Homework Assignment No. 3 - Solutions

Problem 1 – (10 points) (Prob. 7.1 of 3rd and 4th edition)

(a) Transistor parameters are

$$r_\pi = \frac{\beta_o}{g_m} = 200 \times 52 = 10.4\,k\Omega$$

$$\tau_T = \frac{1}{2\pi f_T} = 318\,ps$$

$$C_\pi + C_\mu = g_m \tau_T = \frac{1}{52} \times 318 = 6.12\,pF$$

$$\therefore C_\pi = 6.1 - 0.3 = 5.8\,pF$$

$$C_M = (1 + g_m R_L) C_\mu$$

$$= (1 + \frac{3000}{52}) \times 0.3 = 17.6\,pF$$

In (7.12)

$$f_{-3dB} = \frac{1}{2\pi} \frac{5000+300+10400}{(5000+300)\times 10400} \frac{10^{12}}{5.8+17.6}$$

$$= 1.94\,MHz \quad Hz$$

(b) From (7.27)

$$P_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{R C_\pi} + \frac{1}{R_L C_\pi} + \omega_T\right)$$

$$R = (R_S + r_b) /\!/ r_\pi$$

$$= 5300 \| 10400 = 3511\,\Omega$$

$$\therefore P_2 = -\left(\frac{10^{12}}{3000\times 0.3} + \frac{10^{12}}{3511\times 5.8} + \frac{10^{12}}{3000\times 5.8} + 2\pi \times 500 \times 10^6\right)$$

$$= -(11.1 + 0.49 + 0.57 + 31.4) \times 10^8\ rad/sec$$

$$= -43.6 \times 10^8\ rad/sec$$

$$= -693\,MHz$$

```
COMMON EMITTER GAIN STAGE
VCC 1 0 5V
RL 1 2 3K
Q1 2 3 0 NPN
RS 4 3 5K
VI 4 0 0.7696 AC
.TF V(2) VI
.PLOT AC VDB(2)
.PLOT AC VP(2)
.AC DEC 10 100K 1GIG
.MODEL NPN NPN IS=1E-16A BF=200
+ RB=300 CJC=0.3PF CJS=0 TF=302PS
* ASSUME CJE SMALL COMPARED TO CB
.OPTIONS NOPAGE NOMOD
.WIDTH OUT=80
.OP
.END
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```
0**** BIPOLAR JUNCTION TRANSISTORS
0        Q1
0MODEL   NPN
IB      2.50E-06
IC      5.01E-04
VBE        0.757
VBC       -2.740
VCE        3.497
BETADC   200.000
GM      1.94E-02
RPI     1.03E+04
RX      3.00E+02
RO      1.00E+12
CPI     5.85E-12
CMU     1.81E-13
CBX     0.00E+00
CCS    -0.00E+00
BETAAC   200.000
FT      5.11E+08

0****    SMALL-SIGNAL CHARACTERISTICS
0   V(2)/VI                      = -3.840D+01
0   INPUT RESISTANCE AT VI       =  1.563D+04
0   OUTPUT RESISTANCE AT V(2)    =  3.000D+03

0****    AC ANALYSIS
   FREQ     VDB(2)
            -4.0D+01   -2.0D+01    0.0D+00    2.0D+01  4.0D+01
1.000D+05  3.168D+01
1.259D+05  3.168D+01
1.585D+05  3.167D+01
1.995D+05  3.166D+01
2.512D+05  3.165D+01
3.162D+05  3.163D+01
3.981D+05  3.160D+01
5.012D+05  3.154D+01
6.310D+05  3.146D+01
7.943D+05  3.133D+01
1.000D+06  3.114D+01
1.259D+06  3.085D+01
1.585D+06  3.043D+01
1.995D+06  2.983D+01
2.512D+06  2.903D+01
3.162D+06  2.800D+01
3.981D+06  2.674D+01
5.012D+06  2.529D+01
6.310D+06  2.367D+01
7.943D+06  2.193D+01
1.000D+07  2.011D+01
1.259D+07  1.822D+01
1.585D+07  1.629D+01
1.995D+07  1.434D+01
2.512D+07  1.237D+01
3.162D+07  1.038D+01
3.981D+07  8.391D+00
5.012D+07  6.392D+00
6.310D+07  4.388D+00
7.943D+07  2.377D+00
1.000D+08  3.558D-01
1.259D+08 -1.679D+00
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7.1 - Cont'd

```
1.585D+08 -3.734D+00 .
1.995D+08 -5.821D+00 .
2.512D+08 -7.955D+00 .
3.162D+08 -1.016D+01 .
3.981D+08 -1.247D+01 .
5.012D+08 -1.491D+01 .
6.310D+08 -1.754D+01 .
7.943D+08 -2.037D+01 .
1.000D+09 -2.343D+01 .
```

```
Y
0****   AC ANALYSIS
   FREQ     VP(2)
            0.0D+00    5.0D+01    1.0D+02    1.5D+02  2.0D+02
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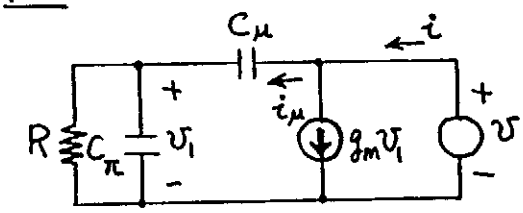
```
1.000D+05  1.779D+02 .
1.259D+05  1.774D+02 .
1.585D+05  1.767D+02 .
1.995D+05  1.758D+02 .
2.512D+05  1.747D+02 .
3.162D+05  1.734D+02 .
3.981D+05  1.717D+02 .
5.012D+05  1.696D+02 .
6.310D+05  1.670D+02 .
7.943D+05  1.637D+02 .
1.000D+06  1.598D+02 .
1.259D+06  1.552D+02 .
1.585D+06  1.498D+02 .
1.995D+06  1.437D+02 .
2.512D+06  1.372D+02 .
3.162D+06  1.306D+02 .
3.981D+06  1.242D+02 .
5.012D+06  1.182D+02 .
6.310D+06  1.130D+02 .
7.943D+06  1.084D+02 .
1.000D+07  1.046D+02 .
1.259D+07  1.013D+02 .
1.585D+07  9.865D+01 .
1.995D+07  9.637D+01 .
2.512D+07  9.440D+01 .
3.162D+07  9.267D+01 .
3.981D+07  9.106D+01 .
5.012D+07  8.952D+01 .
6.310D+07  8.795D+01 .
7.943D+07  8.627D+01 .
1.000D+08  8.440D+01 .
1.259D+08  8.225D+01 .
1.585D+08  7.971D+01 .
1.995D+08  7.667D+01 .
2.512D+08  7.304D+01 .
3.162D+08  6.869D+01 .
3.981D+08  6.356D+01 .
5.012D+08  5.765D+01 .
6.310D+08  5.105D+01 .
7.943D+08  4.396D+01 .
1.000D+09  3.666D+01 .
```

* DOMINANT POLE @ (DC PHASE - 45 DEGREES) = 135 DEGREES
* DOMINANT POLE = 2.5 MEGAHERTZ
* SECOND POLE @ (DC PHASE - 135 DEGREES) = 45 DEGREES
* SECOND POLE = 794 MEGAHERTZ

7.2



$$R = (R_s + r_b) \parallel r_\pi$$

$$v_1 = \frac{Z_\pi}{Z_\pi + \frac{1}{C_\mu s}} v \quad \text{, where } Z_\pi = \frac{R}{1+RC_\pi s}$$

$$g_m v_1 = g_m \frac{Rv}{R + (1+RC_\pi s)\frac{1}{C_\mu s}}$$

$$= \frac{g_m R C_\mu s}{1 + RC_\pi s + RC_\mu s} v$$

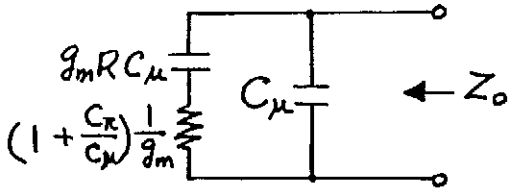
$$i = i_\mu + g_m v_1$$

$$\therefore \frac{1}{Z_o} = \frac{i}{v} = \frac{i_\mu}{v} + \frac{g_m v_1}{v}$$

$$= \frac{i_\mu}{v} + \frac{g_m R C_\mu s}{1 + RC_\pi s + RC_\mu s}$$

But $\frac{i_\mu}{v} \simeq C_\mu s$, because $C_\pi >> C_\mu$

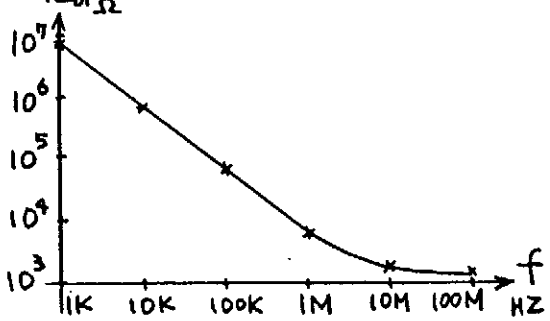
$$\therefore Z_o \simeq \frac{1}{C_\mu s} \parallel \left(\frac{1}{g_m R C_\mu s} + \frac{C_\pi}{C_\mu} \frac{1}{g_m} + \frac{1}{g_m} \right)$$



$C_\mu = 0.3\,pF$

$g_m R C_\mu = \frac{3511}{52} \times 0.3 = 20.3\,PF$

$(1 + \frac{C_\pi}{C_\mu})\frac{1}{g_m} = (1 + \frac{5.8}{0.3})52 = 1057\,\Omega$



7.8

At low freq.

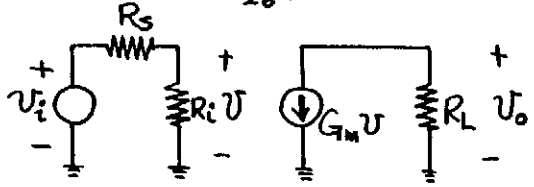
$$\frac{v_o}{v_i} = -\frac{r_\pi}{r_\pi + R_s} g_m R_L$$
$$= -\frac{5.2K}{5.2K+10K}\frac{5000}{26} = -65.8$$

Zero value time constant

$$R_{\pi o} = r_\pi \| (R_s + r_b)$$
$$= 5.2K \| 10K = 3.42K\Omega$$
$$R_{\mu o} = R_{\pi o} + R_L + g_m R_L R_{\pi o}$$
$$= 3.42 + 5 + \frac{5000}{26} \times 3.42$$
$$= 666\,K\Omega$$
$$R_{cso} = R_L = 5K\Omega$$
$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$
$$= \frac{1}{2\pi \times 26 \times 600 \times 10^6} = 10.2\,pF$$
$$\therefore C_\pi = 10\,pF$$
$$\therefore C_\pi R_{\pi o} = 10 \times 3.42 = 34.2\ ns$$
$$C_\mu R_{\mu o} = 0.2 \times 666 = 133.2\ ns$$
$$C_{cs} R_{cso} = 1 \times 5 = 5\ ns$$
$$\therefore \Sigma T_o = 34.2 + 133.2 + 5 = 172.4\ ns$$
$$\therefore f_{-3dB} = \frac{1}{2\pi \Sigma T_o} = 0.923\ MHz$$

7.9

$$G_m = \frac{g_m}{1 + g_m R_E}$$
$$= \frac{1}{26}\frac{1}{1+\frac{300}{26}} = 3.07 \times 10^{-3}\ A/V$$
$$R_i = r_\pi (1 + g_m R_E)$$
$$= 5.2\left(1 + \frac{300}{26}\right) = 65.2\,K\Omega$$

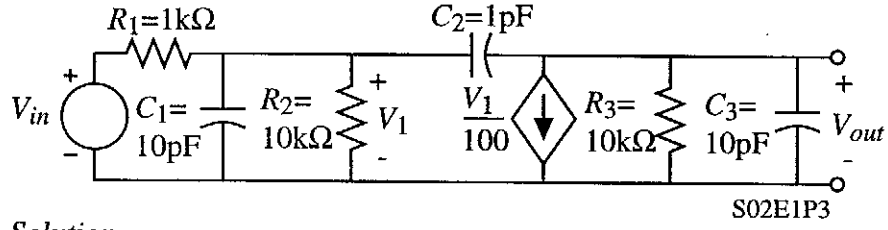


$$\frac{v_o}{v_i} = -\frac{R_i}{R_s + R_i} G_m R_L$$
$$= -\frac{65.2}{10+65.2} \times 3.07 \times 10^{-3} \times 5000$$
$$= -13.31$$

$$R_{\mu o} = R_x + R_L + G_m R_x R_L$$
$$R_x = R_i \| R_s = 10 \| 65.2 = 8.67K\Omega$$
$$\therefore R_{\mu o} = 8.67 + 5 + 3.07 \times 5 \times 8.67$$
$$= 147\,K\Omega$$
$$\therefore C_\mu R_{\mu o} = 0.2 \times 147 = 29.4\ ns$$
$$C_{cs} R_{cso} = 1 \times 5 = 5\ ns$$
$$R_{\pi o} = r_\pi \left\| \frac{R_s + r_b + R_E}{1 + g_m R_E} \right.$$
$$= 5.2K \left\| \frac{10 + 0.3}{1 + \frac{300}{26}} K = 709\,\Omega \right.$$
$$C_\pi R_{\pi o} = 10 \times 0.709 = 7.09\ ns$$
$$\therefore \Sigma T_o = 29.4 + 5 + 7.09 = 41.49\ ns$$
$$\therefore f_{-3dB} = \frac{1}{2\pi \Sigma T_o} = 3.84\ MHz$$

Problem 5 – (10 points)

Find the midband voltage gain and the –3dB frequency in Hertz for the circuit shown.



Solution

The midband gain is given as,

$$\frac{V_{out}}{V_{in}} = -\left(\frac{10k\Omega}{100}\right)\left(\frac{10k\Omega}{11k\Omega}\right) = \underline{\underline{-90.91V/V}}$$

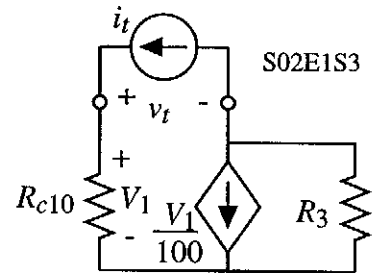
To find the –3dB frequency requires finding the 3 open-circuit time constants.

R_{C10}:

$R_{C10} = 1k\Omega \| 10k\Omega = 0.9091k\Omega$ → $R_{C10}C_1 = 0.9091 \cdot 10ns = 9.09ns$

R_{C20}:

$$v_t = i_t R_{C10} + R_3(i_t + 0.01V_1)$$
$$= i_t(R_{C10} + R_3 + 0.01R_{C10}R_3)$$
$$\therefore R_{C20} = R_{C10} + R_3 + 0.01R_{C10}R_3$$
$$= 0.9091 + 10(1+0.01 \cdot 909.1)k\Omega = 101.82k\Omega$$



$$R_{C20}C_2 = 101.82 \cdot 1ns = 101.82ns$$

R_{C30}:

$$R_{C30} = 10k\Omega \rightarrow R_{C30}C_3 = 10 \cdot 10ns = 100ns$$

$$\Sigma T_0 = (9.091 + 101.82 + 100)ns = 210.91ns \rightarrow \omega_{-3dB} = \frac{1}{\Sigma T_0} = 4.74x10^6 \text{ rad/s}$$

$$f_{-3dB} = \frac{4.74x10^6}{2\pi} = \underline{\underline{754.6kHz}}$$