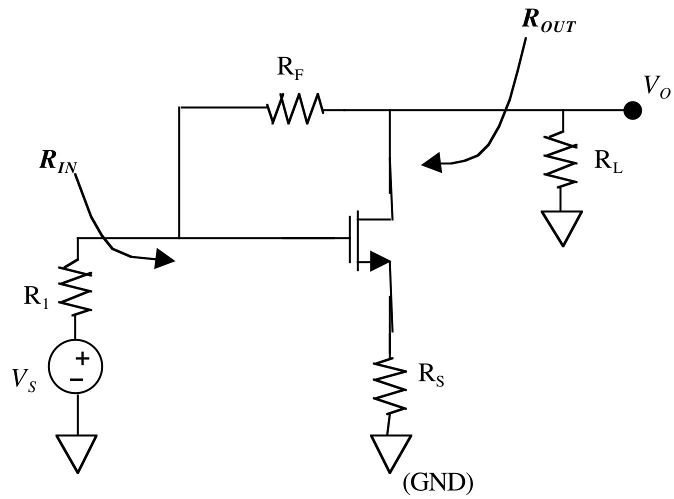
EXAMINATION NO. 3 - SOLUTIONS

(Average Score = 52/100)

Problem 1 - (25 points)

For the feedback circuit shown below:

a. Identify the types of feedback topologies used.
b. Using the Blackman's formula, derive expressions for the input (R_{IN}) and the output (R_{OUT}) resistances of this circuit (neglect the output resistance of the transistor r_o). Simplify your expressions as much as possible with the assumption that $g_mR_S >> 1$.
c. If the source (R_1) and load (R_L) resistances are equal, what relationship will hold between the input and output resistances of this circuit? Explain the role of each feedback loop in achieving this characteristic.



Solution

(a) Two types of feedback are used simultaneously in this circuit (sometimes called *dual-loop feedback*): 1) Shunt-shunt feedback through R_F, and 2) series-series feedback through R_S.

(b)

$$R_{OUT} = R_{OUT}(g_m = 0)\left[\frac{1 + RR(port\,shorted)}{1 + RR(port\,open)}\right]$$

$$R_{OUT}(g_m = 0) = R_F + R_1$$

$$RR(shorted) = g_mR_S$$

$$RR(open) = g_m(R_S + R_1)$$

→ $R_{OUT} = (R_F + R_1)\left[\dfrac{1 + g_mR_S}{1 + g_m(R_S + R_1)}\right]$ → If $g_mR_S >> 1$, then: $\boxed{R_{OUT} = \dfrac{(R_F + R_1)}{1 + \dfrac{R_1}{R_S}}}$

Input resistance: $R_{IN} = R_{IN}(g_m = 0)\left[\dfrac{1 + RR(port\,shorted)}{1 + RR(port\,open)}\right]$

$$R_{IN}(g_m = 0) = R_F + R_L$$

$$RR(shorted) = g_mR_S$$

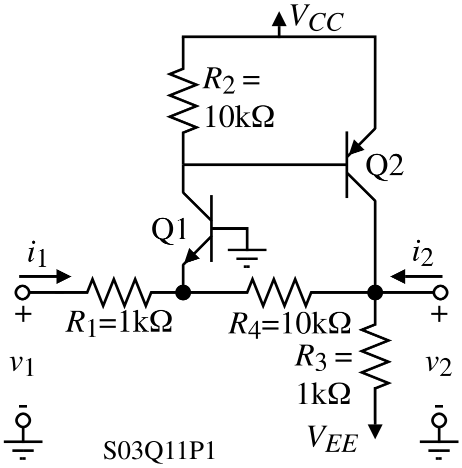
$$RR(open) = g_m(R_S + R_L)$$

→ $R_{IN} = (R_F + R_L)\left[\dfrac{1 + g_mR_S}{1 + g_m(R_S + R_L)}\right]$ → If $g_mR_S >> 1$, then: $\boxed{R_{IN} = \dfrac{(R_F + R_L)}{1 + \dfrac{R_L}{R_S}}}$

c) If $R_1 = R_L$, then $\boldsymbol{R_{IN} = R_{OUT}}$. By choosing the right values of R_F and R_S, the input and output resistances can be matched to the source and load resistances (for maximum power transfer, 50/75Ω in RF circuits) and the voltage gain can be set arbitrarily large. The application of series feedback will increase the output resistance of the transistor. The shunt feedback through R_F will then reduce the input and output resistances of the circuit to achieve impedance matching.

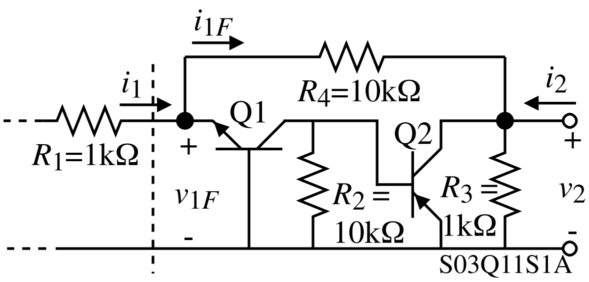
Problem 2 - (25 points)

A shunt-shunt feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of v_2/v_1, v_1/i_1, and v_2/i_2. Assume that all transistors are matched and that V_t= 25mV, β (of the BJT) = 100, $I_{C1} = I_{C2}$ = 100μA, and $r_o = \infty$.



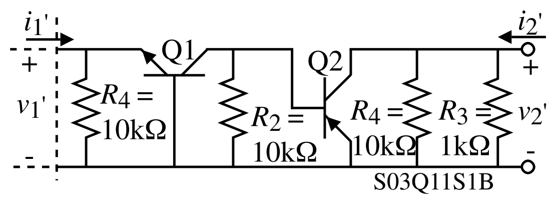
Solution

A simplified ac schematic for $\beta \neq 0$ is given as,

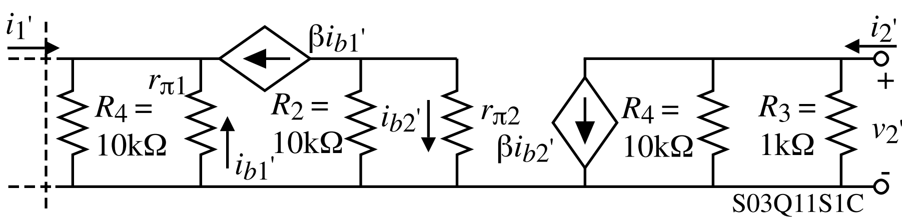


$$\beta = g_{12F} = \frac{i_{1F}}{v_2}\bigg|_{v_{1F}=0} = \frac{-1}{R_4} = \frac{-1}{10\text{k}\Omega}$$

The open-loop (β =0) simplified ac schematic is given as,



The small-signal model for (β = 0) is,



$$\frac{v_2'}{i_1'} = \left(\frac{v_2'}{i_{b2}'}\right)\left(\frac{i_{b2}'}{i_{b1}'}\right)\left(\frac{i_{b1}'}{i_1'}\right) = [-\beta(R_3 \| R_4)]\left(\frac{-\beta R_2}{r_{\pi2}+R_2}\right)\left(\frac{-R_4}{R_4 + 1/g_{m1}}\,\frac{1}{1+\beta}\right)$$

$$= (-100\cdot 1\text{K}\|10\text{K})\left(\frac{-100\cdot 10\text{K}}{35\text{K}}\right)\left(\frac{-10\text{K}}{10\text{K}+0.25\text{K}}\,\frac{1}{101}\right) = (-90.9)(-28.571)(-0.00966)$$

$$R_T = \frac{v_2'}{i_1'} = -25.087\text{k}\Omega \qquad \Rightarrow \qquad \frac{v_2}{i_1} = \frac{R_T}{1+\beta R_T} = \frac{-25.087\text{K}\Omega}{1+2.5087} = -7.15\text{k}\Omega$$

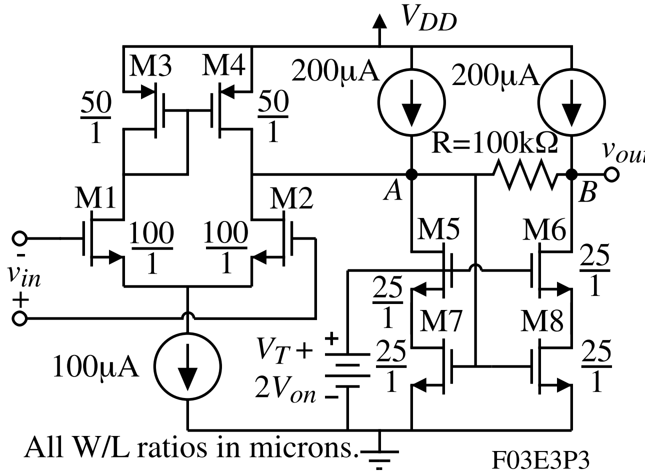
$$R_{in} = R_4 \| (1/g_{m1}) = 10000\|250 = 244\Omega, \quad R_{inF} = \frac{R_{in}}{1+\beta R_T} = \frac{244\Omega}{3.509} = 69.5\Omega$$

$$\therefore \quad \frac{v_1}{i_1} = R_1 + R_{inF} = 1000 + 70 = \underline{\underline{1070\Omega}} \qquad \frac{v_2}{v_1} = \frac{v_2}{i_1}\frac{i_1}{v_1} = \frac{-7.51\text{K}}{1070} = \underline{\underline{-7.02\text{V/V}}}$$

$$R_{out} = R_3 \| R_4 = 909\Omega \qquad \rightarrow \qquad \frac{v_2}{i_2} = \frac{R_{out}}{1+\beta R_T} = \frac{909\Omega}{3.509} = \underline{\underline{259\Omega}}$$

Problem 3 - (25 points)

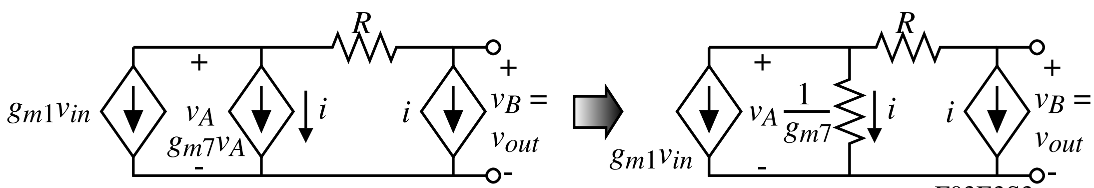
A low-gain, high-bandwidth voltage amplifier is shown. Find the low frequency voltage gain, v_{out}/v_{in}, and the unity-gainbandwidth, GB, if the sum of the capacitance connected to nodes A and B is 0.5pF each. Assume that the independent current sources used have infinite resistance. The transistor model parameters are K_N' =110μA/V^2, V_{TN} = 0.7V, λ_N=0, K_P' =50μA/V^2, V_{TP}= -0.7V, λ_P=0.



F03E3P3

Solution

The low frequency voltage gain can be found by inspection as $0.5g_{m1}R$. For those of you not into "found by inspection" the following small-signal model is useful.



F03E3S3

$$v_{out} = i\left(R + \frac{1}{g_{m7}}\right) = \frac{g_{m1}}{2}\left(R + \frac{1}{g_{m7}}\right)v_{in} \qquad g_{m1} = \sqrt{2\cdot 110\cdot 100\cdot 50} = 1.048\text{mS}$$

$$g_{m7} = \sqrt{2\cdot 110\cdot 25\cdot 200} = 1.048\text{mS} \qquad \therefore\ \frac{v_{out}}{v_{in}} = \frac{1.048}{2}\left(100 + \frac{1}{1.048}\right) = \underline{\underline{52.9\text{ V/V}}}$$

The approach to the second part of the problem will be to find the poles at A and B. The resistance to ground at node A is effectively $R_A \approx 1/g_{m7}$ = 1/1.048mS and at node B to ground is $R_B = R$ = 100kΩ. However, because of the shunt feedback at node B (and A) with a loop gain of 1, the output resistance is really 50kΩ. Therefore,

$$p_A = \frac{2g_{m7}}{R_A} = \frac{2\cdot 1.048\text{mS}}{0.5\text{pF}} = 4.192\text{x}10^9 \text{ rads/sec.}$$

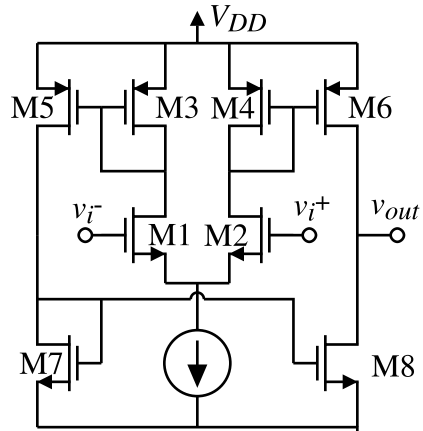
and

$$p_B = \frac{2}{R_B C_B} = \frac{2}{100\text{k}\Omega\cdot 0.5\text{pF}} = 40\text{x}10^6 \text{ rads/sec.}$$

$\therefore$ $GB = 52.9\cdot 40\text{x}10^6 = 2116\text{x}10^6$ rads/sec → $\underline{\underline{GB = 336.8 \text{ MHz}}}$

Problem 4 - (25 points)

For the amplifier shown assume that all transconductances are equal. Find (a.) the equivalent input noise voltage in units of V^2/Hz for thermal noise (k = $1.38\text{x}10^{-23}$ J/K), (b.) the equivalent input noise voltage in units of V^2/Hz for 1/f noise ($B_N = 8\text{x}10^{-22}$ (V-m)^2 and $B_P = 2\text{x}10^{-22}$ (V-m)^2), and (c.) the noise corner frequency in Hz. Using $\int_a^b \frac{1}{f} df = \ln(b) - \ln(a)$, find the rms noise voltage in a bandwidth of 1Hz to 100kHz in V(rms).



All g_m = 300µS and all W/L=10µm/1µm

S03E3P4

Solution

The short-circuit noise current as a function of all 8 of the noise sources in series with the gates can be written as,

$$i_{to}^2 = g_{m1}^2 e_{n1}^2 + g_{m2}^2 e_{n2}^2 + g_{m5}^2(e_{n3}^2 + e_{n5}^2) + g_{m6}^2(e_{n4}^2 + e_{n6}^2) + g_{m8}^2(e_{n7}^2 + e_{n8}^2)$$

The above can be written as,

$$i_{to}^2 = g_m^2[4\, e_{nN}^2 + 4\, e_{nP}^2\,]$$

Dividing by g_m^2 gives the equivalent input noise voltage as,

$$e_{eq}^2 = 4\, e_{nN}^2 + 4\, e_{nP}^2 = 4\, e_{nN}^2 \left(1 + \frac{e_{nP}^2}{e_{nN}^2}\right)$$

(a.) For thermal noise, $e_{nN}^2 = e_{nP}^2$ so that

$$e_{eq}^2 = 8\, e_{nN}^2 = 8\frac{8kT}{3g_{mN}} = 64\,\frac{1.38\text{x}10^{-23}\cdot 300}{3\cdot 300\text{x}10^{-6}} = \underline{\underline{2.944\text{x}10^{-16}\ V^2/\text{Hz}}}$$

(b.) For $1/f$ noise,

$$e_{nN}^2 = \frac{B_N}{fWL} = \frac{8\text{x}10^{-22}}{f10\text{x}10^{-12}} = \frac{8\text{x}10^{-11}}{f} \qquad \text{and} \qquad e_{nP}^2 = \frac{B_P}{fWL} = \frac{2\text{x}10^{-22}}{f10\text{x}10^{-12}} = \frac{2\text{x}10^{-11}}{f}$$

$$\therefore \quad e_{eq}^2 = 4\, e_{nN}^2 \left(1 + \frac{e_{nP}^2}{e_{nN}^2}\right) = \frac{32\text{x}10^{-11}}{f}\left(1 + \frac{2}{8}\right) = \frac{40\text{x}10^{-11}}{f} \qquad \boxed{e_{eq}^2 = \frac{40\text{x}10^{-11}}{f}\ V^2/\text{Hz}}$$

(c.) Equating the above results gives,

$$\frac{40\text{x}10^{-11}}{f} = 2.944\text{x}10^{-16} \quad \rightarrow \quad f_c = \frac{40\text{x}10^{-11}}{2.944\text{x}10^{-16}} = \underline{\underline{1.359\text{MHz}}}$$

Finally, we can find the rms noise by integrating just the $1/f$ noise from 1Hz to 100kHz.

$$V_{eq}^2(\text{rms}) = \int_1^{10^5} \frac{40\text{x}10^{-11}}{f}\,\text{df} = 40\text{x}10^{-11}[\ln(10^5)\text{-}\ln(1)]$$

$$= 40\text{x}10^{-11}(11.513) = 4.605\text{x}10^{9}\ V^2(\text{rms}) \rightarrow \quad V_{eq}(\text{rms}) = \underline{\underline{68\mu\text{V (rms)}}}$$