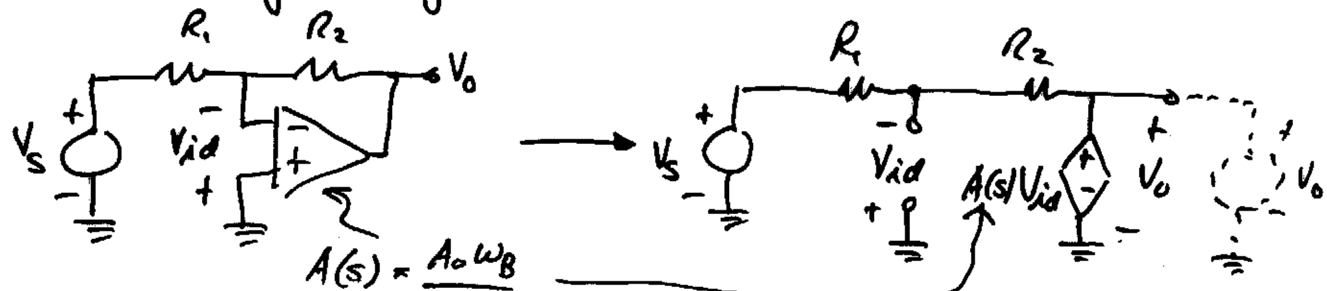
Frequency Response of Op Amps

1.) Inverting Configuration



$$A(s) = \frac{A_o \omega_B}{s + \omega_B}$$

$$V_o(s) = A(s)\,V_{id} = A(s)\left[-V_s \frac{R_2}{R_1+R_2} - V_o \frac{R_1}{R_1+R_2}\right] \quad \leftarrow B$$

$$V_o\left[1 + \frac{A(s) R_1}{R_1+R_2}\right] = -\frac{A(s) R_2}{R_1+R_2} V_s = -A(s)\frac{R_2}{R_1}\frac{R_1}{R_1+R_2} V_s \quad \leftarrow B$$

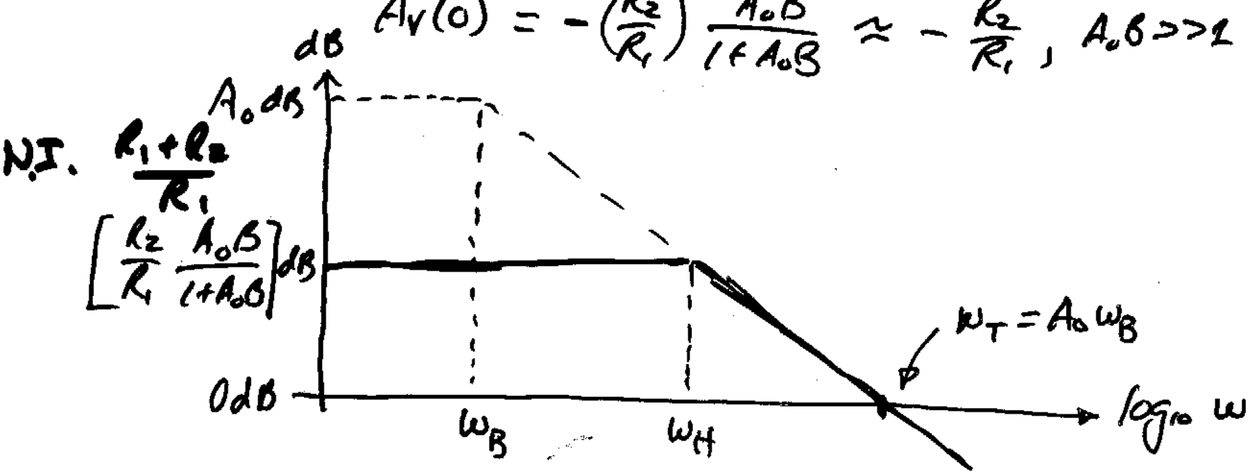
$$A_v(s) = \frac{V_o(s)}{V_s(s)} = \frac{-\frac{R_2}{R_1} A(s) B}{1 + A(s) B} = \frac{-\frac{R_2}{R_1} B}{\frac{1}{A(s)} + B} \qquad B = \frac{R_1}{R_1+R_2}$$

$$A_v(s) = \frac{-\frac{R_2}{R_1} B}{\frac{s+\omega_B}{\omega_B A_o} + B} = \frac{-\frac{R_2}{R_1} B\,\omega_B A_o}{s + \underbrace{\omega_B(1 + A_o B)}_{\omega_H}}$$

$$= -\frac{R_2}{R_1}\frac{\omega_B B A_o}{\omega_B(1+A_oB)} \frac{1}{\frac{s}{\omega_B(1+A_oB)} + 1} = \frac{A_v(0)}{\frac{s}{\omega_H} + 1}$$

$\therefore \quad \omega_H = \omega_B(1 + A_o B) \approx \omega_B A_o B$

$$A_v(0) = -\left(\frac{R_2}{R_1}\right)\frac{A_o B}{1 + A_o B} \approx -\frac{R_2}{R_1}, \quad A_o B >> 1$$



2.) Summary

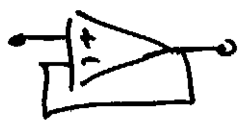
Config.	$A_v(0)$ ← Closed-loop gain	ω_H
Noninverting	$\frac{R_1+R_2}{R_1}$	$\omega_B(1+A_0B)$
Inverting	$-\frac{R_2}{R_1}\left(\frac{A_0B}{1+A_0B}\right)$	$\omega_B(1+A_0B)$

$B = \frac{R_1}{R_1+R_2}$

3.) Examples

a.) An op amp with $A_0 = 100\,dB$ and $f_T = 10\,MHz$ is used to build an amplifier with a gain of +1 and −1. Find f_H.

Noninverting: ($R_1 = \infty$ and $R_2 = 0$)



"Buffer"

$$f_{-3dB} = f_B = \frac{f_T}{A_0} = \frac{10^7}{10^5} = 10^2$$

$$f_H = f_B(1+A_0B) = 10^2(1+10^5 \cdot 1) \approx 10^7 \text{ or } \underline{10\,MHz}$$

Inverting: ($R_1 = R_2 \rightarrow$ Gain −1)

Note that $B = \frac{1}{2}$

$$f_H = f_{-3dB} = 10^2\left(1+10^5\tfrac{1}{2}\right) = \underline{5\,MHz}$$

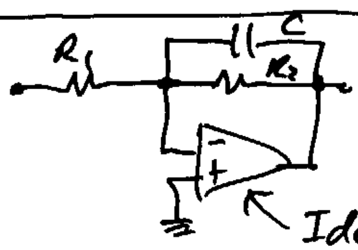
b.) Repeat the above but let the closed-loop amplifier gains be ±100.

Noninverting: $R_1 = 99R_2 \rightarrow B = \frac{1}{100}$

$$f_H = f_B(1+A_0B) = 100(1+10^5 \cdot 10^{-2}) = 100\,kHz$$

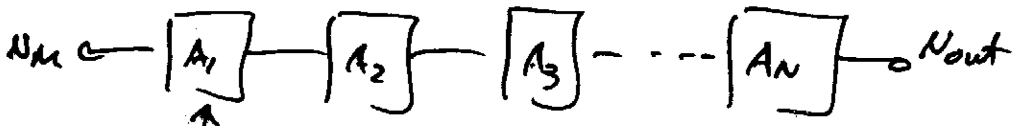
Inverting: $R_1 = 100R_2 \rightarrow B = \frac{1}{101}$

$$f_H = 100\left(1+10^5 \cdot \tfrac{1}{101}\right) \approx \underline{99.5\,kHz} \text{ (Guess)}$$



$$\frac{A_v(0)}{\frac{s}{\omega_H}+1}$$

4.) Frequency response of cascaded amplifiers



$\dfrac{A_{v1}(0)}{\frac{s}{\omega_{H1}}+1}$ $\dfrac{A_{v2}(0)}{\frac{s}{\omega_{H2}}+1}$ etc

$$\frac{V_{out}(s)}{V_{in}(s)} = A_v(s) = ? = \frac{A_v(0)}{\left(\frac{s}{\omega_{H1}}+1\right)\left(\frac{s}{\omega_{H2}}+1\right)\left(\frac{s}{\omega_{H3}}+1\right)\cdots\left(\frac{s}{\omega_{HN}}+1\right)}$$

$$A_v(0) = A_{v1}(0)\,A_{v2}(0)\cdots A_{vN}(0)$$

$$\omega_H = ? \qquad |A_v(j\omega_H)| = \frac{|A_{v1}(0)\cdot A_{v2}(0)\cdots A_{vN}(0)|}{\sqrt{2}}$$

5.) Cascade of identical stages

$$A_v(s) = \left(\frac{A_{v1}(0)}{\frac{s}{\omega_{H1}}+1}\right)^N \rightarrow A_v(0) = [A_{v1}(0)]^N$$

$$\boxed{\omega_H = \omega_{H1}\sqrt{2^{1/N}-1}}$$

N	1	2	3	4	
$\sqrt{2^{1/N}-1}$	1	0.644	0.510	0.435	...

6.) Example – Given an op amp with $f_T = 10\text{MHz}$ and $A_0 = 100\text{dB}$, find the ω_H (−3dB freq.) of the gain of +100 amplifier using 1 op amp, 2 cascaded op amps and 3 cascaded op amps

$$\omega_{H1} = (1+A_0\beta)\omega_B \rightarrow f_{H1} \cong A_0\beta f_B = f_T\beta$$

a.) 1 op amp → $\beta = \frac{1}{100}$ → $f_H = f_{H1} = \frac{10\text{ MHz}}{100} = \underline{100\text{kHz}}$

b.) 2 op amps → $\beta = \frac{1}{10}$ → $f_{H1} = 1\text{MHz}$ → $f_H = \underline{644\text{kHz}}$

c.) 3 op amps → $\beta = \frac{1}{4.692}$ → $f_{H1} = 2.154\text{MHz}$ → $\underline{f_H = 1.099\text{MHz}}$