FINAL EXAMINATION (ALLEN) - SOLUTION

(Average Score = 91/120)

Problem 1 - (20 points - This problem is required)

An open-loop comparator has a gain of 10^4, a dominant pole of 10^5 radians/sec., a slew rate of 5V/μs and an output swing of 1V. (a.) If V_{in} = 1mV find the propagation delay time of this comparator (the time for the output to go halfway from one state to the other). (b.) Repeat part (a.) if V_{in} = 10mV. (c.) Repeat part (a.) if V_{in} = 100mV.

Solution

a.) We know that the linear output voltage of a single-pole comparator can be written as,

$$v_{out}(t) = A(1-e^{-tp_1})V_{in}$$

which can be solved for the propagation delay time as

$$t_p = \tau_1 ln\left(\frac{2k}{2k-1}\right) \quad \text{where} \quad k = \frac{V_{in}}{V_{in}(\text{min})} \quad \text{and } V_{in}(\text{min}) = \frac{(V_{OH}-V_{OL})}{A} = \frac{1\text{V}}{10^4} =$$

0.1mV

We must first determine if the comparator is linear or slewing. The maximum slope occurs at t = 0 and is given as

$$\frac{dv_{out}(t)}{dt} = A\cdot p_1\cdot V_{in}\, e^{-tp_1} \quad \frac{dv_{out}(0)}{dt} = A\cdot p_1\cdot V_{in} = 10^4\cdot 10^5\cdot 10^{-6} = 10^6 = 1\text{V/μs}$$

∴ The comparator is not slewing and t_p is given as

$$t_p = \tau_1 ln\left(\frac{2k}{2k-1}\right) = 10^{-5} ln\left(\frac{20}{20-1}\right) = \underline{\underline{0.513\text{μs}}}$$

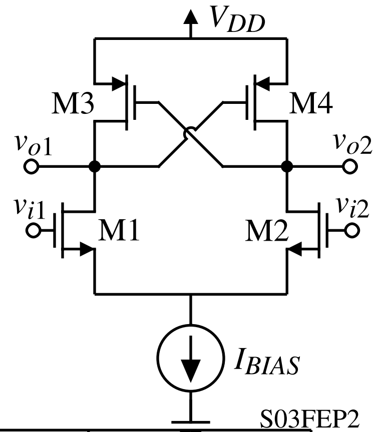
b.) We see the maximum slope for the linear response is 10V/μs which means that the comparator is slewing. Slewing at a rate of 5V/μs requires 0.1μs to go 0.5V. Therefore,

$$t_p = \underline{\underline{0.100\text{μs}}}$$

c.) The answer is the same as b.) namely $t_p = \underline{\underline{0.100\text{μs}}}$

Problem 2 - (20 points - This problem is optional)

Assume the capacitors connected to the drains of M1 and M2 (C_1 and C_2) are initially discharged. Express $\Delta V_{out} = v_{o2} - v_{o1}$ as a function of the applied input, $\Delta V_{in} = v_{i1}$-v_{i2}, in the time domain assuming ΔV_{in} is a step input. If $g_{m1} = g_{m2} =$ 1mS, $g_{m3} = g_{m4} =$ 100µS, and $C_1 = C_2 =$ 1pF, what is the propagation delay time ($\Delta V_{out} = |0.5(V_{OH} - V_{OL})|$) for a step input of $\Delta V_{in} = 0.01(V_{OH} - V_{OL})$?



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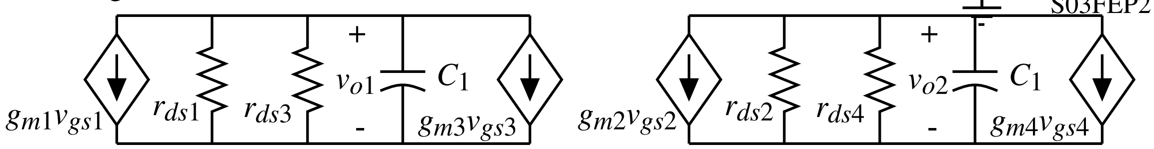
Solution

Small-signal model:



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The nodal equations corresponding two these two circuits are:

$$g_{m1}v_{gs1} + g_{ds1}v_{o1} + g_{d3}v_{o1} + sC_1v_{o1} + g_{m3}v_{gs3} = 0$$

and

$$g_{m2}v_{gs2} + g_{ds2}v_{o2} + g_{d4}v_{o2} + sC_2v_{o2} + g_{m4}v_{gs4} = 0$$

We can write that,

$$(g_{ds1} + g_{ds3} + sC_1)\, v_{o1} = -\, g_{m1}v_{gs1} - g_{m3}v_{gs3}$$

and

$$(g_{ds2} + g_{ds4} + sC_2)\, v_{o2} = -\, g_{m1}v_{gs2} - g_{m4}v_{gs4}$$

Assuming matching, we get

$$(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1) = g_{m1}(v_{gs1} - v_{gs2}) + g_{m3}(v_{gs3} - v_{gs4})$$

or

$$(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1) = g_{m1}(v_{i1} - v_{i2}) + g_{m3}(v_{o2} - v_{o1})$$

$$(v_{o2} - v_{o1})(g_{ds1} + g_{ds3} + sC_1 - g_{m3}) \approx (v_{o2} - v_{o1})(sC_1 - g_{m3}) = g_{m1}(v_{i1} - v_{i2})$$

$$\therefore \quad (v_{o2} - v_{o1}) = \Delta V_{out}(s) = \frac{g_{m1}}{sC_1 - g_{m3}}\Delta V_{in}(s) = \frac{g_{m1}}{g_{m3}}\left(\frac{g_{m3}/C_1}{s - (g_{m3}/C_1)}\right)\frac{\Delta v_{in}}{s}$$

$$\Delta V_{out}(s) = \frac{g_{m1}}{C_1}\Delta v_{in}\left[\frac{k_1}{s} + \frac{k_2}{s - (g_{m3}/C_1)}\right] = \frac{g_{m1}}{g_{m3}}\left(\frac{g_{m3}/C_1}{s - (g_{m3}/C_1)}\right)\frac{\Delta v_{in}}{s}$$

Solving for k_1 and k_2 gives $k_1 = -(C_1/g_{m3})$ and $k_2 = (C_1/g_{m3})$. Thus,

$$\Delta V_{out}(s) = \frac{g_{m1}}{g_{m3}}\left(\frac{1}{s - (g_{m3}/C_1)} - \frac{1}{s}\right)\Delta_{in} \rightarrow \boxed{\Delta v_{out}(t) = \frac{g_{m1}}{g_{m3}}\Delta V_{in}[e^{(g_{m3}/C_1)t} - 1]}$$

If $\Delta V_{in} = 0.01(V_{OH} - V_{OL})$, then

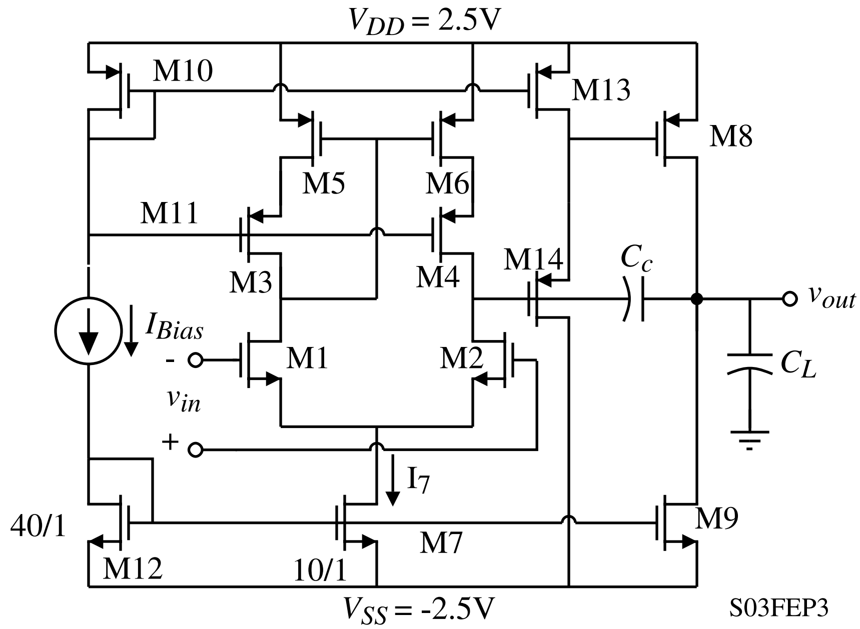
$$0.5(V_{OH} - V_{OL}) = 0.1(V_{OH} - V_{OL})[e^{10^8 t_p} - 1] \rightarrow 5 = e^{10^8 t_p} - 1$$

or

$$e^{10^8 t_p} = 6 \rightarrow t_p = \frac{1}{10^8}\ln(6) = \frac{1.792}{10^8} = \underline{\underline{19.92 \text{ ns}}}$$

Problem 3 - (20 points - This problem is optional)

An internally-compensated, cascode op amp is shown. (a) Derive an expression for the common-mode input range. (b) Find W_1/L_1, W_2/L_2, W_5/L_5, and W_6/L_6 when I_{BIAS} is 80 A and the input CMR is -1.25 V to +2 V. Use K'_N = 110 A/V^2, K'_p = 50 A/V^2 and $|V_T|$ = 0.6 to 0.8V. (c.) Develop an expression for the small-signal differential-voltage gain and output resistance of the cascode op amp.



Solution

(a.) $V_{icm}(\text{min}) = V_{SS} + V_{DS7}(\text{sat}) + V_{DS1}(\text{sat}) + V_{T1}(\text{max})$

and

$V_{icm}(\text{max}) = V_{DD} - V_{SD5}(\text{sat}) - V_{T5}(\text{max}) + V_{T1}(\text{min})$ (we will ignore that M8 and M14 cause a more severe upper ICM limit)

$\therefore$ $ICMR = V_{icm}(\text{max}) - V_{icm}(\text{min})$

$$\boxed{ICMR = (V_{DD} - V_{SS}) + [V_{DS7}(\text{sat}) + V_{DS1}(\text{sat}) + V_{T1}(\text{max})] - [V_{SD5}(\text{sat}) + V_{T5}(\text{max}) - V_{T1}(\text{min})]}$$

(b.) $I_7 = \frac{1}{4} I_{BIAS} = 20$ A Using $V_{DS}(\text{sat}) = \sqrt{\frac{2I}{K'(W/L)}}$ and $\frac{W}{L} = \frac{2I}{K'[V_{DS}(\text{sat})]^2}$ gives,

$V_{DS7}(\text{sat}) = 0.191\text{V} \rightarrow V_{icm}(\text{min}) = -2.5\text{V} + 0.191\text{V} + V_{DS1}(\text{sat}) + 0.8\text{V} \rightarrow V_{DS1}(\text{sat}) = 0.259\text{V}$

$$\therefore \quad \frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{20\text{ A}}{110\text{Æ}0.259^2} = \underline{\underline{2.70}}$$

$V_{icm}(\text{max}) = 2.0\text{V} = 2.5\text{V} - V_{SD5}(\text{sat}) - 0.6\text{V} + 0.8\text{V} \rightarrow V_{SD5}(\text{sat}) = 0.3\text{V}$

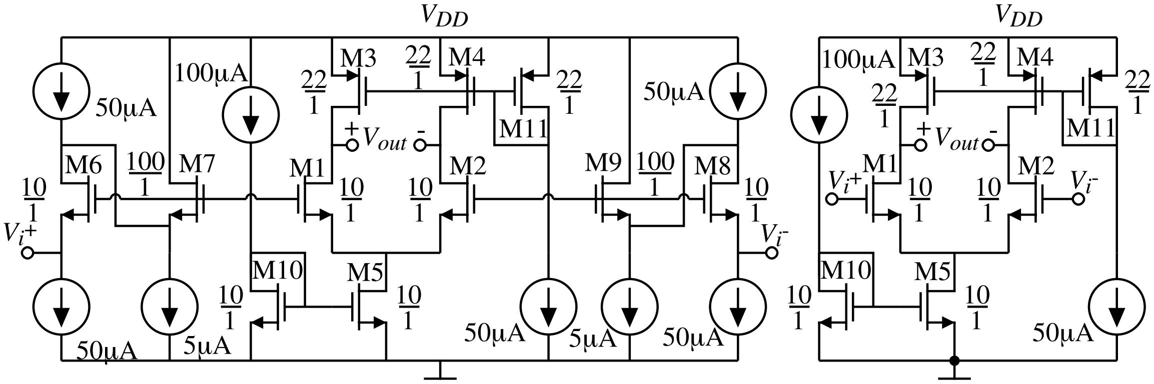
$$\therefore \quad \frac{W_5}{L_5} = \frac{W_6}{L_6} = \frac{20\text{ A}}{50\text{Æ}0.3^2} = \underline{\underline{4.44}}$$

(c.) By inspection, we can write:

$$\boxed{A_v = (-g_{m1} r_{ds2}) \left(\frac{g_{m14}}{g_{m14} + g_{ds13} + g_{ds14}}\right) \left(\frac{-g_{m8}}{g_{ds8} + g_{ds9}}\right)}$$

Problem 4 - (20 points - This problem is optional)

George P. Burdell has submitted the following input stage for the design challenge problem in ECE 6412. Assuming that the transistor model parameters are K_N' =110µA/V^2, V_{TN} = 0.7V, λ_N=0.04V^{-1}, K_P' =50µA/V^2, V_{TP}= -0.7V, λ_P=0.05V^{-1}, your job is to check this op amp out. In particular, what is the upper and lower input common mode voltages, what is the minimum power supply that gives zero input common mode range, what is the small-signal voltage gain, and compare this input stage with the classical differential input stage (list advantages and disadvantages).



S03FEP4 The George P. Burdell Approach Classical Approach

Solution

GPB Differential Input Stage:

$$V_{icm}^{+} = V_{DD} — V_{SD3}(\text{sat}) + V_{T1} - V_{GS6}(50\ \text{A}) = V_{DD} — V_{SD3}(\text{sat}) + V_{T1} — V_{GS6}(\text{sat}) — V_{T6}$$

$$V_{icm}^{+} = V_{DD} — V_{SD3}(\text{sat}) — V_{GS6}(\text{sat})$$

$$V_{icm}^{-} = V_{DS5}(\text{sat}) + V_{GS1}(50\ \text{A}) - V_{GS6}(50\ \text{A}) = V_{DS5}(\text{sat}) + V_{DS1}(\text{sat}) - V_{DS6}(\text{sat})$$

$$V_{DS1}(\text{sat}) = V_{DS6}(\text{sat}) = \sqrt{\frac{2\text{Æ}50}{110\text{Æ}10}} = 0.301\text{V},\quad V_{DS5}(\text{sat}) = \sqrt{\frac{2\text{Æ}100}{110\text{Æ}10}} = 0.426\text{V},$$

and $V_{SD3}(\text{sat}) = \sqrt{\frac{2\text{Æ}50}{50\text{Æ}22}} = 0.301\text{V}$

$\therefore$ $V_{icm}^{+} = \underline{V_{DD} — 0.602\text{V}}$ and $V_{icm}^{-} = 0.426 + 0.301 — 0.301 = \underline{0.426\text{V}}$

Note that $V_{DS6} = V_{GS6}\text{-}V_{GS7}$ = 1.001-0.73 = 0.271 < V_{DS6}(sat) so that M6 is slightly in the active region but this is not a problem.

The minimum V_{DD} is found by letting $V_{icm}^{+} = V_{icm}^{-}$ which gives V_{DD}(min) = 1.028V

The small-signal voltage gain is $A_{vd} = \dfrac{V_{out}}{V_i^{+}\text{-}°V_i^{-}} = \dfrac{-g_{m1}}{g_{ds1}+°g_{ds3}} = \dfrac{-g_{m2}}{g_{ds2}+°g_{ds4}}$

Classical Differential Input Stage:

$$V_{icm}^{+} = V_{DD} — V_{SD3}(\text{sat}) + V_{T1} = V_{DD} — 0.301 + 0.7 = \underline{V_{DD} + 0.4\text{V}}$$

$$V_{icm}^{-} = V_{DS5}(\text{sat}) + V_{GS1}(50\ \text{A}) = \underline{1.427\text{V}}$$

V_{DD}(min) = 1.027V and the small-signal voltage gain is the same.

Problem 4 – Continued

Comparison between the two approaches:

Characteristic	GBP Differential Amplifier	Classical Differential Amplifier
V_{icm}^{+}	V_{DD} — 0.602V	V_{DD} + 0.4V
V_{icm}^{-}	0.426V	1.427V
V_{DD}(min)	1.028V	1.027V
P_{diss}	V_{DD}(360 A)	V_{DD}(250 A)
Noise	Higher	Lower
Input Offset Voltage	Larger	Smaller
Small-signal gain	Same	Same
Useable ICMR	Within power supply	Outside of power supply