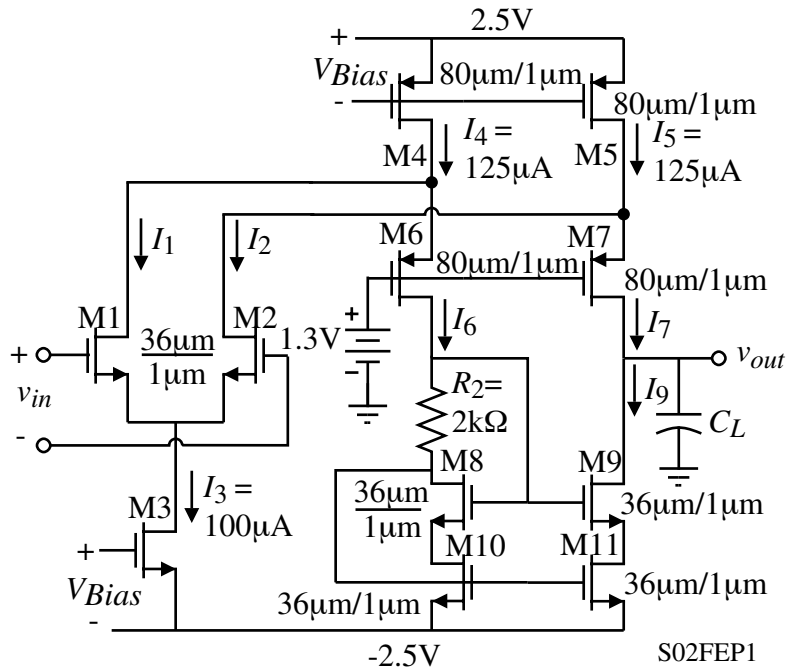


FINAL EXAMINATION - SOLUTIONS

(Average = 100, High = 120, and Low = 69)

Problem 1 - (20 points - This problem is required)

If the folded-cascode op amp shown having a small-signal voltage gain of 7464V/V is used as a comparator, find the dominant pole if $C_L = 5\text{pF}$. If the input step is 10mV , determine whether the response is linear or slewing and find the propagation delay time. Assume the parameters of the NMOS transistors are $K_N' = 110\text{V}/\mu\text{A}^2$, $V_{TN} = 0.7\text{V}$, $\lambda_N = 0.04\text{V}^{-1}$ and for the PMOS transistors are $K_P' = 110\text{V}/\mu\text{A}^2$, $V_{TP} = -0.7\text{V}$, $\lambda_P = 0.05\text{V}^{-1}$.

**Solution**

V_{OH} and V_{OL} can be found from many approaches. The easiest is simply to assume that V_{OH} and V_{OL} are 2.5V and -2.5V , respectively. However, no matter what the input, the values of V_{OH} and V_{OL} will be in the following range,

$$(V_{DD} - 2V_{ON}) < V_{OH} < V_{DD} \quad \text{and} \quad V_{DD} < V_{OH} < (V_{SS} + 2V_{ON})$$

The reasoning is as follows, suppose $V_{in} > 0$. This gives $I_1 > I_2$ which gives $I_6 < I_7$ which gives $I_9 < I_7$. V_{out} will increase until I_7 equals I_9 . The only way this can happen is for M5 and M7 to leave saturation. The same reasoning holds for $V_{in} < 0$.

Therefore assuming that V_{OH} and V_{OL} are 2.5V and -2.5V , respectively, we get

$$V_{in(\text{min})} = \frac{5\text{V}}{7464} = 0.67\text{mV} \quad \rightarrow \quad k = \frac{10\text{mV}}{0.67\text{mV}} = 14.93$$

Problem 1 – Continued

The folded-cascode op amp as a comparator can be modeled by a single dominant pole. This pole is found as,

$$p_1 = \frac{1}{R_{out}C_L} \text{ where } R_{out} \approx g_{m9}r_{ds9}r_{ds11} \parallel [g_{m7}r_{ds7}(r_{ds2} \parallel r_{ds5})]$$

$$g_{m9} = \sqrt{2 \cdot 75 \cdot 110 \cdot 36} = 771 \mu\text{S}, \quad g_{ds9} = g_{ds11} = 75 \times 10^{-6} \cdot 0.04 = 3 \mu\text{S}, \quad g_{ds2} = 50 \times 10^{-6} (0.04) = 2 \mu\text{S}$$

$$g_{m7} = \sqrt{2 \cdot 75 \cdot 50 \cdot 80} = 775 \mu\text{S}, \quad g_{ds5} = 125 \times 10^{-6} \cdot 0.05 = 6.25 \mu\text{S}, \quad g_{ds7} = 50 \times 10^{-6} (0.05) = 3.75 \mu\text{S}$$

$$g_{m9}r_{ds9}r_{ds11} = (771 \mu\text{S}) \left(\frac{1}{3 \mu\text{S}} \right) \left(\frac{1}{3 \mu\text{S}} \right) = 85.67 \text{M}\Omega$$

$$g_{m7}r_{ds7}(r_{ds2} \parallel r_{ds5}) \approx (775 \mu\text{S}) \left(\frac{1}{3.75 \mu\text{S}} \right) \left(\frac{1}{2 \mu\text{S}} \parallel \frac{1}{6.25 \mu\text{S}} \right) = 25.05 \text{M}\Omega,$$

$$R_{out} \approx 85.67 \text{M}\Omega \parallel 25.05 \text{M}\Omega = 19.4 \text{M}\Omega$$

The dominant pole is found as, $p_1 = \frac{1}{R_{out}C_L} = \frac{1}{19.4 \times 10^6 \text{pF}} = 10,318 \text{ rps}$

The time constant is $\tau_1 = 96.9 \mu\text{s}$.

For a dominant pole system, the step response is, $v_{out}(t) = A_{vd}(1 - e^{-t/\tau_1})V_{in}$

The slope is the largest at $t = 0$. Evaluating this slope gives,

$$\frac{dv_{out}}{dt} = \frac{A_{vd}}{\tau_1} e^{-t/\tau_1} V_{in} \text{ For } t = 0, \text{ the slope is } \frac{A_{vd}}{\tau_1} V_{in} = \frac{7464}{96.9 \mu\text{s}} (10 \text{mV}) = 0.77 \text{V}/\mu\text{s}$$

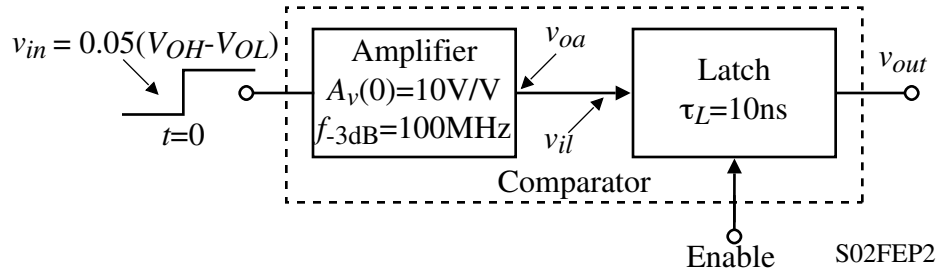
The slew rate of this op amp/comparator is $SR = \frac{I_3}{C_L} = \frac{100 \mu\text{A}}{5 \text{pF}} = 20 \text{V}/\mu\text{s}$

Therefore, the comparator does not slew and its propagation delay time is found from the linear response as,

$$t_P = \tau_1 \ln \left(\frac{2k}{2k-1} \right) = 96.9 \mu\text{s} \cdot \ln \left(\frac{2 \cdot 14.93}{2 \cdot 14.93 - 1} \right) = (96.9 \mu\text{s})(0.0341) = \underline{\underline{3.3 \mu\text{s}}}$$

Problem 2 - (20 points - This problem is required)

A comparator consists of an amplifier cascaded with a latch as shown below. The amplifier has voltage gain of 10V/V and $-f_{-3dB} = 100\text{MHz}$ and the latch has a time constant of 10ns. The maximum and minimum voltage swings of the amplifier and latch are V_{OH} and V_{OL} . When should the latch be enabled after the application of a step input to the amplifier of $0.05(V_{OH}-V_{OL})$ to get minimum overall propagation time delay? What is the value of the minimum propagation time delay? It may be useful to recall that the propagation time delay of the latch is given as $t_p = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right)$ where v_{il} is the latch input (ΔV_i of the text).

**Solution**

The solution is based on the figure shown.
We note that,

$$v_{oa}(t) = 10[1 - e^{-\omega \cdot 3dB t}]0.05(V_{OH} - V_{OL}).$$

If we define the input voltage to the latch as,

$$v_{il} = x \cdot (V_{OH} - V_{OL})$$

then we can solve for t_1 and t_2 as follows:

$$x \cdot (V_{OH} - V_{OL}) = 10[1 - e^{-\omega \cdot 3dB t_1}]0.05(V_{OH} - V_{OL}) \rightarrow x = 0.5[1 - e^{-\omega \cdot 3dB t_1}]$$

This gives,

$$t_1 = \frac{1}{\omega_{3dB}} \ln\left(\frac{1}{1-2x}\right)$$

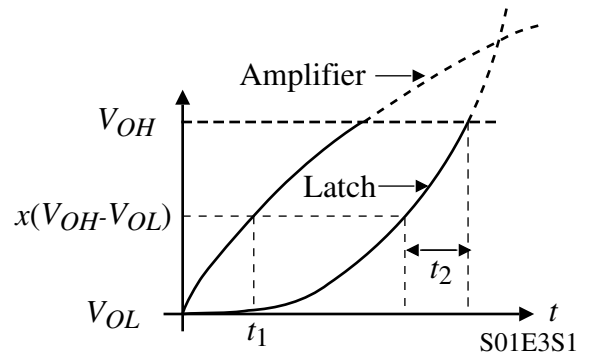
From the propagation time delay of the latch we get,

$$t_2 = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right) = \tau_L \ln\left(\frac{1}{2x}\right)$$

$$\therefore t_p = t_1 + t_2 = \frac{1}{\omega_{3dB}} \ln\left(\frac{1}{1-2x}\right) + \tau_L \ln\left(\frac{1}{2x}\right) \rightarrow \frac{dt_p}{dx} = 0 \text{ gives } x = \frac{\pi}{1+2\pi} = 0.4313$$

$$t_1 = \frac{10\text{ns}}{2\pi} \ln(1+2\pi) = 1.592\text{ns} \cdot 1.9856 = \underline{\underline{3.16\text{ns}}} \text{ and } t_2 = 10\text{ns} \ln\left(\frac{1+2\pi}{2\pi}\right) = 1.477\text{ns}$$

$$\therefore t_p = t_1 + t_2 = 3.16\text{ns} + 1.477\text{ns} = \underline{\underline{4.637\text{ns}}}$$



Problem 3 - (20 points - This problem is optional)

If $R_1 = R_2$ of the circuit shown, find an expression for the small-signal output resistance R_{out} ignoring R_L . Repeat including the influence of R_L on the output resistance. Let $R_1=R_2$ and $R_L = 1\text{k}\Omega$, dc currents through M1 and M2 be $500\mu\text{A}$, $W_1/L_1 = 100\mu\text{m}/1\mu\text{m}$ and $W_2/L_2 = 200\mu\text{m}/1\mu\text{m}$. Find the value of R_{out} .

Solution

The loop-gain for this network can be written from inspection as,

$$LG = \left(\frac{R_1}{R_1+R_1} \right) \left(\frac{g_{m1}+g_{m2}}{g_{ds1}+g_{ds2}+G_L} \right)$$

Therefore, since the output is shunt feedback we can solve for output resistance as,

$$R_{out} = \frac{r_{ds1} \parallel r_{ds2} \parallel R_L}{1 + \left(\frac{R_1}{R_1+R_1} \right) \left(\frac{g_{m1}+g_{m2}}{g_{ds1}+g_{ds2}+G_L} \right)}$$

Setting $G_L = 0$ ($R_L = \infty$) gives the output resistance not including the load resistor, R_L , as

$$R_{out} = \frac{r_{ds1} \parallel r_{ds2}}{1 + \left(\frac{R_1}{R_1+R_1} \right) \left(\frac{g_{m1}+g_{m2}}{g_{ds1}+g_{ds2}} \right)}$$

Calculating the small-signal parameters gives,

$$g_{m1} = \sqrt{2 \cdot 110 \cdot 500 \cdot 100} = 3.316\text{mS}, \quad g_{m2} = \sqrt{2 \cdot 50 \cdot 500 \cdot 200} = 3.162\text{mS},$$

$$r_{ds1} = \frac{10^6}{0.04 \cdot 500} = 50\text{k}\Omega \quad \text{and} \quad r_{ds2} = \frac{10^6}{0.05 \cdot 500} = 40\text{k}\Omega$$

$$\therefore R_{out}(R_L=\infty) = \frac{50\text{k}\Omega \parallel 40\text{k}\Omega}{1 + 0.5 \left(\frac{3316+3162}{25+20} \right)} = \frac{22.22\text{k}\Omega}{72.98} = 304\Omega$$

$$R_{out}(R_L=1\text{k}\Omega) = \frac{50\text{k}\Omega \parallel 40\text{k}\Omega \parallel 1\text{k}\Omega}{1 + 0.5 \left(\frac{3316+3162}{25+20+1000} \right)} = \frac{957\Omega}{4.0995} = \underline{\underline{233\Omega}}$$

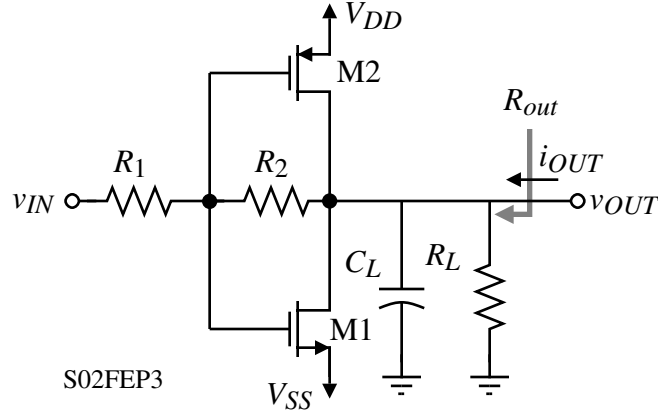
As usual, straight-forward small-signal models are faster, summing the currents at the output,

$$I_{out} = V_{out} \left[\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}} + \frac{1}{R_1+R_2} + \frac{g_{m1}+g_{m2}}{2} + \frac{1}{R_L} \right] \approx V_{out} \left[\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}} + \frac{g_{m1}+g_{m2}}{2} + \frac{1}{R_L} \right]$$

$$\therefore R_{out} = \left[\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}} + \frac{g_{m1}+g_{m2}}{2} + \frac{1}{R_L} \right]^{-1} = 304\Omega \quad (R_L=\infty)$$

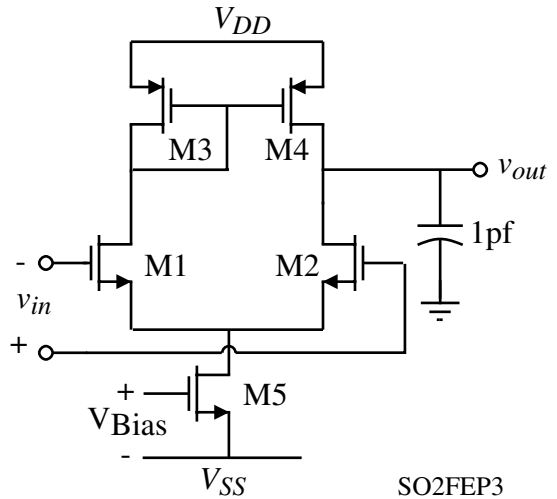
$$R_{out} = \underline{\underline{233\Omega}} \quad (R_L=1\text{k}\Omega)$$

(Principle: Feedback is a great concept tool but a terrible analysis tool.)

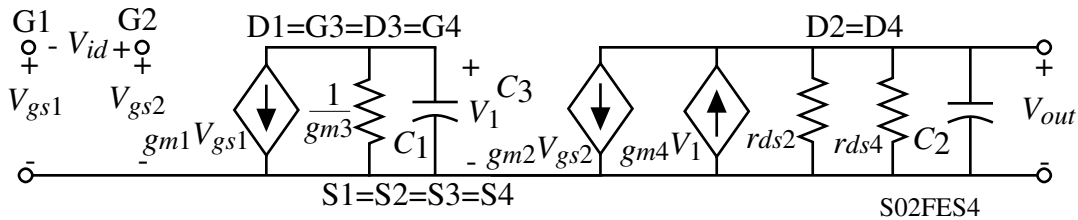


Problem 4 - (20 points - This problem is optional)

A current mirror load, CMOS differential amplifier is shown. The current in M5 is $100\mu\text{A}$. Assume the parameters of the NMOS transistors are $K_N' = 110\text{V}/\mu\text{A}^2$, $V_{TN} = 0.7\text{V}$, $\lambda_N = 0.04\text{V}^{-1}$ and for the PMOS transistors are $K_P' = 110\text{V}/\mu\text{A}^2$, $V_{TP} = 0.7\text{V}$, $\lambda_P = 0.04\text{V}^{-1}$. (a.) Find the small-signal output resistance and voltage gain if the W/L ratio of M1 and M2 is $100\mu\text{m}/1\mu\text{m}$. (b.) If the W/L ratio of M3 and M4 is $50\mu\text{m}/1\mu\text{m}$ and $C_{ox} = 24.7 \times 10^{-4}\text{F}/\text{m}^2$, and the effective output capacitance is 1pF , find all roots of this amplifier (ignore the influence of C_{gd4}). (c.) What is the -3dB frequency in Hertz?

**Solution**

The small-signal model suitable for this problem is shown below.



$$C_1 = 2(0.667)(50 \times 10^{-12}\text{m}^2)(24.7 \times 10^{-4}\text{F}/\text{m}^2) = 0.1647\text{pF} \quad g_{m3} = \sqrt{2 \cdot 50 \cdot 50 \cdot 50} = 500\mu\text{S}$$

$$\begin{aligned} V_{out} &= (g_{m4}V_1 - g_{m2}V_{gs2})Z_{out} = \left(\frac{g_{m4}g_{m1}V_{gs1}}{g_{m3} + sC_1} - g_{m2}V_{gs2} \right) Z_{out} \\ &= \left[\left(\frac{1}{s\frac{C_1}{g_{m3}} + 1} \right) \left(\frac{-g_{m1}V_{in}}{2} \right) - \frac{g_{m2}V_{in}}{2} \right] \left(\frac{1}{sC_L + g_{ds2} + g_{ds4}} \right) \\ &= -g_{md} \left(\frac{\frac{C_1}{s\frac{C_1}{g_{m3}} + 2}}{\frac{C_1}{s\frac{C_1}{g_{m3}} + 1}} \right) \left(\frac{1}{sC_2 + g_{ds2} + g_{ds4}} \right) \frac{V_{in}}{2} = -g_{md} \left(\frac{\frac{C_1}{s\frac{C_1}{g_{m3}} + 1}}{\frac{C_1}{s\frac{C_1}{g_{m3}} + 1}} \right) \left(\frac{1}{sC_2 + g_{ds2} + g_{ds4}} \right) V_{in} \end{aligned}$$

The small-signal output resistance and voltage gain is,

$$R_{out} = \frac{1}{g_{ds2} + g_{ds4}} = \frac{10^6}{50(0.05 + 0.04)} = \underline{\underline{222\text{k}\Omega}} \quad A_{vd} = -g_{m1}R_{out}$$

$$g_{m1} = g_{m1} = \sqrt{2 \cdot 110 \cdot 100 \cdot 50} = 1.049\text{mS} \Rightarrow A_{vd} = -g_{m1}R_{out} = (1.049)(222) = \underline{\underline{-233\text{V/V}}}$$

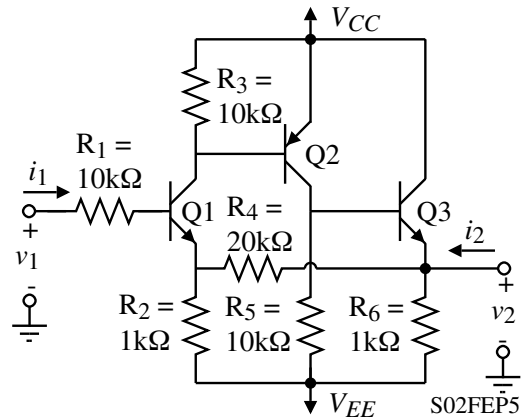
The roots of this circuit are,

$$p_1 = -\frac{g_{m3}}{C_1} = -\frac{500\mu\text{S}}{0.1647\text{pF}} = \underline{\underline{-3.036 \times 10^9\text{rps}}}, \quad z_1 = 2p_1 = \underline{\underline{-6.072 \times 10^9\text{rps}}},$$

$$\text{and } p_2 = -\frac{g_{ds2} + g_{ds4}}{C_2} = -\frac{1}{222\text{k}\Omega \cdot 1\text{pF}} = \underline{\underline{-4.504 \times 10^6\text{rps}}} \Rightarrow f_{-3\text{dB}} = \frac{4.504 \times 10^6}{2\pi} = \underline{\underline{717\text{kHz}}}$$

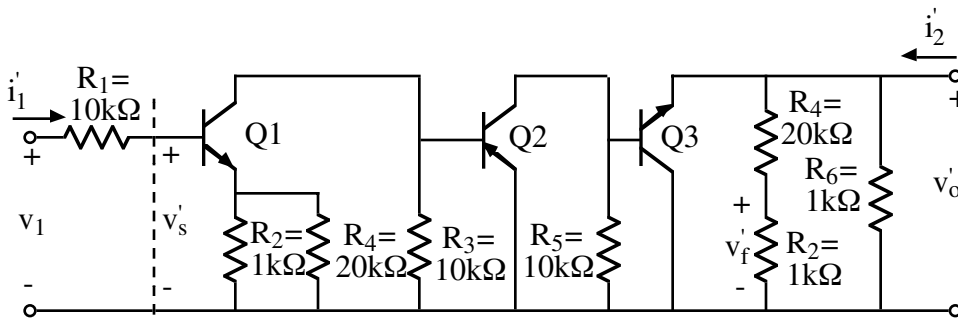
Problem 5 - (20 points - This problem is optional)

The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find v_2/v_1 , $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $\beta = 100$, $r_{\pi} = 5k\Omega$ and $r_o = \infty$.

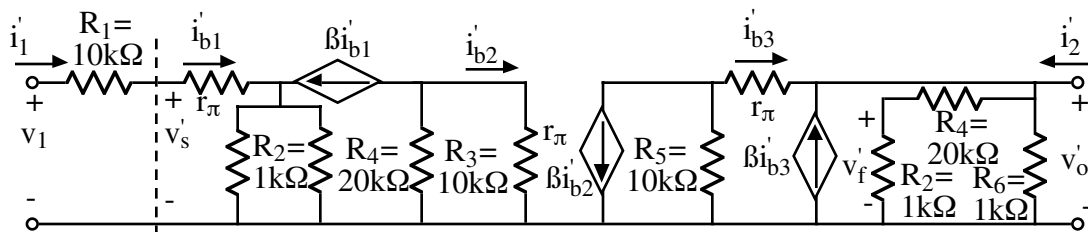


Solution

Open-loop, quasi-ac model:



Small-signal, open-loop model:



$$R_i = \frac{v_s'}{i_{b1}'} = r_{\pi} + (1+\beta)(R_2 \parallel R_4) = 5k\Omega + (101)(1k\Omega \parallel 20k\Omega) = 101.19k\Omega$$

$$a = \frac{v_o'}{v_s'} = \left(\frac{v_o'}{i_{b3}'}\right) \left(\frac{i_{b3}'}{i_{b2}'}\right) \left(\frac{i_{b2}'}{i_{b1}'}\right) \left(\frac{i_{b1}'}{v_s'}\right)$$

$$= (1+\beta)[R_6 \parallel (R_2+R_4)] \left(\frac{-\beta R_5}{R_5+r_{\pi}+(1+\beta)[R_6 \parallel (R_2+R_4)]}\right) \left(\frac{-\beta R_3}{R_3+r_{\pi}}\right) \left(\frac{1}{R_1}\right)$$

$$= [(101)(954.55)](-8.976)(-66.67)(1/101.19k\Omega) = 570.16 \text{ V/V}$$

$$f = \frac{R_2}{R_2+R_4} = \frac{1}{21} = 0.0476 \quad \text{and} \quad R_o = \frac{v_o'}{i_2'} = R_6 \parallel (R_2+R_4) \parallel \left(\frac{r_{\pi}+R_5}{1+\beta}\right) = 128.5\Omega$$

Closed-loop quantities are:

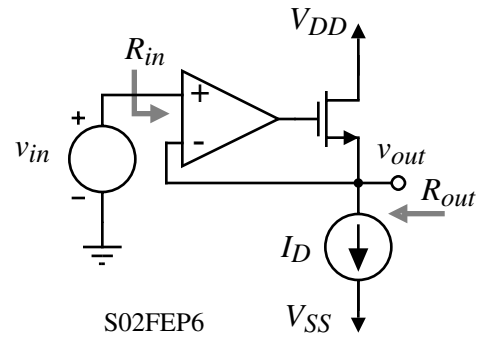
$$A_{vf} = \frac{v_o}{v_s} = \frac{a}{1+af} = \frac{570.16}{1+27.15} = 20.25, \quad R_{if} = (1+af)R_i = 2.848M\Omega$$

$$\therefore \boxed{R_{out} = R_{of} = \frac{R_o}{1+A_v\beta_f} = \frac{128.5\Omega}{28.15} = 4.565\Omega}$$

$$\boxed{R_{in} = R_1 + R_{if} = 2.858M\Omega} \quad \text{and} \quad \boxed{\frac{v_2}{v_1} = A_{vf} \frac{R_{if}}{R_1 + R_{if}} = 20.18 \text{ V/V}}$$

Problem 6 - (20 points - This problem is optional)

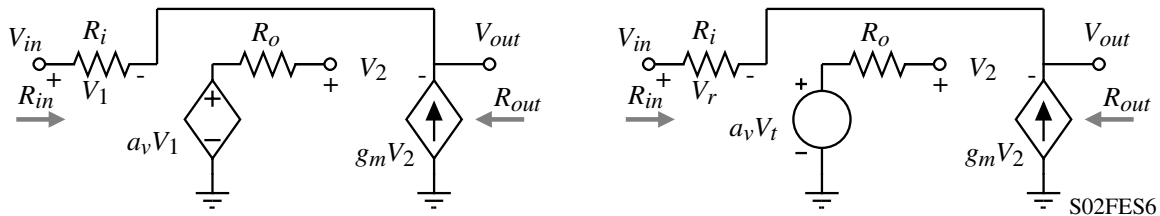
A voltage follower feedback circuit is shown. For the MOS transistor, $I_D = 0.5\text{mA}$, $K' = 180\mu\text{A}/\text{V}^2$, $r_{ds} = \infty$, and $W/L = 100$. Although, the bulk effect, g_{mbs} , should be considered, for simplicity ignore the bulk effects in this problem. For the op amp, assume that $R_i = 1\text{M}\Omega$, $R_o = 10\text{k}\Omega$, and $a_v = 1000$. Calculate the input resistance and output resistance using Blackman's formula given below.



$$R_{out} = R_{out} (\text{Controlled Source Gain}=0) \left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})} \right]$$

Solution

Circuit for calculating the return ratios.



Input Port:

$$R_{in}(a_v=0) = R_i + (1/g_m), \quad g_m = \sqrt{2 \cdot 500 \cdot 100 \cdot 180} = 4.243\text{mS}$$

$$R_{in}(a_v=0) = 1\text{M}\Omega + 236\Omega \approx 1\text{M}\Omega$$

$RR(\text{input port shorted})$:

$$V_r = -g_m R_i V_2 \quad \text{and} \quad V_2 = a_v V_t - g_m R_i V_2 \quad \rightarrow \quad V_r = \frac{-a_v g_m R_i}{1 + g_m R_i} V_t$$

$$RR(\text{input port shorted}) = -\frac{V_r}{V_t} = \frac{-a_v g_m R_i}{1 + g_m R_i} = -\frac{1000 \cdot 4.243\text{mS} \cdot 1\text{M}\Omega}{1 + 4.243\text{mS} \cdot 1\text{M}\Omega} = -999.8$$

$RR(\text{input port open})$:

$$RR(\text{input port open}) = 0 \quad \text{because} \quad V_r = 0$$

$$\therefore R_{in} = R_{in}(a_v=0) \left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})} \right] = 1\text{M}\Omega(1+999.8) = \underline{\underline{1000.8\text{M}\Omega}}$$

Output Port:

$$R_{out}(a_v=0) = R_i \parallel (1/g_m) \approx 236\Omega$$

$RR(\text{output port shorted})$:

$$RR(\text{output port shorted}) = 0 \quad \text{because} \quad V_r = 0$$

$RR(\text{output port open})$:

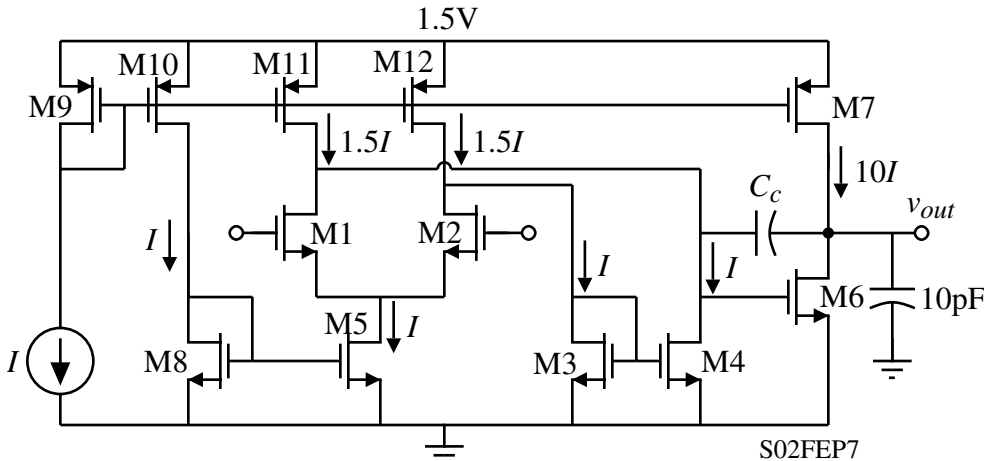
Same as the RR for the input port shorted.

$$\therefore R_{out} = R_{out}(a_v=0) \left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})} \right] = 236\Omega \left(\frac{1+0}{1+999.8} \right) = \underline{\underline{0.236\Omega}}$$

Problem 7 – (20 points – This problem is optional)

A CMOS op amp capable of operating from 1.5V power supply is shown. All device lengths are 1 μm and are to operate in the saturation region. Design all of the W values of every transistor of this op amp to meet the following specifications.

Slew rate = $\pm 10\text{V}/\mu\text{s}$	$V_{\text{out(max)}} = 1.25\text{V}$	$V_{\text{out(min)}} = 0.75\text{V}$
$V_{\text{ic(min)}} = 1\text{V}$	$V_{\text{ic(max)}} = 2\text{V}$	GB = 10MHz
Phase margin = 60° when the output pole = 2GB and the RHP zero = 10GB. Keep the mirror pole $\geq 10\text{GB}$ ($C_{\text{ox}} = 0.5\text{fF}/\mu\text{m}^2$).		



Your design should meet or exceed these specifications. Ignore bulk effects in this problem and summarize your W values to the nearest micron, the value of C_c (pF), and I (μA) in the following table. Use the following model parameters: $K_N' = 24\mu\text{A}/\text{V}^2$, $K_P' = 8\mu\text{A}/\text{V}^2$, $V_{TN} = -V_{TP} = 0.75\text{V}$, $\lambda_N = 0.01\text{V}^{-1}$ and $\lambda_P = 0.02\text{V}^{-1}$.

Solution

$$1.) p_2 = 2\text{GB} \Rightarrow g_{m6}/C_L = 2g_{m1}/C_c \text{ and } z = 10\text{GB} \Rightarrow g_{m6} = 10g_{m1}. \therefore \boxed{C_c = C_L/5 = 2\text{pF}}$$

$$2.) I = C_c \cdot \text{SR} = (2 \times 10^{-12}) \cdot 10^7 = 20\mu\text{A} \quad \therefore \boxed{I = 20\mu\text{A}}$$

$$3.) \text{GB} = g_{m1}/C_c \Rightarrow g_{m1} = 20\pi \times 10^6 \cdot 2 \times 10^{-12} = 40\pi \times 10^{-6} = 125.67\mu\text{S}$$

$$\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}^2}{2K_N(I/2)} = \frac{(125.67 \times 10^{-6})^2}{2 \cdot 24 \times 10^{-6} \cdot 10 \times 10^{-6}} = 32.9 \Rightarrow \boxed{W_1 = W_2 = 33\mu\text{m}}$$

$$4.) V_{\text{ic(min)}} = V_{\text{DS5(sat)}} + V_{\text{GS1}}(10\mu\text{A}) = 1\text{V} \rightarrow V_{\text{DS5(sat)}} = 1 - \sqrt{\frac{2 \cdot 10}{24 \cdot 33}} - 0.75 = 0.0908$$

$$V_{\text{DS5(sat)}} = 0.0908 = \sqrt{\frac{2 \cdot I}{K_N S_5}} \rightarrow W_5 = \frac{2 \cdot 20}{24 \cdot (0.0908)^2} = 201.9\mu\text{m} \quad \boxed{W_5 = 202\mu\text{m}}$$

$$5.) V_{\text{ic(max)}} = V_{\text{DD}} - V_{\text{SD11(sat)}} + V_{\text{TN}} = 1.5 - V_{\text{SD11(sat)}} + 0.75 = 2\text{V} \rightarrow V_{\text{SD11(sat)}} = 0.25\text{V}$$

$$V_{\text{SD11(sat)}} \leq \sqrt{\frac{2 \cdot 1.5I}{K_P \cdot S_{11}}} \rightarrow S_{11} = W_{11} \geq \frac{2 \cdot 30}{(0.25)^2 \cdot 8} = 120 \rightarrow \underline{\underline{W_{11} = W_{12} \geq 120\mu\text{m}}}$$

Problem 7 - Continued

6.) Choose $S_3(S_4)$ by satisfying $V_{ic}(\max)$ specification then check mirror pole.

$$V_{ic}(\max) \geq V_{GS3}(20\mu A) + V_{TN} \rightarrow V_{GS3}(20\mu A) = 1.25V \geq \sqrt{\frac{2 \cdot I}{K_N \cdot S_3}} + 0.75V$$

$$S_3 = S_4 = \frac{2 \cdot 20}{(0.5)^2 \cdot 2.24} = 6.67 \Rightarrow \boxed{W_3 = W_4 = 7\mu m}$$

7.) Check mirror pole ($p_3 = g_{m3}/C_{Mirror}$).

$$p_3 = \frac{g_{m3}}{C_{Mirror}} = \frac{g_{m3}}{2 \cdot 0.667 \cdot W_3 \cdot L_3 \cdot C_{ox}} = \frac{\sqrt{2 \cdot 24 \cdot 6.67 \cdot 20 \times 10^{-6}}}{2 \cdot 0.667 \cdot 6.67 \cdot 0.5 \times 10^{-15}} = 17.98 \times 10^9$$

which is much greater than 10GB (0.0628×10^9). Therefore, W_3 and W_4 are OK.

8.) $g_{m6} = 10g_{m1} = 1256.7\mu S$

$$a.) g_{m6} = \sqrt{2K_N S_6 I_0} \Rightarrow W_6 = 164.5\mu m$$

$$b.) V_{out}(\min) = 0.5 \Rightarrow V_{DS6}(\text{sat}) = 0.5 = \sqrt{\frac{2 \cdot 10I}{K_N S_6}} \Rightarrow W_6 = 66.67\mu m$$

Therefore, use $\boxed{W_6 = 165\mu m}$

Note: For proper mirroring, $S_4 = \frac{I_4}{I_6} S_6 = 8.25\mu m$ which is close enough to $7\mu m$.

9.) Use the $V_{out}(\max)$ specification to design W_7 .

$$V_{out}(\max) = 0.25V \geq V_{DS7}(\text{sat}) = \sqrt{\frac{2 \cdot 200\mu A}{8 \times 10^{-6} \cdot S_7}}$$

$$\therefore S_7 \geq \frac{400\mu A}{8 \times 10^{-6} (0.25)^2} \Rightarrow \boxed{W_7 = 800\mu m}$$

10.) Now to achieve the proper currents from the current source I gives,

$$S_9 = S_{10} = \frac{S_7}{10} = 80 \rightarrow \boxed{W_9 = W_{10} = 80\mu m}$$

and

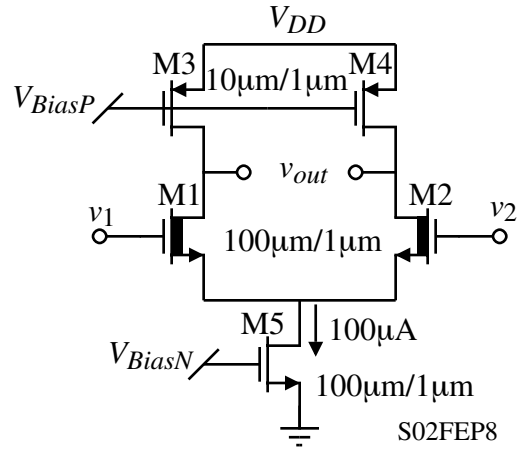
$S_{11} = S_{12} = \frac{1.5 \cdot S_7}{10} = 120 \rightarrow W_{11} = W_{12} = 120\mu m$. We saw in step 5 that W_{11} and W_{12} had to be greater than $120\mu m$ to satisfy $V_{ic}(\max)$. $\therefore \boxed{W_{11} = W_{12} = 120\mu m}$

11.) $P_{diss} = 15I \cdot 1.5V = 300\mu A \cdot 1.5V = 450\mu W$

C_c	I	$W_1=W_2$	$W_3=W_4$	$W_5=W_8$	W_6	W_7	$W_9=W_{10}$	$W_{11}=W_{12}$	P_{diss}
2pF	20 μ A	33 μ m	7 μ m	202 μ m	165 μ m	800 μ m	80 μ m	120 μ m	450 μ W

Problem 8 – (20 points – This problem is optional)

A differential CMOS amplifier using depletion mode input devices is shown. Assume that the normal MOSFETs parameters are $K_N' = 110\text{V}/\mu\text{A}^2$, $V_{TN} = 0.7\text{V}$, $\lambda_N = 0.04\text{V}^{-1}$ and for the PMOS transistors are $K_P' = 110\text{V}/\mu\text{A}^2$, $V_{TP} = 0.7\text{V}$, $\lambda_P = 0.04\text{V}^{-1}$. For the depletion mode NMOS transistors, the parameters are the same as the normal NMOS except that $V_{TN} = -0.5\text{V}$. (a.) What is the maximum input common-mode voltage, $V_{icm}^+(\text{max})$? (b.) What is the minimum input common-mode voltage, $V_{icm}^-(\text{min})$? (c.) What value of V_{DD} gives an $ICMR = 0.5V_{DD}$?

**Solution**

$$(a.) \quad V_{icm}^+(\text{max}) = V_{DD} - V_{SD3}(\text{sat}) - V_{DS1}(\text{sat}) + V_{GS1}(50\mu\text{A})$$

$$i_D = \frac{\beta}{2} (V_{GS1} - V_{T1})^2 \rightarrow V_{GS1} = \sqrt{\frac{2i_D}{\beta}} + V_{T1} = V_{DS1}(\text{sat}) + V_{T1}$$

$$\therefore V_{icm}^+(\text{max}) = V_{DD} - V_{SD3}(\text{sat}) + V_{T1} = V_{DD} - \sqrt{\frac{2I_{D3}}{\beta_3}} + V_{T1}$$

$$V_{icm}^+(\text{max}) = V_{DD} - 0.4472 - 0.5 = \underline{\underline{V_{DD} - 0.9472}}$$

$$(b.) \quad V_{icm}^-(\text{min}) = V_{DS5}(\text{sat}) + V_{GS1}(50\mu\text{A}) = V_{DS5}(\text{sat}) + V_{DS1}(\text{sat}) + V_{T1}$$

$$V_{icm}^-(\text{min}) = \sqrt{\frac{2I_{D5}}{\beta_5}} + \sqrt{\frac{2I_{D1}}{\beta_1}} + V_{T1} = 0.1348 + 0.0953 - 0.5 = \underline{\underline{-0.2698\text{V}}}$$

$$(c.) \quad ICMR = V_{icm}^+(\text{max}) - V_{icm}^-(\text{min}) = V_{DD} - 0.9472 + 0.2698 = V_{DD} - 0.6774$$

$$\therefore V_{DD} - 0.6774 = 0.5V_{DD} \rightarrow V_{DD} = 2(0.6774) = \underline{\underline{1.355\text{V}}}$$