:
\[
B_n = \int_0^\infty |H(j2\pi f)|^2 df = \int_0^\infty \frac{K^2}{K^2 + (2\pi f)^2} df = \frac{K}{2\pi} \int_0^\infty \frac{K}{K^2 + (2\pi f)^2} d(2\pi f) = \frac{K \pi}{2\pi} = \frac{K}{4} \text{ (Hz)}
\]

Hold Range:
\[ \Delta \omega_H = \beta NK \]

Lock (Capture) Range:
\[ \Delta \omega_L = \Delta \omega_H = \beta NK \]

Type-I, First-Order Loop ($F(0) = 1$) - Continued

Steady-State Phase Error:

For a sinusoidal phase detector, 
\[ \varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1}\left(\frac{\Delta \omega_{osc}}{NK}\right) \]

For a nonsinusoidal (digital) phase detector, 
\[ \varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \frac{\Delta \omega_{osc}}{NK} \leq \beta \]

The steady-state error is never larger than \( \beta \). A larger error indicates a failure to lock.

Frequency Acquisition Time:
\[ T_a = \frac{1}{K} \text{ (sec.)} \]

For a Type-I loop,

Lock Range and Acquisition Time = Hold Range and Acquisition Time.
**Type-I, Second-Order Loop**

This type of loop is generally implemented with a lag-lead filter as shown below.

![Filter Diagram](Fig. 090-01)

Filter Transfer Function:

\[ F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} \]

where \( \tau_1 = (R_1 + R_2)C \) and \( \tau_2 = R_2C \)

(Note: The definition for \( \tau_1 = (R_1 + R_2)C \) which is different from that in Lecture 050)

System Parameters:

\[
\omega_n = \sqrt{\frac{K}{\tau_1}} \quad \text{and} \quad \zeta = \frac{\omega_n}{2} \left( \frac{1}{\tau_2} + \frac{1}{K} \right) = \frac{1}{2} \sqrt{\frac{K}{\tau_1}} (1 + \tau_2 K)
\]

Note that because \( \tau_2 < \tau_1 \), we see that

\[
\frac{\omega_n}{2K} < \zeta < \frac{K^2 + \omega_n^2}{2\omega_n K}
\]

**Type-I, Second-Order Loop – Continued**

Crossover Frequency:

The general close-loop frequency response for high-gain loops is,

\[
H(s) = \frac{2s\xi\omega_n + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{1 + \frac{2\xi\omega_n s + \omega_n^2}{s^2}} = \frac{1}{1 + \text{Loop Gain}}
\]

The crossover frequency, \( \omega_c \), is the frequency when the loop gain is unity.

\[
\omega_c^4 - (4\xi^2\omega_n^2)\omega_c^2 - \omega_n^4 = 0
\]

Solving for \( \omega_c \) gives,

\[
\omega_c = \omega_n \sqrt{2\xi^2 + \sqrt{4\xi^4 + 1}}
\]

3dB Bandwidth:

\[
\omega_{3\text{dB}} = \omega_n \sqrt{b + \sqrt{b^2 + 1}} \quad \text{where} \quad b = 2\xi^2 + 1 - \frac{\omega_n}{K} \left(4\xi - \frac{\omega_n}{K}\right)
\]

Noise Bandwidth:

\[
B_n = \frac{\omega_n}{2} \left(\zeta + \frac{1}{4\xi}\right) \text{ (Hz)}
\]
Type-I, Second-Order Loop – Continued

Hold Range:
\[
\Delta \omega_H = \beta NK \text{ at the output} \quad \Delta \omega_H = \beta K \text{ at the input}
\]

Lock Range:
\[
\Delta \omega_L = \frac{\tau_2}{\tau_1} \Delta \omega_H = \frac{\tau_2}{\tau_1} \beta NK
\]

Lock Time:
The lock time is set by the loop natural frequency, \( \omega_n \) and is
\[
T_L = \frac{2\pi}{\omega_n}
\]

Pull-in Range:
\[
\Delta \omega_P = N\beta\sqrt{2\zeta\omega_n KF(0)-\omega_n^2} \text{ at the output} \quad \Delta \omega_P = \beta\sqrt{2\zeta\omega_n KF(0)-\omega_n^2} \text{ at the input}
\]

This formula is only valid for moderate or high loop gains, i.e. \( KF(0) \leq 0.4\omega_n \).

Pull-in Time:
\[
T_P \approx \frac{4\left(\frac{\Delta f_{osc}}{N}\right)^2}{B_n^3} \approx \frac{\pi^2}{16} \frac{\Delta \omega_{osc}^2}{\zeta\omega_n^3}
\]

Note that \( \Delta \omega_H \leq \Delta \omega_{osc} \leq \Delta \omega_P \)

Type-I, Second-Order Loop – Continued

Frequency Acquisition Time:
\[
T_d = T_P + T_L
\]

If the frequency step is within the lock limit (one beat), then the pull-in time is zero.

Steady-State Phase Error to a frequency step of \( \Delta \omega_{osc} \):

For a sinusoidal phase detector, \( \epsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1}\left(\frac{\Delta \omega_{osc}}{NK}\right) \)

For a nonsinusoidal (digital) phase detector, \( \epsilon_{ss} = \lim_{t \to \infty} \theta(t) = \frac{\Delta \omega_{osc}}{NK} \leq \beta \)

The steady-state error is \( \leq \beta \). A larger error indicates a failure to lock.

Maximum Sweep Rate of the Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.
\[
\frac{d(\Delta \omega)}{dt} = \omega_n^2 \text{ (radians/sec)}
\]

Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.
\[
\frac{d(\Delta \omega)}{dt} = \frac{\omega_n^2}{2} \text{ (radians/sec)}
\]
**Type-2, Second-Order Loop**

This type of PLL system generally uses the active PI filter as shown below.

![Type-2, Second-Order Loop Diagram](image)

**Filter Transfer Function:**

\[
F(s) = \frac{1 + \frac{1}{\tau_2}}{\tau_1 s} = -\left(\frac{R_2}{R_1}\right)\frac{s + 1/\tau_2}{s} \quad \text{where} \quad \tau_1 = R_1 C \text{ and } \tau_2 = R_2 C
\]

**System Parameters:**

\[
\omega_n = \sqrt{\frac{K}{\tau_1}} \quad \text{and} \quad \zeta = \frac{1}{2} \sqrt{K\tau_2 \frac{R_2}{R_1}} = \frac{1}{2} \tau_2 \sqrt{\frac{K}{\tau_1}} = \frac{\tau_2 \omega_n}{2}
\]

**3dB Bandwidth:**

\[
\omega_{-3\text{dB}} = \omega_n \sqrt{2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}}
\]

**Type-2, Second-Order Loop – Continued**

**Noise Bandwidth:**

\[
B_n = \frac{\omega_n}{2} \left(\zeta + \frac{1}{4\zeta^2}\right) \quad \text{or} \quad B_n = \frac{1}{4} \left(\frac{R_2}{R_1} + 1\right)
\]

**Hold Range:**

Limited by the dynamic range of the loop components.

**Lock (Capture) Range:**

\[
\Delta \omega_H = \beta N 2\zeta \omega_n
\]

**Lock (Capture) Time:**

\[
T_L = \frac{2\pi}{\omega_n}
\]

**Pull-in Range:**

The pull-in range is the frequency range beyond the lock (capture) range over which the loop will lock after losing lock (skipping cycles).

- The pull-in range for a 2nd or higher order, type-2 loop is theoretically infinite and limited by the amplifier and phase detector offsets and by the dynamic range of the loop.
- A system with large offsets and a large frequency error may never lock.
Type-2, Second-Order Loop – Continued

Pull-in Time:

\[ T_P = \tau_2 \left( \frac{\Delta \omega_{osc}}{N} - \sin \theta_o \right) \]

where \( \theta_o \) is the initial phase difference between the reference and VCO signals.
Assume \( \sin \theta_o = -1 \) for the worst case.

Pull-out Range:

\[ \Delta \omega_{PO} \approx 1.8N\beta \omega_n(1 + \zeta) \] at the output
\[ \Delta \omega_{PO} \approx 1.8\beta \omega_n(1 + \zeta) \] at the input

Frequency Acquisition Time:

\[ T_a = T_P + T_L \]

If the frequency step is within the lock limit (one beat), then the pull-in time is zero.

Steady-state Phase Error:

The steady-state phase error of a type-2 system is zero for both a phase step and a frequency step.

Steady-state Phase Error – Continued:

For a sinusoidal phase detector,

\[ \varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1} \left( \frac{R_1}{R_2} \tau_2 \frac{d \Delta \omega_{osc}}{NK} \right) \]

For a nonsinusoidal (digital) phase detector,

\[ \varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \left[ \frac{R_1}{R_2} \tau_2 \frac{d \Delta \omega_{osc}}{NK} \right] \leq \beta \]

The steady-state error is \( \leq \beta \). A larger error indicates a failure to lock.

Maximum Sweep Rate of Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.

\[ \frac{d(\Delta \omega_{in})}{dt} = \beta \omega_n^2 \frac{(\text{radians/sec})}{\text{sec}} \]

Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.

\[ \frac{d(\Delta \omega_{osc})}{dt} = \frac{N\beta}{2\tau_2} \left( 4B_n - \frac{1}{\tau_2} \right) \frac{(\text{radians/sec})}{\text{sec}} \]
DESIGN OF A 450-475 MHz DPLL FREQUENCY SYNTHESIZER

Specifications

Design a DPLL frequency synthesizer that meets the following specifications:
Frequency Range: 450 – 475 MHz
Channel Spacing: 25 kHz
Modulation: FM from 300 to 3000 Hz
Modulation Deviation: ±5kHz
Loop Type: Type 2
Loop Order: Second order
VCO Gain: \( K_o = 1.25\text{MHz/V} = 7.854\text{Mradians/sec./V} \)
Phase Detector Type: PFD (\( \beta = 2\pi \))
Phase Detector Gain: \( K_d = 0.796\text{V/radian} \)

(Note this example will be continued later in more detail concerning phase noise and spurs)

Note on channel spacing:
Carson’s rule \( \rightarrow \) BW of an FM signal is \( \approx 2[\Delta f_c + f_m(\text{max})] = 2(\pm5\text{kHz}+3\text{kHz}) = 16\text{kHz} \)
If we assume a 9 kHz guard band, then Channel Spacing = 9 kHz + 16 kHz = 25 kHz

PLL System

Block Diagram:

The pertinent transfer function for this problem is given as \( \frac{\omega_2(s)}{V_{fm}(s)} \) which is found as

\[
\omega_2(s) = K_o \left[ V_{fm}(s) + F(s)K_d \left( \theta_1 - \frac{\omega_2(s)}{sN} \right) \right] = K_o \left[ V_{fm}(s) + F(s)K_d \theta_1 - \frac{F(s)K_d}{sN} \omega_2(s) \right]
\]

Setting \( \theta_1 = 0 \) gives

\[
\frac{\omega_2(s)}{V_{fm}(s)} = \frac{K_o}{1 + \frac{F(s)K_d K_o}{sN}}
\]
PLL System - Continued

The charge-pump + filter combination has a transfer function given as

\[ F(s) = \frac{1 + \tau_2 s}{\tau_1 s} \]

The final form of the closed-loop transfer is given as

\[ \frac{\omega_2(s)}{V_{fm}(s)} = \frac{1}{1 + (1 + \tau_2 s)K_dK_o} \frac{K_o}{s^2N\tau_1} = \frac{s^2K_o}{s^2 + \frac{K_dK_o\tau_2}{N\tau_1} s + \frac{K_dK_o}{N\tau_1}} = \frac{s^2K_o}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

where,

\[ \omega_n = \sqrt{\frac{K_dK_o}{N\tau_1}} \quad \text{and} \quad \zeta = \frac{\tau_2}{2} \sqrt{\frac{K_dK_o}{N\tau_1}} \]

Finding the Loop Parameters

1.) Division Ratio

\[ N_{\min} = \frac{450 \text{ MHz}}{25 \text{ kHz}} = 18,000 \quad \text{and} \quad N_{\max} = \frac{475 \text{ MHz}}{25 \text{ kHz}} = 19,000 \]

2.) Loop Bandwidth

To pass the 300Hz lower frequency limit, we require that the maximum –3dB frequency is 300Hz. Therefore, \( B_L = 300 \text{Hz} \).

3.) Damping Constant

For reasons discussed previously, we select \( \zeta = 0.707 \). Let us check to see if this is consistent with the design.

We know that,

\[ \zeta = \frac{\tau_2}{2} \sqrt{\frac{K_dK_o}{N\tau_1}} \quad \rightarrow \quad \zeta = \frac{k}{\sqrt{N}} \]

\[ \therefore \quad \zeta_{\max} = \frac{k}{\sqrt{N_{\min}}} \quad \text{and} \quad \zeta_{\min} = \frac{k}{\sqrt{N_{\max}}} \quad \rightarrow \quad \zeta_{\max} = \zeta_{\min} \sqrt{\frac{N_{\max}}{N_{\min}}} = 1.0274\zeta_{\min} \]

Also, \( \zeta = \sqrt{\zeta_{\max}\zeta_{\min}} = 0.707 \), which gives

\[ \zeta_{\min}^2(1.0274) = 0.5 \quad \rightarrow \quad \zeta_{\min} = 0.6976 \quad \text{and} \quad \zeta_{\max} = 1.0274\cdot0.6976 = 0.7167 \]
Finding the Loop Parameters – Continued

4.) Natural frequency, $\omega_n$

$$\omega_{3dB} = \omega_n \sqrt{2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}} \quad \rightarrow \quad \omega_n = \frac{\omega_{3dB}}{\sqrt{2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}}}$$

The maximum $\omega_n$ will occur at the minimum value of $N$ and the minimum damping factor. Therefore,

$$\omega_n(max) = \frac{\omega_{3dB}}{\sqrt{2\zeta_{\min}^2 + 1 + \sqrt{(2\zeta_{\min}^2 + 1)^2 + 1}}$$

$$= \frac{\omega_{3dB}}{\sqrt{2(0.6976)^2 + 1 + \sqrt{(2(0.6976)^2 + 1)}}} = 980 \text{ radians/sec.}$$

$$\omega_n(min) = \frac{\omega_{3dB}}{\sqrt{2\zeta_{\max}^2 + 1 + \sqrt{(2\zeta_{\max}^2 + 1)^2 + 1}}}$$

$$= \frac{\omega_{3dB}}{\sqrt{2(0.7167)^2 + 1 + \sqrt{(2(0.7167)^2 + 1)}}} = 910 \text{ radians/sec.}$$

∴ $\omega_n = \sqrt{\omega_n(max) \cdot \omega_n(min)} = 944$

Loop Parameter Summary:

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>N</th>
<th>$\omega_n$ (rads./sec.)</th>
<th>$\zeta$</th>
<th>Bandwidth (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>450.00</td>
<td>18,000</td>
<td>910</td>
<td>0.7167</td>
<td>300</td>
</tr>
<tr>
<td>475.00</td>
<td>19,000</td>
<td>980</td>
<td>0.6976</td>
<td>300</td>
</tr>
</tbody>
</table>

Design of the Loop Filter

The loop filter selected is the active PI using the single-ended realization below.

The transfer function is,

$$F(s) = \frac{sR_2C + 1}{sR_1C} = \frac{sT_2 + 1}{sT_1} \quad \rightarrow \quad T_1 = R_1C \quad \text{and} \quad T_2 = R_2C$$

1.) Time constants

We will use the date for $N = 18,000$ to design the filter.

$$\tau_1 = \frac{K_dK_o}{N\omega_n^2} = \frac{0.796 \cdot 7.854 \times 10^6}{18,000(910)^2} = 0.419 \text{ ms}$$

$$\tau_2 = \frac{2\zeta}{\omega_n} = \frac{2 \cdot 0.7167}{910} = 1.575 \text{ ms}$$

2.) Loop filter design

Select $R_1 = 2.4k\Omega$ which gives

$$C = \frac{\tau_1}{R_1} = \frac{0.419 \times 10^{-3}}{2.4 \times 10^{-3}} = 0.175 \mu F \quad \text{and} \quad R_2 = \frac{\tau_2}{C} = \frac{1.575 \times 10^{-3}}{0.175 \times 10^{-6}} = 9.0 \text{ k}\Omega$$
**Design of the Loop Filter – Continued**

3.) Simulated response of the filter.

4.) Differential version of the filter.

![Figure 090-04](image1.png)

![Figure 090-05](image2.png)

**Loop Stability**

1.) Loop Gain.

The loop gain for $N = 18,000$ is given by

$$LG(s) = \frac{K_dK_oF(s)}{Ns} = \frac{K(s\tau_2 + 1)}{s^2N\tau_1} = \frac{7.854 \times 10^6 \times 0.796 (1 + 1.575 \times 10^{-3}s)}{0.419 \times 10^{-3} \times 18,000 \times s^2} = \frac{828.83 \times 10^3 (1 + 1.575 \times 10^{-3}s)}{s^2}$$

2.) Bode Plot

![Figure 090-06](image3.png)
**Closed Loop Gain**

A plot of the closed-loop transfer function of \( \frac{\omega_2(j\omega)}{V_{fm}(j\omega)} \) is shown below.

![Graph of Closed Loop Gain](image)

(We will continue this example later.)

---

**MEASUREMENT OF PLL PERFORMANCE**

(The device under test in this section is the Exar XR-S200.)

**Measurement of Center Frequency, \( f_0 \)**

![Diagram of Measurement Setup](image)

Results:

\[ v_f = 0 \quad \Rightarrow \quad T/2 = 76 \mu s \]
\[ \text{or} \quad T = 152 \mu s \]
\[ \therefore \quad f_0 = 1/T = 6.54 \text{ kHz} \]
**Measurement of the VCO Gain, $K_o$**

Use the same measurement configuration as for $f_o$. Vary $v_f$ and measure the output frequency of the VCO.

Calculation of $K_o$.

$$K_o = \frac{\Delta \omega}{\Delta v_f} = \frac{2\pi(6.54-5.88)}{1-0} \times 10^3 = 4.13 \times 10^3 \text{ V} \cdot \text{sec}$$

$T/2 = 85 \mu s$

or $T = 170 \mu s$

∴ $f_o = 1/T = 5.88 \text{ kHz}$

---

**Measurement of the Phase Detector Gain, $K_d$**

**Test circuit:**

**Measurement Principle:**

The above measurement assumes that $\theta_e = 90^\circ$ so that $v_d = K_d \sin 90^\circ = K_d$

For a sinewave input, the dc output of the PD is $2/\pi$ of the peak sinusoidal voltage.

If a dc voltage is applied at the PD input of $(\sqrt{2}/1) \times (2/\pi)$ $V_{peak} = 0.9V_{peak}$, then $K_d$ is simply $1/2$ of the peak-to-peak output of the phase detector.
Measurement of $K_d$ – Continued

$v_1 = 10, 20, 30$ and $40$ mV (rms)

More values indicate that the PD saturates at 0.4V (rms)

Measurement of the Hold Range, $\Delta \omega_H$, and the Pull In Range, $\Delta \omega_p$

Measurement Circuit:

The measurement requires a full functional PLL. The loop filter must be added which in this case consists of both on-chip and external components.

1.) To measure $\Delta f_H$, start with a value of $f_1$ where the loop is locked and slowly vary $f_1$ to find the upper and lower values where the system unlocks.

2.) To measure $\Delta f_p$, start with $f_1$ at approximately the center frequency, then increase $f_1$ until the loop locks out. Decrease $f_1$ until the loop pulls in. The difference between this value of $f_1$ and $f_0$ is $\Delta f_p$. 
**Measurement of \( \Delta \omega_H \) and \( \Delta \omega_p \) – Continued**

Loop out of lock

Loop on the threshold of lock

Loop locked

**Measurement of \( \omega_n \), \( \zeta \), and the Lock Range \( \Delta \omega_L \)**

Test circuit:

Waveforms:

Parameter extraction:

With \( A_1=1.9, A_2=1.4 \), \( \zeta = \frac{\ln(A_1/A_2)}{\sqrt{\pi^2 + \left( \ln(A_1/A_2) \right)^2}} \) and \( \omega_n = \frac{2\pi}{T \sqrt{1 - \zeta^2}} \Rightarrow \zeta = 0.8 \) and \( f_n = 4.1 \text{kHz} \)
Measurement of $\omega_n$, $\zeta$, and the Lock Range $\Delta \omega_L$ – Continued

Measurement of $\Delta \omega_L$:
1.) The signal generator is adjusted to generate two frequencies, $\omega_{\text{high}}$ and $\omega_{\text{low}}$ such that,
   $$\omega_{\text{high}} > \omega_o + \Delta \omega_p$$
2.) Set $\omega_{\text{low}} = \omega_{\text{high}}$ (the amplitude of the square wave generator will be zero)
3.) Decrease $\omega_{\text{low}}$.
4.) When $\omega_{\text{low}} \approx \omega_o + \Delta \omega_L$, the PLL will lock.
   \[ \therefore \Delta \omega_L \approx \omega_{\text{low}} - \omega_o \]

Measurement of the Phase Transfer Function, $H(j\omega)$

Since most signal generators are not phase modulated, use a frequency modulated signal generator instead as follows.

Test circuit:

```
\[ \begin{array}{c}
\text{Input}\quad \omega_m \\
\text{Output} \\
\text{PD} \\
\text{Lowpass Filter} \\
\text{VCO} \\
\text{To oscilloscope or spectrum analyzer}
\end{array} \]
```

Principle:

\[ \omega_1 = \omega_o + \Delta \omega \sin \omega_mt \quad \Rightarrow \quad \theta_1(t) = \int_{0}^{t} \frac{\Delta \omega}{\omega_m} \cos \omega_m t \quad \Rightarrow \quad |\theta_1(j\omega)| = \frac{\Delta \omega}{\omega_m} \]

\[ H(j\omega) = \frac{\theta_2(j\omega)}{\theta_1(j\omega)} \quad \text{and} \quad \text{VCO gain at } \omega_m \rightarrow \theta_2(j\omega) = \frac{K_o}{j\omega_m} V_f(j\omega_m) \]

\[ |H(j\omega)| = \frac{K_o V_f(j\omega_m)}{\Delta \omega} \]

What about $\Delta \omega$?

As long as $\Delta \omega$ is small enough, the PD operates in its linear region and $v_f(t)$ is an undistorted sinewave (see next slide).
Measurement of $H(j\omega)$ – Continued

$\Delta\omega$ small enough for linear operation.  $\Delta\omega$ too large for linear operation.

Implementation:
1.) Can plot the frequency response point-by-point.
2.) Use a spectrum analyzer
   - Sweep generator rate $\ll$ Spectrum analyzer sweep rate
   - Watch out that resonance peaks in the response don’t cause nonlinear operation.

Typical results: →

![Frequency Response](image1)

Timing relationship between the sweep generator and the spectrum analyzer:

![Sweep Generator vs. Spectrum Analyzer](image2)

Basically, $f_m$ should approximate a constant during one sweep period of the analyzer.

Problem due to resonance peaks that cause nonlinear operation:

![Resonance Peaks](image3)
SUMMARY

- PLL Design Equations
  - Basic design equations for
    - Type-I, first-order loop
    - Type-I, second-order loop
    - Type-II, second-order loop

- Design of a 450-475 MHz DPLL Frequency Synthesizer
  - PFD plus Charge Pump
  - Design of active PI filter
  - Stability

- Measurements of PLL Performance
  - How to experimentally measure the various performance parameters of a PLL