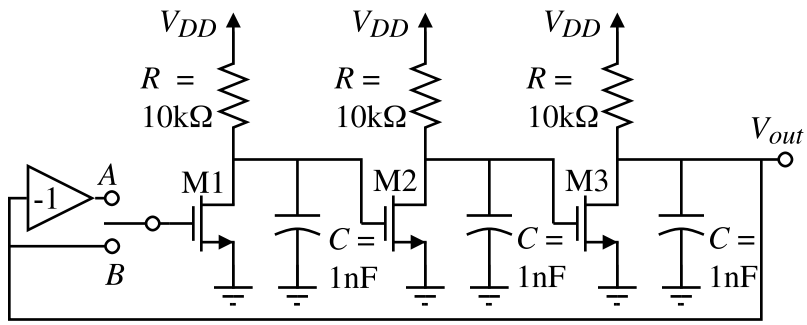
# FINAL EXAMINATION - SOLUTIONS

(Average Score = 58/100)

Problem 1 - (20 points - This problem must be attempted)

The circuit shown is to be an oscillator. The transistors are identical with a $g_m$ = 1mS and $r_{ds} = \infty$. (a.) Should the switch at the gate of M1 be connected to point $A$ or $B$ in order to oscillate? (b.) Find the frequency of oscillation in Hertz and the value of $g_mR$ necessary for oscillation.




F02FEP1

*Solution*

(a.) Assuming the switch is connected to B, the gain from the gate of M1 to $V_{out}$ can be expressed as,

$$\frac{V_{out}}{V_{g1}} = T(s) = \left(\frac{-g_mR}{sRC+1}\right)^3 = \frac{(-g_mR)^3}{(sRC)^3 + 3s^2R^2C^2 + 3sRC + 1}$$

$$T(j\omega) = \frac{-(g_mR)^3}{[1-3\omega^2R^2C^2]+j\omega RC\,[3-3\omega^2R^2C^2]} = 1+j0$$

(b.) From the above equation, we get,

$$\omega_{osc} = \frac{\sqrt{3}}{RC} = \frac{1.732}{10\text{x}10^3\cdot 1\text{x}10^{-9}} = 173.2\text{Krad/sec} \quad \rightarrow \quad f_{osc} = 27.57\text{kHz}$$

Also, from the above equation, we get

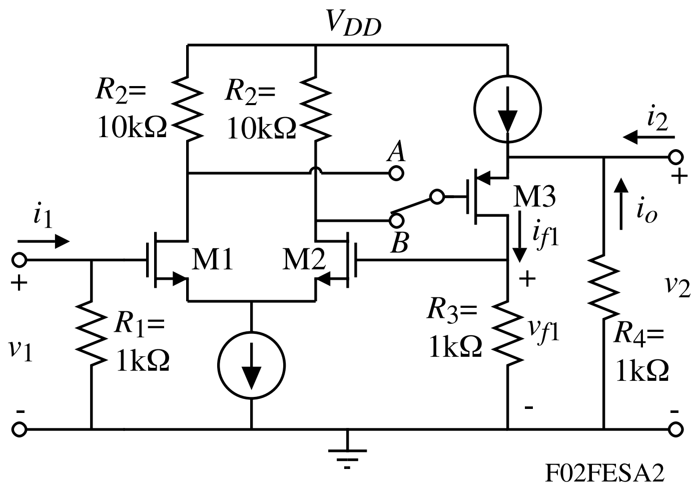
$$-(g_mR)^3 = 1 - 3\omega^2R^2C^2 = 1 - 9 = -8$$

$$\therefore \quad (g_mR)^3 = 8 \quad \rightarrow \quad g_mR = 8^{0.33} = 2$$

We see that the switch should be connected to B. Otherwise, $g_mR$ would be negative.

Problem 2 – (20 points – This problem must be attempted)

The simplified schematic of a feedback amplifier is shown. Assume that all transistors are matched and $g_m$ = 1mA/V and $r_{ds} = \infty$. (a.) Where should the switch be connected for negative feedback? (b.) Use the method of feedback analysis to find $v_2/v_1$, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$.



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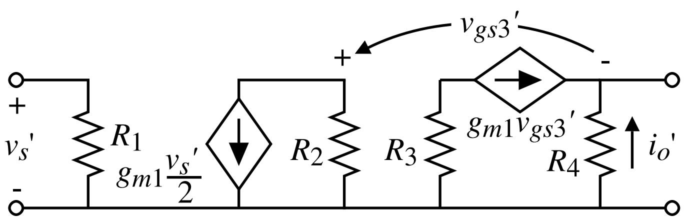
*Solution*

(a.) A quick check of the ac voltage changes around the loop show that the switch should be connected to A.

(b.) This feedback circuit is series-series. The units of $A$ are A/V and the units of $\beta$ are V/A.

$$\beta = z_{12f} = \left.\frac{v_{1f}}{i_{2f}}\right|_{i_{1f}=0} = R_3 = 1\text{k}\Omega$$

The circuit for calculating the small-signal open-loop gain is,



F02FES2

$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs3}'}\right)\left(\frac{v_{gs3}'}{v_{g3}'}\right)\left(\frac{v_{id}'}{v_s'}\right) = (-g_{m3})\left(\frac{1}{1+g_{m3}R_4}\right)\left(\frac{-g_{m1}R_2}{2}\right)$$

$$A = \frac{i_o'}{v_s'} = (1\text{mS})(0.5)(5) = 2.5\text{mS} \;\rightarrow\; A_F = \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{2.5\text{mS}}{1+2.5\cdot 1} = 0.714 \text{ mS}$$

$$\frac{v_2}{v_1} = \frac{v_2}{v_s} = \left(\frac{v_2}{i_o}\right)\left(\frac{i_o}{v_s}\right) = -R_4(0.714\text{mS}) = -0.714 \text{ V/V}$$

$$\boxed{\frac{v_2}{v_s} = -0.714 \text{ V/V}}$$

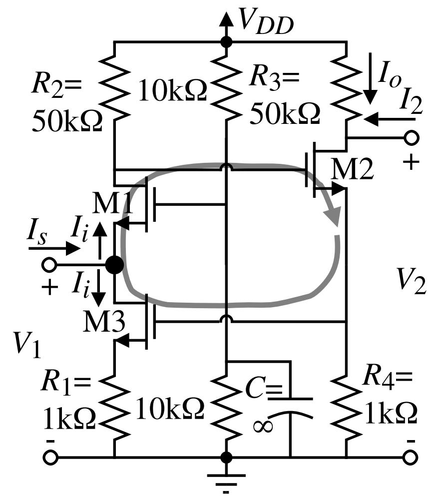
$R_1$ is not influenced by feedback so $\boxed{\frac{v_1}{i_1} = R_1 = 1\text{k}\Omega}$

$$R_o = R_4+(1/g_{m3}) = 1\text{k}\Omega+1\text{k}\Omega = 2\text{k}\Omega \rightarrow \qquad R_{oF} = 2\text{k}\Omega(1+2.5) = 7\text{k}\Omega$$

$$R_{out} = \frac{v_2}{i_2} = (R_{oF} - R_4)||R_4 = 7\text{k}\Omega||1\text{k}\Omega = 875\Omega \qquad \boxed{\frac{v_2}{i_2} = 875\Omega}$$

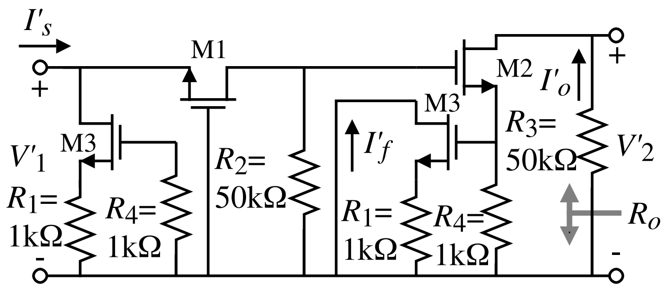
## Problem 3 - (20 points - This problem is optional)

The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find $v_2/v_1$, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $g_m$ = 1mA/V and $r_{ds} = \infty$.

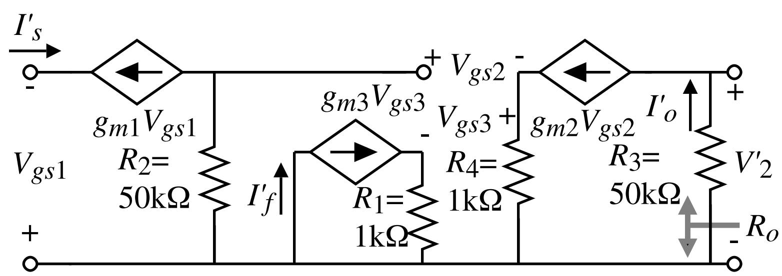



*Solution*

This feedback circuit is shunt-series. We begin with the open-loop, quasi-ac model shown below.




The small-signal, open-loop model is:




$$\frac{I'_o}{I'_s} = \left(\frac{I'_o}{V_{gs2}}\right)\left(\frac{V_{gs2}}{V_{gs1}}\right)\left(\frac{V_{gs1}}{I'_s}\right)$$

$V_{gs2} =$

$-g_{m1}V_{gs1}R_2 - g_{m2}V_{gs2}R_4$

or

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1}R_2}{1 + g_{m2}R_4} = -\frac{50}{2} = -25 \qquad \therefore \frac{I'_o}{I'_s} = (g_{m2})(-25)\left(\frac{-1}{g_{m1}}\right) = 25\text{A/A}$$

$$ß = \frac{I'_f}{I'_o} = \left(\frac{I'_f}{V_{gs3}}\right)\left(\frac{V_{gs3}}{I'_o}\right) = (g_{m3})\left(\frac{R_1}{1 + g_{m3}R_4}\right) = (1\text{mA/V})(0.5\text{k}\Omega) = 0.5$$

$$\therefore A\beta = 25 \cdot 0.5 = 12.5$$

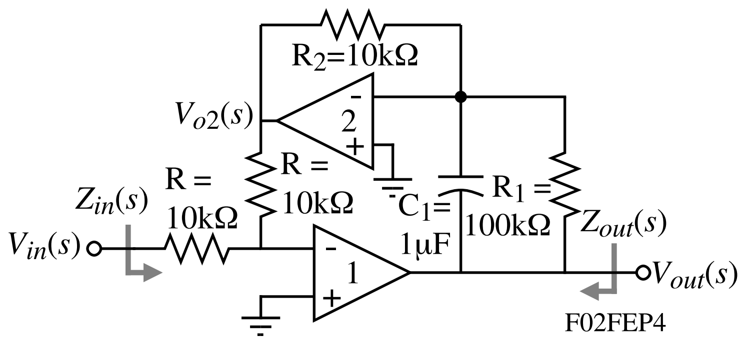
$$R_i = \frac{v'_1}{I'_s} = \frac{1}{g_{m1}} = 1\text{k}\Omega \rightarrow \boxed{R_{in} = R_{if} = \frac{R_i}{1 + A\beta} = \frac{1000}{13.5} = 74.07\Omega}$$

$$\boxed{R_{out} = 50\text{k}\Omega \ \ (R_3 \text{ is outside the feedback loop})}$$

$$\frac{I_o}{I_s} = \frac{A}{1+A\beta} = \frac{25}{1+12.5} = 1.852 \text{ A/A} \rightarrow \boxed{\frac{v_2}{v_1} = \frac{I_o(-50\text{k}\Omega)}{I_s(74.07\Omega)} = -1240.1 \text{ V/V}}$$

Problem 4 - (20 points - This problem is optional)

If the op amps shown are ideal (infinite voltage gain, infinite differential input resistance, and zero output resistance) find the voltage transfer function, $V_{out}(s)/V_{in}(s)$, the input impedance, $Z_{in}(s)$, and the output impedance, $Z_{out}(s)$. Sketch an asymptotic plot for the magnitude and phase shift of the voltage transfer function, $V_{out}(i\omega)/V_{in}(j\omega)$ as a function of $\log_{10}\omega$.

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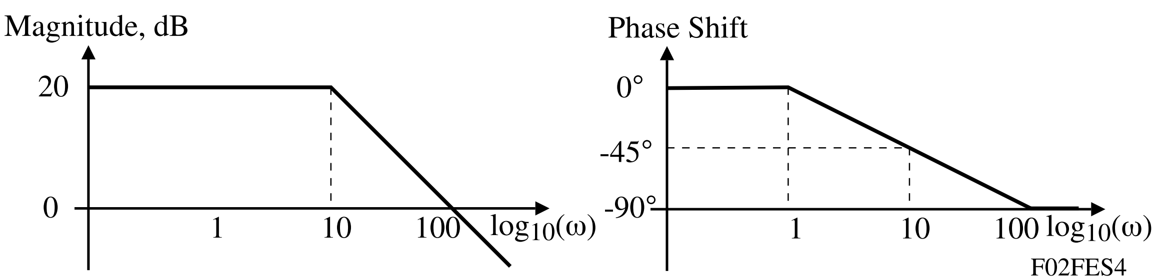
*Solution*

$V_{o2}(s)$ can be written as $V_{o2}(s) = -\frac{R_2}{Z_1}v_{out}(s)$ . Thus, the currents flowing toward the inverting terminal of the 1st op amp are, $\frac{V_{in}(s)}{R}+\frac{V_{o2}(s)}{R}=\frac{V_{in}(s)}{R}-\frac{R_2}{Z_1(s)}\frac{V_{out}(s)}{R}=0$

$$\therefore \frac{V_{out}(s)}{V_{in}(s)}=\frac{Z_1(s)}{R_2}=\frac{1}{R_2}\frac{R_1(1/sC_1)}{R_1+(1/sC_1)}=\frac{R_1}{R_2}\frac{1}{sR_1C_1+1} \quad \boxed{\frac{V_{out}(s)}{V_{in}(s)}=\frac{R_1}{R_2}\frac{1}{sR_1C_1+1}}$$

By inspection, $\boxed{Z_{in}(s) = R = 10\text{k}\Omega \text{ and } Z_{out}(s) = 0}$

For the Bode plot we want to plot the magnitude and phase of $\frac{V_{out}(j\omega)}{V_{in}(j\omega)}=\frac{10}{1+j\omega/10}$ .

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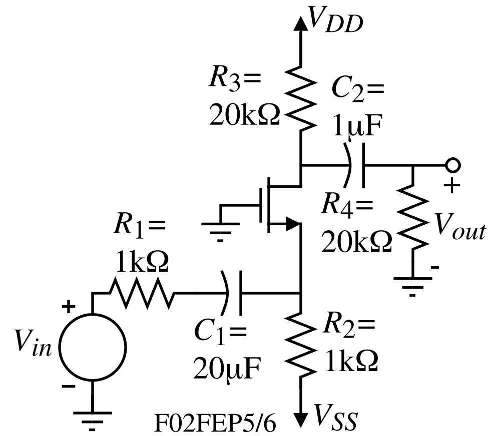
Problem 5 - (20 points - This problem is optional)

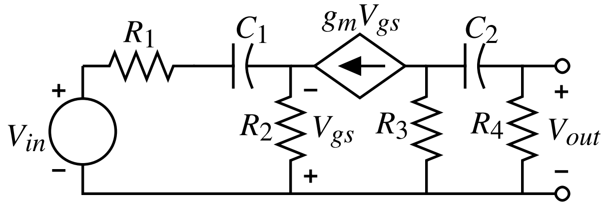
1.) If $g_m$ = 2mA/V, what is the midband voltage gain of the amplifier shown? Assume $r_d = \infty$.

2.) Find the lower -3dB frequency ($f_L$) of the amplifier shown.




*Solution*

The small signal model for this problem is:




$$\frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_{in}}\right)$$

$$\therefore \frac{V_{out}}{V_{gs}} = \frac{-g_m R_3 R_4}{R_3+R_4+\frac{1}{sC_2}} = \left(\frac{-g_m R_3 R_4}{R_3+R_4}\right)\left(\frac{s}{s + \frac{1}{C_2(R_3+R_4)}}\right) = (-20)\left(\frac{s}{s+25}\right)$$

Next, find $V_{gs}/V_{in}$:

$$\frac{V_{in}+V_{gs}}{R_1+\frac{1}{sC_1}} + \frac{V_{gs}}{R_2} + g_m V_{gs} = 0 \quad\rightarrow\quad \frac{-V_{in}}{R_1 + \frac{1}{sC_1}} = V_{gs}\left(\frac{1}{R_1 + \frac{1}{sC_1}} + \frac{1}{R_2 \| (1/g_m)}\right)$$

or

$$-V_{in}\left(R_2\|\frac{1}{g_m}\right) = V_{gs}\left(R_2\|\frac{1}{g_m} + R_1 + \frac{1}{sC_1}\right) \rightarrow \frac{V_{gs}}{V_{in}} = \left(\frac{R_2\|\frac{1}{g_m}}{R_1 + R_2\|\frac{1}{g_m}}\right)\left(\frac{s}{s + \frac{1}{C_1\left[R_1+R_2\|\frac{1}{g_m}\right]}}\right)$$

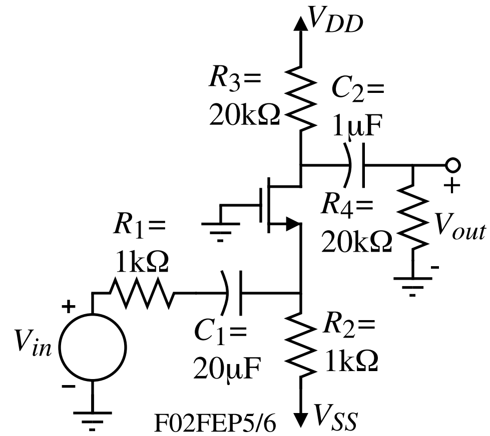
$$\frac{V_{gs}}{V_{in}} = \frac{-0.33}{1+0.33}\left(\frac{s}{s + 37.5}\right) = (-0.25)\left(\frac{s}{s+37.5}\right)$$

Thus, $\frac{V_{out}(s)}{V_{in}(s)} = 5\left(\frac{s}{s+25}\right)\left(\frac{s}{s+37.5}\right)$

$\therefore$ **MBG = 5, $\omega_L \approx \sqrt{(25)^2+(37.5)^2}$ = 45.07 rads/sec → $f_L$ = 7.17 Hz**

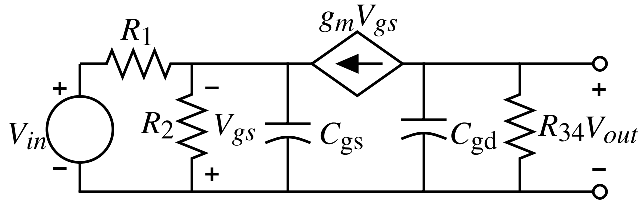
Problem 6 - (20 points - This problem is optional)

The FET in the amplifier shown has $g_m$ = 1mA/V, $r_d = \infty$, $C_{gd}$ = 0.5pF, and $C_{gs}$ = 10pF. (a.) Find the midband gain, $V_{out}/V_{in}$. (b.) Find the upper -3dB frequency, $f_H$, in Hz. (Note: You cannot use the Miller's theorem on this problem because there is no bridging capacitor.)




*Solution*

The small signal model for the high frequency range is shown where $R_{34} = R_3 || R_4$ = 10kΩ.




Because the capacitors are independent, probably the best way to work this problem is by superposition (open-circuit time constants). Therefore,

$C_{gs}$:

$$R_{Cgs} = R_1 || R_2 || (1/g_m) = 1\text{K}||1\text{K}||1\text{K} = 333\Omega \;\rightarrow\; \omega_{Cgs} = \frac{1}{C_{gs}\cdot 333\Omega} = 300 \text{ Mrads/sec.}$$

$C_{gd}$:

$$R_{Cgd} = R_{34} = 10\text{k}\Omega \;\rightarrow\; \omega_{Cgd} = \frac{1}{C_{gd}\cdot 10\text{k}\Omega} = 200 \text{ Mrads/sec.}$$

$$\therefore\; \omega_H \approx \frac{1}{\sqrt{\left(\frac{1}{300\text{Mrads/sec}}\right)^2 + \left(\frac{1}{200\text{Mrads/sec}}\right)^2}} = 166 \text{ Mrads/sec.}$$

$$\boxed{f_L = 26.48\text{MHz}}$$

The midband gain is given as

$$\boxed{\text{MBG} = \left(\frac{R_2 || \frac{1}{g_m}}{R_1 + R_2 || \frac{1}{g_m}}\right)\left(\frac{-g_m R_3 R_4}{R_3 + R_4}\right) = \left(\frac{-0.5}{1.5}\right)(-10) = 3.33\text{V/V}}$$

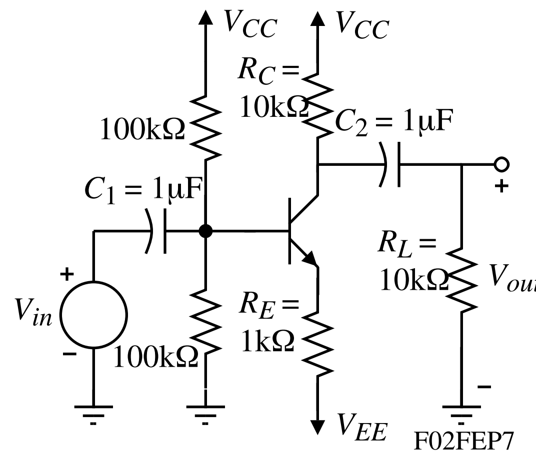
Problem 7 - (20 points - This problem is optional).

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of $g_m$ = 50mA/V, $r_\pi$ = 2kΩ, and $r_o = \infty$.

a.) Find the midband voltage gain of this amplifier, $V_{out}/V_{in}$.

b.) Find the numerical value of all poles and zeros of the low frequency response.

c.) Find the value of the lower -3dB frequency, $f_L$, in Hz.



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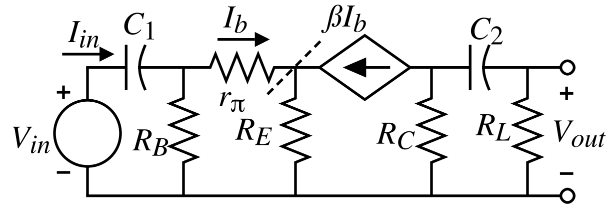
*Solution*

The low-frequency, small signal model for this problem is shown where $R_B$ = 50kΩ.




The algebraic approach to this problem is:

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_b}\right)\left(\frac{I_b}{I_{in}}\right)\left(\frac{I_{in}}{V_{in}}\right) = \left(\frac{-\beta R_L R_C}{R_C+R_L+\frac{1}{sC_2}}\right)\left(\frac{R_B}{R_B+r_\pi+(1+\beta)R_E}\right)\left(\frac{1}{\frac{1}{sC_1}+R_B\|[r_\pi+(1+\beta)R_E]}\right)$$

$$= \left(\frac{-\beta R_L R_C}{(R_C+R_L)[r_\pi+(1+\beta)R_E]}\right)\left(\frac{s}{s+\frac{1}{C_2(R_C+R_L)}}\right)\left(\frac{s}{s+\frac{1}{C_1(R_B\|[r_\pi+(1+\beta)R_E])}}\right)$$

$$= \frac{-100\cdot 10K\cdot 10K}{20K\cdot 103K}\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right) = -4.854\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right)$$

The midband gain is $\boxed{MBG = 4.854\text{ V/V}}$

$\therefore \quad \omega_L \approx \sqrt{(29.7)^2 + (50)^2} = 58.2$ rads/sec. $\rightarrow$ $\boxed{f_L = 9.26\text{Hz}}$

The poles and zeros are,

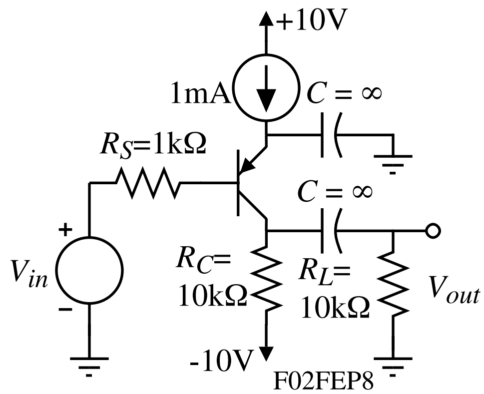
**Two zeros at s = 0, a pole at s = -29.7 rads/sec. and a pole at s = -50 rads/sec.**

Problem 8 – (20 points, this problem is optional)

A common-emitter BJT amplifier is shown. Assume that the BJT has a $\beta = h_{fe} = 100$, $C_\mu = 2\text{pF}$, $V_t = 25\text{mV}$, $f_T = 500\text{MHz}$, $r_b = 0\Omega$, and $r_o = \infty$.

a.) Find the numerical values of $r_\pi$, $g_m$, and $C_\pi$.

b.) If $r_\pi = 1\text{k}\Omega$, $g_m = 0.01\text{A/V}$ and $C_\pi = 10\text{pF}$ for the above amplifier, find the value of the upper -3dB frequency, $f_H$, in Hz.

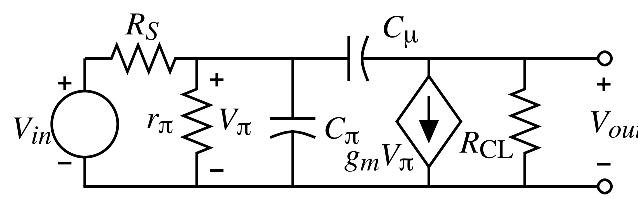



*Solution*

a.) $$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 0.04 \text{ A/V}$$ $$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{0.04} = 2500\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{0.04}{2\pi\cdot 500\text{x}10^6} - 2\text{pF} = 12.732\text{pF} - 2\text{pF} = 10.732 \text{ pF}$$

b.) The high-frequency, small-signal model for this problem is shown where $R_{CL} = R_C||R_L = 5\text{k}\Omega$.




The midband gain of this amplifier is given by

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_\pi}\right)\left(\frac{V_\pi}{V_{in}}\right) = -g_m R_C||R_L\left(\frac{r_\pi}{r_\pi + R_S}\right) = (-0.01\cdot 5\text{k}\Omega)(0.5) = -25\text{V/V}$$

∴ $\text{MBG} = -25 \text{ V/V}$

Using Miller's theorem on this problem:

If $\frac{1}{\omega_H C} >> R_C||R_L$, then $C_{eq} \approx C_\pi + C(1 + g_m R_C||R_L) = 10\text{pf} + 2\text{pF}(1+50) = 112\text{pF}$

We know that, $\omega_H = \frac{1}{C_{eq}(r_\pi||R_S)} = \frac{1}{(112\text{pF}\cdot 500\Omega)} = 17.86$ Mrads/sec.

∴ $$f_H = \frac{\omega_H}{2\pi} = 2.842 \text{ MHz}$$

Note that:

$$\frac{1}{\omega_H C} = \frac{10^6}{17.86\cdot 2} = 28.06\text{k}\Omega > 5\text{k}\Omega$$ so that the Miller approximation (neglecting $C_\mu$) is valid.