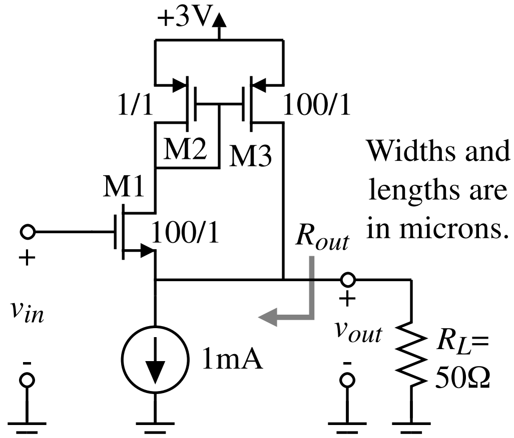
EXAMINATION NO. 1 – SOLUTIONS

(Average score = 60/100)

Problem 1 - (25 points)

Find an algebraic expression for the voltage gain, v_{out}/v_{in}, and the output resistance, R_{out}, of the source follower shown in terms of the small-signal model parameters, g_m and R_L (ignore r_{ds}). If the bias current is 1mA find the numerical value of the voltage gain and the output resistance. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $K_P' = 50\mu A/V^2$, $V_{TP} = -0.7V$.



S03E1P1

Solution

A small-signal model for this circuit is shown below neglecting r_{ds} of the transistors.

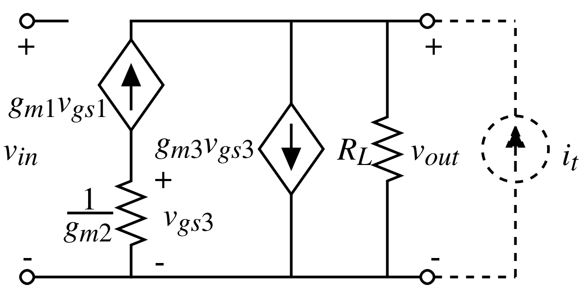


Fig. S03E1S1

Summing currents at the output node gives,

$$g_{m1}v_{gs1} = g_{m3}v_{gs3} + G_Lv_{out}$$

Also, $v_{gs3} = -g_{m1}v_{gs1}(1/g_{m2})$

$$\therefore \quad g_{m1}v_{gs1} = g_{m3}\left(-\frac{g_{m1}}{g_{m2}}\right)v_{gs1} + G_Lv_{out}$$

$$g_{m1}v_{gs1}\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_Lv_{out} \quad \rightarrow$$

$$g_{m1}(v_{out} - v_{in})\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_Lv_{out}$$

$$\therefore \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right) + G_L}}$$

Setting $v_{in} = 0$ and applying i_t and solving for v_{out} and ignoring R_L gives,

$$i_t = g_{m3}v_{gs3} + g_{m1}v_{out} = g_{m3}\left(\frac{g_{m1}}{g_{m2}}\right)v_{out} + g_{m1}v_{out}$$

$$\therefore \quad \frac{v_{out}}{i_t} = \boxed{R_{out} = \frac{1}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}}$$

Note that the 1mA splits between M1(M2) and M3 in a ratio of 1 to 100. Therefore, $I_{D1} = I_{D2} = 9.9\mu A$ and $I_{D3} = 990.1\mu A$.

$$\therefore \; g_{m1} = \sqrt{2\cdot 110\cdot 100\cdot 9.9} = 466.71\mu S, \; g_{m2} = \sqrt{2\cdot 50\cdot 1\cdot 9.9} = 31.47\mu S$$

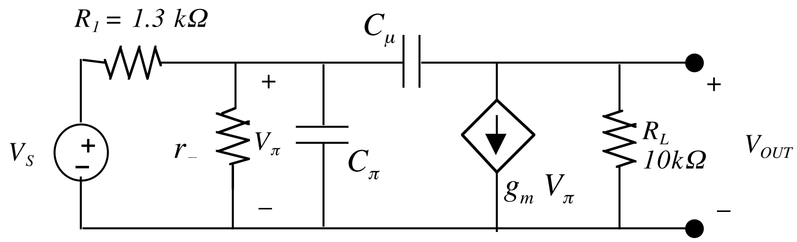
and $g_{m3} = \sqrt{2\cdot 110\cdot 100\cdot 990.1} = 3146.7\mu S$

$$\frac{v_{out}}{v_{in}} = \frac{466.71\cdot 101}{466.71\cdot 101 + 1/50} = \frac{47.137}{47.137 + 20} = \underline{\underline{0.702 \text{ V/V}}}$$

$$R_{out} = \frac{1000}{47.137} = \underline{\underline{21.2\Omega}}$$

Problem 2 - (25 points)

Using the open-circuit and short-circuit time constant methods, find the two poles of the circuit shown below (assuming that the two poles are far apart from each other). β_0=100, I_C=0.5mA, f_T=1GHz, C_μ=0.2pF, V_T=26mV.



Solution

$r_\pi=\beta_0/g_m=100\times26/0.5=5.2k\Omega$

$\tau_T=1/(2\pi f_T)=159ps$

$C_\pi=g_m\tau_T-C_\mu=(0.5/26)\times159pF-0.2pF=2.86pF$

$R_I=R_1||r_\pi=1.04k\Omega$

Open circuit time constant method:

$P_1\cong1/\Sigma\tau=[R_IC_\pi+(R_I+R_L+g_mR_IR_L)C_\mu]^{-1}=22.13\times10^6rad/s=3.52MHz$

Short circuit time constant method:

$P_2\cong\Sigma(1/\tau)=[(R_I||1/g_m||R_L)C_\pi]^{-1}+[R_LC_\mu]^{-1}=7.224\times10^9rad/s=1.15GHz$

Problem 3 - (25 points)

a) For the emitter follower output stage shown below, find the value of R_1 for maximum efficiency and find the value of that efficiency. $V_{CC}=-V_{EE}=2.5V$, $V_{CE}(sat)=0.2V$, $R_L=10k\Omega$, $V_{BE}(on)=0.7V$.

b) A load capacitor of 100pF is attached to the output voltage. If the input voltage suddenly drops from 2.5V to -2.5V, explain what happens at the output and accurately sketch the output voltage as a function of time, specifying its initial and final values and times.

Solution

The I_Q for maximum efficiency is found as,

$$I_Q = \left(\frac{V_{CC} - V_{CE}(sat)}{R_L}\right) = 230\mu A$$

$$R_1 = \left(\frac{-V_{EE} - V_{BE}}{I_Q}\right) = 7.826k\Omega$$

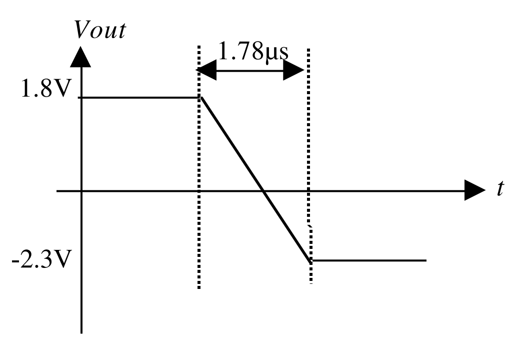
$$P_L(\max) = \left(\frac{V_{CC} - V_{CE}(sat)}{\sqrt{2}}\right)\left(\frac{I_Q}{\sqrt{2}}\right) = 0.5(2.3V)(0.23mA) = 0.2645mW$$

$$P_{supply} = 2V_{CC}I_Q = 2(2.5)(0.23mA) = 1.15mW$$

$$\eta = \frac{P_{L(\max)}}{P_{\sup ply}} = \frac{1}{4}\left(1 - \frac{V_{CE(sat)}}{V_{CC}}\right) = 23\%$$

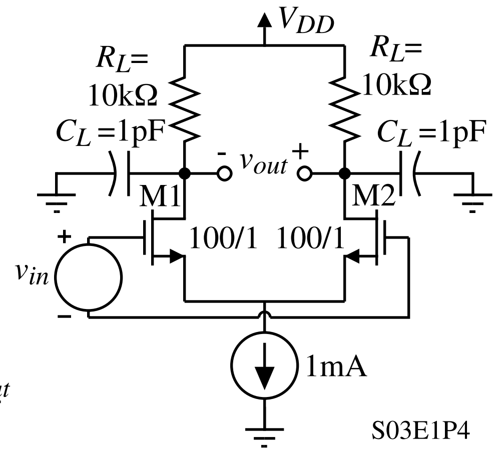
b) The output would slew under such condition. The current will be limited by the bias current:

Slew rate=0.23mA/100pF=2.3V/µs



Problem 4 - (25 points)

Find the numerical values of all roots and the midband gain of the transfer function v_{out}/v_{in} of the differential amplifier shown. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $\lambda_N = 0.04V^{-1}$. The values of $C_{gs} = 0.2pF$ and $C_{gd} = 20fF$.



S03E1P4

Solution

A small-signal model appropriate for this circuit is shown.

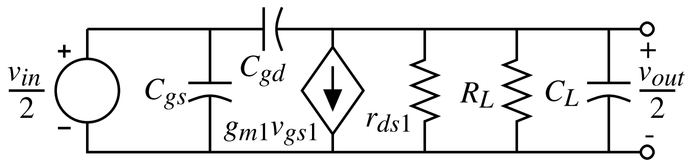


Fig. S03E1S4

Summing the currents at the output nodes gives,

$$g_{m1}v_{gs1} + sC_{gd}(v_{out}-v_{in}) + (g_{ds1} + G_L)v_{out} + sC_L\, v_{out} = 0$$

(Note: we are ignoring the fact that v_{out} and v_{in} should be divided by two since it makes no difference in the results and is easier to write.) Replacing v_{gs1} by v_{in} gives

$$-(g_{m1} - sC_{gd})v_{in} = [(g_{ds1} + G_L) + sC_L + sC_{gd}]\, v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{-(g_{m1} - sC_{gd})}{s(C_L + C_{gd}) + (g_{ds1} + G_L)} = \left(\frac{-g_{m1}}{g_{ds1} + G_L}\right)\left(\frac{1 - \dfrac{sC_{gd}}{g_m}}{1 + s\,\dfrac{C_L + C_{gd}}{g_{ds1} + G_L}}\right)$$

$\therefore$ $\quad$ MGB = $-\, g_{m1}(r_{ds}||R_L)$, $\quad$ Zero = $\dfrac{g_m}{C_{gd}}$ $\quad$ and $\quad$ Pole = $-\dfrac{g_{ds} + G_L}{C_{gd} + C_L}$

$$g_m = \sqrt{2\cdot 110\cdot 100\cdot 500} = 3316.7\mu S \quad \text{and} \quad r_{ds} = \frac{1}{\lambda I_D} = \frac{25}{500\mu A} = 50\ k\Omega$$

$\therefore$ $\quad$ MGB = $-3.3167mS\cdot(10k\Omega||50k\Omega)$ = <u>-27.64 V/V</u>

$$\text{Zero} = \frac{3.3167\text{x}10^{-3}}{20\text{x}10^{-15}} = \underline{1.658\text{x}10^{11}\ \text{radians/sec.}}$$

$$\text{Pole} = \frac{-1}{1.02\text{x}10^{-12}(10k\Omega||50k\Omega)} = \underline{-1.1176\text{x}10^{8}\ \text{radians/sec.}}$$