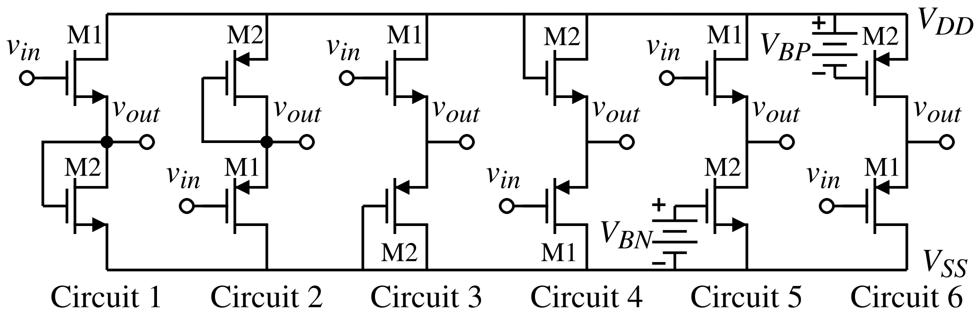
EXAMINATION NO. 1- SOLUTIONS

(Average = 60, High = 88, Low = 46)

Problem 1 - (25 points)

Six versions of a source follower are shown below. Assume that $K'_N = 2K'_P$, $\lambda_P = 2\lambda_N$, all W/L ratios of all devices are equal, and that all bias currents in each device are equal. Neglect bulk effects in this problem and assume no external load resistor. Identify which circuit or circuits have the following characteristics: (a.) highest small-signal voltage gain, (b.) lowest small-signal voltage gain, (c.) the highest output resistance, (d.) the lowest output resistance, (e.) the highest v_{out}(max) and (f.) the lowest v_{out}(max).



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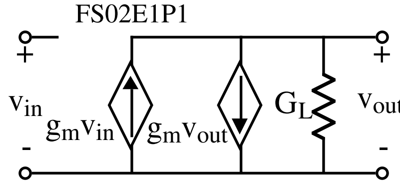
Solution

(a.) and (b.) - Voltage gain.

Small signal model:



The voltage gain is found as: $\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + G_L}$

where G_L is the load conductance. Therefore we get:

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{g_{mN}}{g_{mN}+g_{mN}}$	$\frac{g_{mP}}{g_{mP}+g_{mP}}$	$\frac{g_{mN}}{g_{mN}+g_{mP}}$	$\frac{g_{mP}}{g_{mP}+g_{mN}}$	$\frac{g_{mN}}{g_{mN}+g_{dsN}+g_{dsP}}$	$\frac{g_{mP}}{g_{mP}+g_{dsN}+g_{dsP}}$

But $g_{mN} = \sqrt{2}\, g_{mP}$ and $g_{dsN} = 0.5 g_{dsP}$, therefore

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{1}{2}$	$\frac{1}{2}$	0.5858	0.4142	$\frac{g_{mP}}{g_{mP}+(g_{dsP}+g_{dsN})/\sqrt{2}}$	$\frac{g_{mP}}{g_{mP}+g_{dsP}+g_{dsN}}$

Thus, circuit 5 has the highest gain and circuit 4 the lowest gain

(c.) and (d.) - Output resistance.

The denominators of the first table show the following:

Ckt.6 has the highest output resistance and Ckt. 1 the lowest output resistance.

(e.) Assuming no current has to be provided by the output, circuits 2, 4, and 6 can pull the output to V_{DD}. ∴ **Circuits 2, 4 and 6 have the highest output swing.**

(f.) Assuming no current has to be provided by the output, circuits 1, 3, and 5 can pull the output to ground. ∴ **Circuits 1, 3 and 5 have lowest output swing.**

Summary

(a.) Ckt. 5 has the highest voltage gain

(b.) Ckt. 4 has the lowest voltage gain

(c.) Ckt. 6 has the highest output resistance

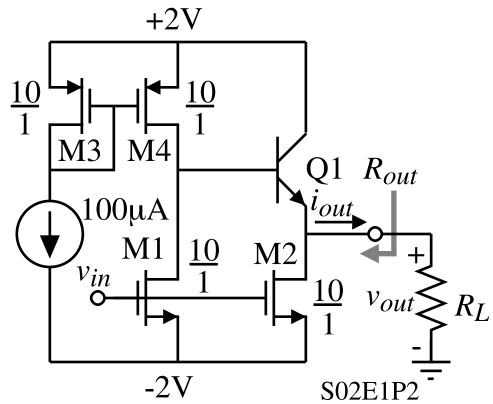
(d.) Ckt. 1 has the lowest output resistance

(e.) Ckts. 2,4 and 6 have the highest output

(f.) Ckts. 1,3 and 5 have the lowest output

Problem 2 - (25 points)

An output stage using both MOSFETs and a BJT is shown. Assume the transistor parameters are K_N' = 110μA/V^2, V_T = 0.7V, and λ_N = 0.04V^{-1} for the NMOS; K_P' = 50μA/V^2, V_T = -0.7V, and λ_P = 0.05V^{-1} for the PMOS and β_F = 100, V_t = 0.025V, and I_s = 10fA for the NPN BJT. (a.) If v_{in} can vary between ±2V, what is the maximum positive and negative value of i_{out} when R_L = 0Ω? (b.) If v_{in} can vary between ±2V, what is the maximum and minimum output voltage when R_L = 100Ω?



Solution

(a.) The maximum i_{out} occurs when v_{in} = -2V. All of the 100μA through M4 is base current giving a maximum $i_{out} = (1+\beta)100\mu A = 10.1mA$ → $i_{out}(\max)$ = 10.1mA

The maximum $-i_{out}$ occurs when v_{in} = +3V. Since V_{DS} = 2V and $V_{GS} - V_T$ = 3.3V, M2 is in the triode region. Under these conditions, we assume M1 absorbs all of the 100μA of M4 and therefore the BJT is off and maximum $-i_{out}$ is,

$$-i_{out}(\max) = \frac{K_N'W}{L}[(V_{GS2}-V_T)v_{DS} - 0.5v_{DS}^2] = 110\cdot 10[3.3\cdot 2 - 0.5(2)^2] = 5.06\text{mA}$$

∴ $-i_{out}(\max)$ = -5.06mA

(b.) There are 2 possible answers for the maximum v_{out}. The current limited max. v_{out} is

$$\text{Max. } v_{out} = i_{out}(\max)R_L = 10.1\text{mA}\cdot 0.1\text{k}\Omega = 1.01\text{V}$$

The voltage limited $v_{out}(\max)$ is,

$$\text{Max. } v_{out} = 2\text{V} - V_{SD4}(\text{sat}) - V_{BE1}(10.1\text{mA}) = 2-\sqrt{\frac{2\cdot 100}{50\cdot 10}} - 0.025\ln\left(\frac{10\text{mA}}{10\text{fA}}\right)$$

$$= 2-0.6325-0.6908 = 0.6768\text{V} \qquad \therefore \text{ Max. } v_{out} = 0.6768\text{V}$$

For the maximum $-v_{out}$ we see that the V_{GS2} = 4V which strongly suggests that M2 will be in the triode region. Equating the current in the 100Ω resistor with that in M2 gives,

$$\frac{2-v_{DS}}{100} = \frac{K_N'W}{L}[(V_{GS2}-V_T)v_{DS} - 0.5v_{DS}^2]$$

$$0.02-0.01v_{DS} = 1.1\text{x}10^{-3}[3.3v_{DS} - 0.5v_{DS}^2] \rightarrow v_{DS}^2 - 24.782v_{DS} + 36.36 = 0$$

∴ $v_{DS} = +12.391 \pm 10.8247 \rightarrow v_{DS} = 1.5662\text{V}$

∴ Max. $-v_{out}$ = -2V+1.5662V = -0.4338V

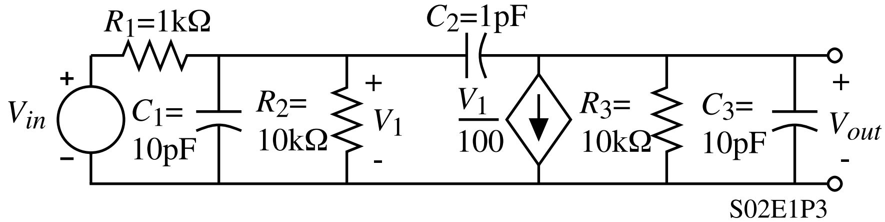
The current through M2 under this condition is $\frac{110\cdot 10}{2\cdot 1}(1.5662\text{V})^2 = 1.349\text{mA}$

It can be shown that if M2 remains saturated that Max. $-v_{out} = I\cdot 100\Omega$ = -0.1815V

So our assumption that M2 was in the triode region is valid.

Problem 3 - (25 points)

Find the midband voltage gain and the –3dB frequency in Hertz for the circuit shown.



Solution

The midband gain is given as,

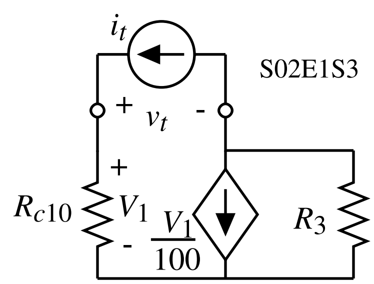
$$\frac{V_{out}}{V_{in}} = -\left(\frac{10\text{k}\Omega}{100}\right)\left(\frac{10\text{k}\Omega}{11\text{k}\Omega}\right) = \underline{\underline{-90.91\text{V/V}}}$$

To find the –3dB frequency requires finding the 3 open-circuit time constants.

R_{C10}:

$$R_{C10} = 1\text{k}\Omega \| 10\text{k}\Omega = 0.9091\text{k}\Omega \quad \rightarrow \quad R_{C10}C_1 = 0.9091\cdot 10\text{ns} = 9.09\text{ns}$$

R_{C20}:



$$v_t = i_t R_{C10} + R_3(i_t + 0.01V_1)$$

$$= i_t(R_{C10} + R_3 + 0.01R_{C10}R_3)$$

$$\therefore\ R_{C20} = R_{C10} + R_3 + 0.01R_{C10}R_3$$

$$= 0.9091 + 10(1 + 0.01\cdot 909.1)\text{k}\Omega = 101.82\text{k}\Omega$$

$$R_{C20}C_2 = 101.82\cdot 1\text{ns} = 101.82\text{ns}$$

R_{C30}:

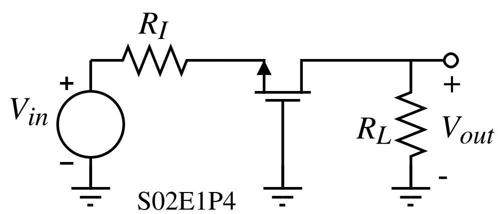
$$R_{C30} = 10\text{k}\Omega \quad \rightarrow \quad R_{C30}C_3 = 10\cdot 10\text{ns} = 100\text{ns}$$

$$\Sigma T_0 = (9.091 + 101.82 + 100)\text{ns} = 210.91\text{ns} \quad \rightarrow \quad \omega_{-3\text{dB}} = \frac{1}{\Sigma T_0} = 4.74\text{x}10^6 \text{ rad/s}$$

$$f_{-3\text{dB}} = \frac{4.74\text{x}10^6}{2\pi} = \underline{\underline{754.6\text{kHz}}}$$

Problem 4 - (25 points)

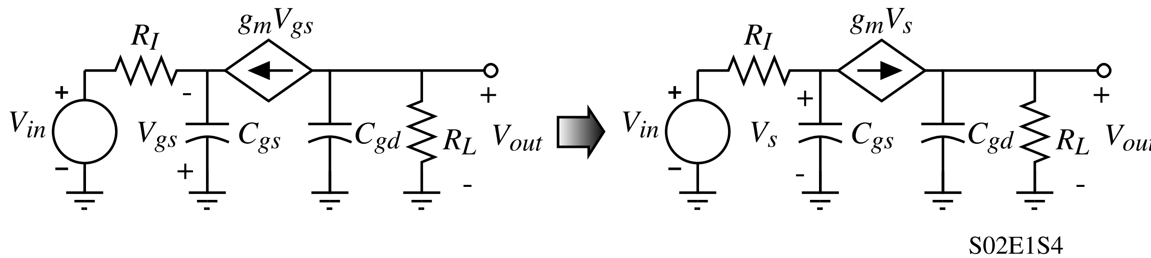
Find the midband voltage gain and the exact value of the two poles of the voltage transfer function for the circuit shown. Assume that $R_I = 1\text{k}\Omega$, $R_L = 10\text{K}\Omega$, $g_m = 1\text{mS}$, $C_{gs} = 5\text{pF}$ and $C_{gd} = 1\text{pF}$. Ignore r_{ds}.



Solution

The best approach to this problem is a direct analysis.

Small-signal model:



$$V_{out} = g_m Z_L V_s \quad \text{where} \quad Z_L = \frac{1}{sR_LC_{gd}+1} \quad \text{and} \quad \frac{V_{in}-V_s}{R_I} = g_mV_s + sC_{gs}V_s$$

Solving for V_s from the second equation gives,

$$V_s = \frac{V_{in}}{1+g_mR_I+sC_{gs}R_I}$$

Substituting V_s in the first equation gives,

$$V_{out} = g_mZ_L\frac{V_{in}}{1+g_mR_I+sC_{gs}R_I} \quad \rightarrow \quad \frac{V_{out}}{V_{in}} = g_m\left(\frac{1}{sR_LC_{gd}+1}\right)\left(\frac{1}{1+g_mR_I+sC_{gs}R_I}\right)$$

$$= \left(\frac{g_mR_L}{1+g_mR_I}\right)\left(\frac{1}{sR_LC_{gd}+1}\right)\left(\frac{1}{\frac{sC_{gd}R_I}{1+g_mR_I}+1}\right) = \text{MBG}\left(\frac{1}{1-\frac{s}{p_1}}\right)\left(\frac{1}{1-\frac{s}{p_2}}\right)$$

$$\therefore \ \text{MBG} = \left(\frac{g_mR_L}{1+g_mR_I}\right) = \left(\frac{1\cdot 10}{1+1\cdot 1}\right) = \underline{\underline{5\text{V/V}}}$$

$$p_1 = -\frac{1}{R_LC_{gd}} = -\frac{1}{10\cdot 1\text{ns}} = \underline{\underline{-10^8\text{ rad/s}}} \quad \text{and} \quad p_2 = -\frac{1+g_mR_I}{R_IC_{gs}} = -\frac{1+1}{1\cdot 5\text{ns}} = \underline{\underline{-4\text{x}10^8\text{ rad/s}}}$$