Homework Assignment No. 12 - Solutions

Problem 1 - (10 points) - Problem 7.3-7 of Allen and Holberg, 2nd edition

(a.) If all transistors in Fig. 7.3-12 have a dc current of 50µA and a *W/L* of 10µm/1µm, find the gain of the common mode feedback loop. (b.) If the output of this amplifier is cascoded, then repeat part (a.).

Solution

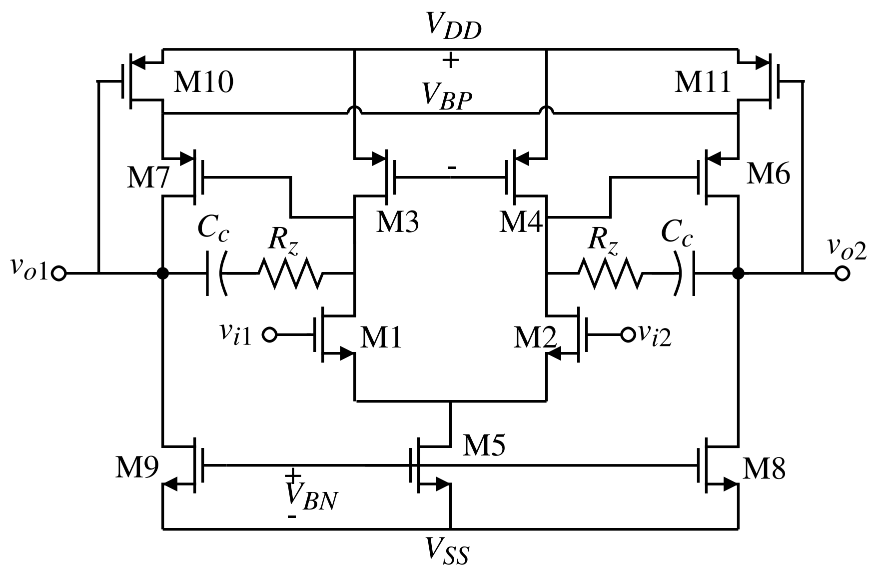


Figure 7.3-12 Two-stage, Miller, differential-in, differential-out op amp with common-mode stabilization.

The loop gain of the common-mode feedback loop is,

$$\text{CMFB Loop gain} \approx -\frac{g_{m10}}{g_{ds9}} = -g_{m10}r_{ds9} \quad \text{or} \quad -\frac{g_{m11}}{g_{ds8}} = -g_{m11}r_{ds8}$$

With I_D = 50µA and W/L = 10µm/1µm, $g_{m10} = \sqrt{\frac{2K_P'WI_D}{L}} = \sqrt{2\cdot 50\cdot 10\cdot 50)}$ = 223.6µS,

$$r_{dsN} = \frac{1}{\lambda_N I_D} = \frac{25}{50\mu A} = 0.5\text{M}\Omega \quad \text{and} \quad r_{dsP} = \frac{1}{\lambda_P I_D} = \frac{20}{50\mu A} = 0.4\text{M}\Omega$$

∴ **CMFB Loop gain ≈ $-g_{m10}r_{ds9}$ = -223.6(0.5) = -111.8V/V**

If the output is cascoded, the gain becomes,

$$\text{CMFB Loop gain with cascoding} \approx -\frac{g_{m10}}{g_{ds9}}\ g_m(\text{cascode})r_{ds}(\text{cascode})$$

$$= -g_{m10}\{[r_{ds9}\ g_m(\text{cascode})r_{ds}(\text{cascode})]||[g_{m7}r_{ds7}\ (r_{ds10}||r_{ds10}]\}$$

$$g_{mP} = \sqrt{\frac{2K_N'WI_D}{L}} = \sqrt{2\cdot 110\cdot 10\cdot 50)} = 331.67\mu S$$

$$= -(223.6)[(0.5\cdot 331.67\cdot 0.5)||(223.6)(0.4)(0.2)] = 223.6(14.7) = -3{,}290 \text{ V/V}$$

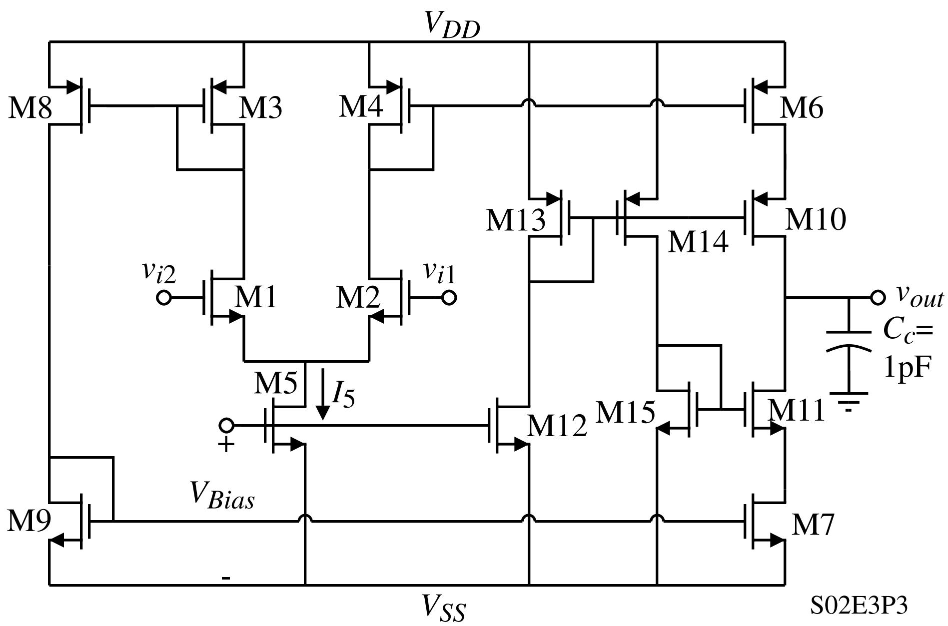
∴ **CMFB Loop gain with cascoding ≈ -3.290V/V**

Problem 2 – (10 points)

Calculate the small-signal voltage gain, the SR (C_L = 1pF), and the P_{diss} for the op amp shown where I_5 = 100nA and all transistors M1-M11 have a W/L of 10µm/1µm and V_{DD} = -V_{SS} = 1.5V. If the minimum voltage across the drain-source of M6 and M7 are to be 0.1V, design the W/L ratios of M12-M15 that give the maximum plus and minus output voltage swing assuming that transistors M12 and M15 have a current of 50nA. The transistors are working in weak inversion and are modeled by the large signal model of

$$i_D = \frac{W}{L} I_{DO} \exp\left(\frac{v_{GS}}{nV_t}\right)$$

where I_{DO} = 2nA for PMOS and NMOS and n_P = 2.5 and n_N = 1.5. Assume V_t = 26mV and λ_N=0.4V^{-1} and λ_P=0.5V^{-1}.



S02E3P3

Soluton

The small-signal voltage gain, A_v, is $g_{m1}R_{out}$ (see end of solution) where,

$$R_{out} = (r_{ds6}g_{m10}r_{ds10})||(r_{ds7}g_{m11}r_{ds11})$$

With the currents and W/L ratios of transistors M1 through M11 known, we get

$$g_{m1} = g_{m11} = \frac{50\text{nA}}{1.5\cdot 26\text{mV}} = 1.282\mu\text{S} \quad \text{and} \quad r_{ds7} = r_{ds11} = \frac{10^9}{0.04\cdot 50} = 0.5\text{x}10^9\Omega$$

$$g_{m10} = \frac{50\text{nA}}{2.5\cdot 26\text{mV}} = 0.769\mu\text{S} \text{ and } \quad r_{ds6} = r_{ds10} = \frac{10^9}{0.05\cdot 50} = 0.4\text{x}10^9\Omega$$

$$R_{out} = (0.4\text{x}10^9\cdot 1.282\text{x}10^{-6}\cdot 0.4\text{x}10^9)||(0.5\text{x}10^9\cdot 0.769\text{x}10^{-6}\cdot 0.5\text{x}10^9) = 9.924\text{x}10^{10}\Omega$$

$$\therefore \quad A_v = 1.282\text{x}10^{-6}\cdot 9.924\text{x}10^{10} = \underline{\underline{127{,}726 \text{ V/V}}}$$

$$SR = \frac{100\text{nA}}{1\text{pF}} = \underline{\underline{0.1\text{V}/\mu\text{s}}}$$

$$P_{diss} = 3\text{V}(50\text{nA}\cdot 6) = \underline{\underline{0.9\mu\text{W}}}$$

Problem 2 - Continued

Design of the *W/L*'s of M12 through M15:

To get 50nA in M12 means the $W_{12}/L_{12} = 0.5(W_5/L_5) = \underline{5\mu m/1\mu m}$

M15:

$$V_{GS11} = n_N V_t \ln\left(\frac{I_D}{I_{DO}(W/L)}\right) = 1.5 \cdot 0.026 \ln\left(\frac{50}{2\cdot 10}\right) = 0.0357\text{V}$$

$$\therefore \qquad V_{GS15} = 0.1 + 0.0357 = 0.1357\text{V} \rightarrow \frac{W_{15}}{L_{15}} = \frac{50\text{nA}}{2\text{nA}\cdot\exp\left(\frac{135.7}{1.5\cdot 26}\right)} = \underline{0.77\mu m/1\mu m}$$

M13 and M14:

$$V_{SG10} = n_P V_t \ln\left(\frac{I_D}{I_{DO}(W/L)}\right) = 2.5 \cdot 0.026 \ln\left(\frac{50}{2\cdot 10}\right) = 0.0596\text{V}$$

$$\therefore \qquad V_{GS13} = 0.1 + 0.0596 = 0.1596\text{V} \rightarrow \frac{W_{13}}{L_{13}} = \frac{50\text{nA}}{2\text{nA}\cdot\exp\left(\frac{159.6}{1.5\cdot 26}\right)} = 2.146\mu m/1\mu m$$

Thus

$$\frac{W_{13}}{L_{13}} = \frac{W_{14}}{L_{14}} = \underline{2.146\mu m/1\mu m}$$

Comments on the small-signal gain:

It is much easier to use the expression $g_{m1}R_{out}$ for the small-signal voltage gain. However, some prefer the following expression,

$$v_{out} = \left(\frac{g_{m1}\cdot g_{m8}\cdot g_{m7}}{2g_{m3}\cdot g_{m9}} + \frac{g_{m2}\cdot g_{m6}}{2g_{m4}}\right) R_{out}$$

which is equivalent since $g_{m3}=g_{m8}$, $g_{m7}=g_{m9}$, $g_{m4}=g_{m6}$, and $g_{m1}=g_{m2}$.

<u>Problem 3 – (10 points) - Problem 7.4-3 of Allen and Holberg, 2nd edition</u>

Derive Eq. (17). If $A = 2$, at what value of v_{in}/nV_t will $i_{out} = 5I_5$ or $5I_b$ if b=1 ?

<u>Solution</u>

Start with the following relationships:

$$i_1 + i_2 = I_5 + A(i_2 - i_1) \qquad \text{Eq. (15)}$$

and $$\frac{i_2}{i_1} = \exp\left(\frac{v_{in}}{nV_t}\right) \qquad \text{Eq. (16)}$$

Defining $i_{out} = b(i_2 - i_1)$ solve for i_2 and I_1.

$$i_1 + i_1 \exp\left(\frac{v_{in}}{nV_t}\right) = I_5 + Ai_1 \exp\left(\frac{v_{in}}{nV_t}\right) - Ai_1$$

or $$i_1[(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)] = I_5 \quad \rightarrow \quad i_1 = \frac{I_5}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

Similarly for i_2,

$$i_1 = \frac{I_5 \exp\left(\frac{v_{in}}{nV_t}\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

$$\therefore \quad i_{out} = b(i_2 - I_1) = i_{out} = (i_2 - I_1) = \frac{I_5\left(\exp\left(\frac{v_{in}}{nV_t}\right) - 1\right)}{(1+A) + (1-A)\exp\left(\frac{v_{in}}{nV_t}\right)} \qquad \text{Eq. (17)}$$

Setting $i_{out} = 5I_5$ and solving for $\frac{v_{in}}{nV_t}$ gives,

$$5[3 - \exp\left(\frac{v_{in}}{nV_t}\right)] = \exp\left(\frac{v_{in}}{nV_t}\right) - 1 \quad \rightarrow \quad 16 = 6\exp\left(\frac{v_{in}}{nV_t}\right) \quad \rightarrow \quad \exp\left(\frac{v_{in}}{nV_t}\right) = 2.667$$

$$\therefore \quad \frac{v_{in}}{nV_t} = \ln(2.667) = \underline{\underline{0.9808}}$$

Problem 4 - (10 points) - Problem 7.5-5 of Allen and Holberg, 2nd edition

Find the equivalent rms noise voltage of the op amp designed in Example 6.5-2 over a bandwidth of 1Hz to 100kHz. Use the values for *KF* of Example 7.5-1.

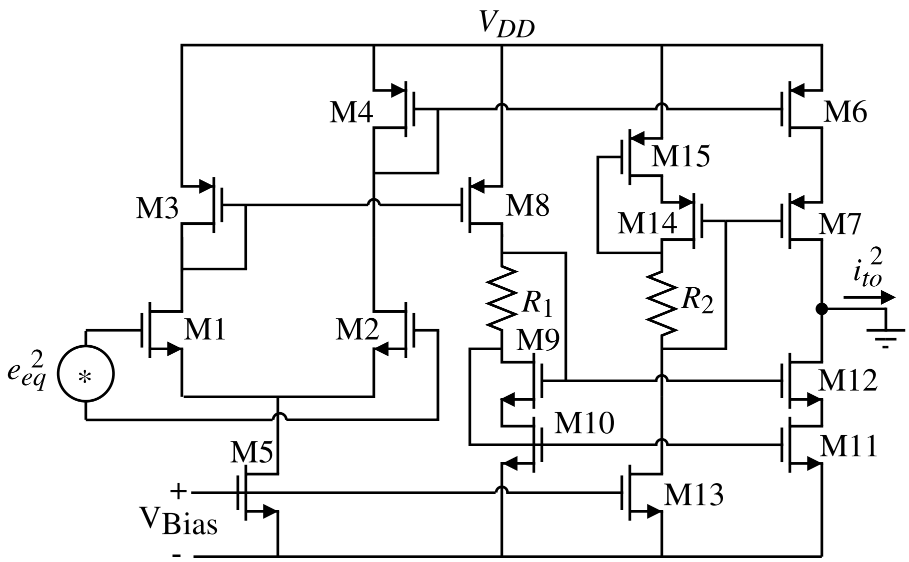


Fig. S7.5-05

Solution

The circuit for this amplifier is shown.

The *W/L* ratios in microns are:

$S_1 = S_2 = 12/1$

$S_3 = S_4 = 16/1$

$S_5 = 7/1$

$S_5 = 8.75/1$

$S_6 = S_7 = S_8 = S_{14} = S_{15} = 40/1$

$S_9 = S_{10} = S_{11} = S_{12} = 18.2/1$

Find the short circuit noise current at the output, i_{to}^2, due to each noise-contributing transistor in the circuit (we will not includeM7, M9, M12 and M14 because they are cascodes and their effective g_m is small. The result is,

$$i_{to}^2 = 2g_{m1}^2 e_{n1}^2\left(\frac{g_{m8}^2}{g_{m3}^2}\right) + 2g_{m8}^2 e_{n3}^2 + 2g_{m8}^2 e_{n8}^2 + + 2g_{m11}^2 e_{n10}^2$$

where we have assumed that $g_{m1}=g_{m2}$, $g_{m3}=g_{m4}$, $g_{m6}=g_{m8}$, and $g_{m10}=g_{m11}$ and $e_{n1}=e_{n2}$, $e_{n3}=e_{n4}$, $e_{n6}=e_{n8}$, and $e_{n10}=e_{n11}$. Dividing i_{to}^2 by the transconductance gain gives

$$e_{eq}^2 = \frac{i_{to}^2}{g_{m1}^2 g_{m8}^2 / g_{m3}^2} = 2e_{n1}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n3}^2 + 2\left(\frac{g_{m3}^2}{g_{m1}^2}\right)e_{n8}^2 + 2\left(\frac{g_{m3}^2 g_{m11}^2}{g_{m1}^2 g_{m8}^2}\right)e_{n10}^2$$

The values of the various parameters are:

$g_{m1} = 251\mu S$, $g_{m3} = 282.5\mu S$, $g_{m8} = 707\mu S$, and $g_{m11} = 707\mu S$.

Problem 4 - Problem 7.5-05 of Allen and Holberg, 2nd edition – Continued

$$\therefore \qquad e_{eq}^2 = 2e_{n1}^2\left[1 + 1.266\left(\frac{e_{n3}^2}{e_{n1}^2} + \frac{e_{n8}^2}{e_{n1}^2} + \frac{e_{n10}^2}{e_{n1}^2}\right)\right]$$

1/f Noise:

Using the results of Ex. 7.5-1 we get $B_N = 7.36\text{x}10^{-22}(\text{V}\cdot\text{m})^2$ and $B_p = 2.02\text{x}10^{-22}(\text{V}\cdot\text{m})^2$

$$e_{n1}^2 = \frac{B_N}{fW_1L_1} = \frac{7.36\text{x}10^{-22}}{f\cdot 12\text{x}10^{-12}} = \frac{6.133\text{x}10^{-11}}{f}\ V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_3L_3} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 16} = 0.2058$$

$$\frac{e_{n8}^2}{e_{n1}^2} = \frac{B_P\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_8L_3} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{2.02\cdot 12}{7.36\cdot 40} = 0.0823$$

$$\frac{e_{n10}^2}{e_{n1}^2} = \frac{B_N\cdot f\cdot W_1L_1}{B_N\cdot f\cdot W_{10}L_{10}} = \frac{B_P\cdot W_1L_1}{B_N\cdot W_3L_3} = \frac{12}{18.2} = 0.6593$$

$$e_{eq}^2 = 2\,\frac{6.133\text{x}10^{-11}}{f}\,[1+1.266(0.2058+0.0823+0.6593)] = 2\,\frac{6.133\text{x}10^{-11}}{f}\,2.1995$$

$$e_{eq}^2 = \frac{2.1995\text{x}10^{-10}}{f}\ V^2/\text{Hz}$$

Thermal noise:

$$e_{n1}^2 = \frac{8kT}{3g_{m1}} = \frac{8\cdot 1.38\text{x}10^{-23}\cdot 300}{3\cdot 251\text{x}10^{-6}} = 4.398\text{x}10^{-17}\ V^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m3}} = \frac{251}{282.4} = 0.8888 \quad \text{and} \quad \frac{e_{n8}^2}{e_{n1}^2} = \frac{e_{n10}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m8}} = \frac{251}{707} = 0.0.355$$

The corner frequency is $f_c = 2.698\text{x}10^{-10}/2.66\text{x}10^{-16} = 1.01\text{x}10^6$ Hz. Therefore in a 1Hz to 100kHz band, the noise is 1/*f*. Solving for the *rms* value gives,

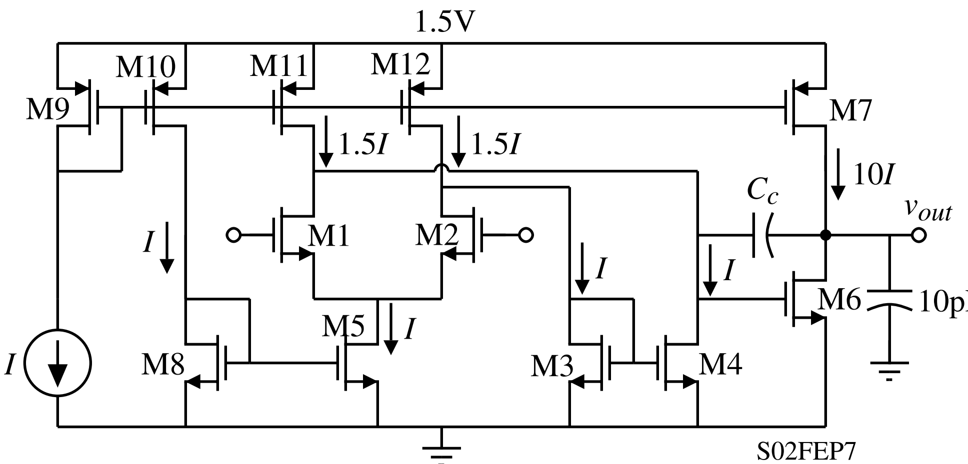
$$V_{eq}^2\,(\text{rms}) = \int_1^{100,000} \frac{2.698\text{x}10^{-10}}{f}df = 2.698\text{x}10^{-10}[\ln(100,000)-\ln(1)] = 3.1062\text{x}10^{-9}\ \text{V}^2(\text{rms})$$

$V_{eq}(\text{rms})$ = 55.73μV(rms)

Problem 5 - (10 points)

A CMOS op amp capable of operating from 1.5V power supply is shown. All device lengths are 1μm and are to operate in the saturation region. Design all of the W values of every transistor of this op amp to meet the following specifications.

Slew rate = ±10V/μs	V_{out}(max) = 1.25V	V_{out}(min) = 0.75V
V_{ic}(min) = 1V	V_{ic}(max) = 2V	GB = 10MHz
Phase margin = 60° when the output pole = 2GB and the RHP zero = 10GB. Keep the mirror pole ≥ 10GB (C_{ox} = 0.5fF/μm²).		



Your design should meet or exceed these specifications. Ignore bulk effects in this problem and summarize your W values to the nearest micron, the value of C_c(pF), and I(μA) in the following table. Use the following model parameters: K_N'=24μA/V², K_P' = 8μA/V², $V_{TN} = -V_{TP}$ = 0.75V, $\lambda_N = 0.01V^{-1}$ and $\lambda_P = 0.02V^{-1}$.

Solution

1.) $p_2 = 2GB \Rightarrow g_{m6}/C_L = 2g_{m1}/C_c$ and $z = 10GB \Rightarrow g_{m6} = 10g_{m1}$. $\therefore$ $\boxed{C_c = C_L/5 = 2pF}$

2.) $I = C_c \cdot SR = (2\text{x}10^{-12}) \cdot 10^7 = 20\mu A$ $\therefore$ $\boxed{I = 20\mu A}$

3.) $GB = g_{m1}/C_c \Rightarrow g_{m1} = 20\pi\text{x}10^6 \cdot 2\text{x}10^{-12} = 40\pi\text{x}10^{-6} = 125.67\mu S$

$$\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}^2}{2K_N(I/2)} = \frac{(125.67\text{x}10^{-6})^2}{2 \cdot 24\text{x}10^{-6} \cdot 10\text{x}10^{-6}} = 32.9 \Rightarrow \boxed{W1 = W_2 = 33\mu m}$$

4.) $V_{ic}(min) = V_{DS5}(sat.) + V_{GS1}(10\mu A) = 1V \rightarrow V_{DS5}(sat.) = 1 - \sqrt{\frac{2 \cdot 10}{24 \cdot 33}} - 0.75 = 0.0908$

$$V_{DS5}(sat) = 0.0908 = \sqrt{\frac{2 \cdot I}{K_N S_5}} \rightarrow W_5 = \frac{2 \cdot 20}{24 \cdot (0.0908)^2} = 201.9\mu m \quad \boxed{W_5 = 202\mu m}$$

5.) $V_{ic}(max) = V_{DD} - V_{SD11}(sat) + V_{TN} = 1.5 - V_{SD11}(sat) + 0.75 = 2V \rightarrow V_{SD11}(sat) = 0.25V$

$$V_{SD11}(sat) \le \sqrt{\frac{2 \cdot 1.5I}{K_P \cdot S_{11}}} \rightarrow S_{11} = W_{11} \ge \frac{2 \cdot 30}{(0.25)^2 \cdot 8} = 120 \rightarrow \underline{\underline{W_{11} = W_{12} \ge 120\mu m}}$$

Problem 5 - Continued

6.) Choose $S_3(S_4)$ by satisfying $V_{ic}(max)$ specification then check mirror pole.

$$V_{ic}(max) \geq V_{GS3}(20\mu A) + V_{TN} \rightarrow V_{GS3}(20\mu A) = 1.25V \geq \sqrt{\frac{2\cdot I}{K_N\cdot S_3}} + 0.75V$$

$$S_3 = S_4 = \frac{2\cdot 20}{(0.5)^2\cdot 24} = 6.67 \Rightarrow \boxed{W_3 = W_4 = 7\mu m}$$

7.) Check mirror pole ($p_3 = g_{m3}/C_{Mirror}$).

$$p_3 = \frac{g_{m3}}{C_{Mirror}} = \frac{g_{m3}}{2\cdot 0.667\cdot W_3\cdot L_3\cdot C_{ox}} = \frac{\sqrt{2\cdot 24\cdot 6.67\cdot 20}\text{x}10^{-6}}{2\cdot 0.667\cdot 6.67\cdot 0.5\text{x}10^{-15}} = 17.98\text{x}10^9$$

which is much greater than 10GB ($0.0628\text{x}10^9$). Therefore, W_3 and W_4 are OK.

8.) $g_{m6} = 10g_{m1} = 1256.7\mu S$

a.) $g_{m6} = \sqrt{2K_N S_6 10I} \Rightarrow W_6 = 164.5\mu m$

b.) $V_{out}(min) = 0.5 \Rightarrow V_{DS6}(sat) = 0.5 = \sqrt{\frac{2\cdot 10I}{K_N S_6}} \Rightarrow W_6 = 66.67\mu m$

Therefore, use $\boxed{W_6 = 165\mu m}$

Note: For proper mirroring, $S_4 = \frac{I_4}{I_6} S_6 = 8.25\mu m$ which is close enough to $7\mu m$.

9.) Use the $V_{out}(max)$ specification to design W_7.

$$V_{out}(max) = 0.25V \geq V_{DS7}(sat) = \sqrt{\frac{2\cdot 200\mu A}{8\text{x}10^{-6}\cdot S_7}}$$

$$\therefore\ S_7 \geq \frac{400\mu A}{8\text{x}10^{-6}(0.25)^2} \Rightarrow \boxed{W_7 = 800\mu m}$$

10.) Now to achieve the proper currents from the current source I gives,

$$S_9 = S_{10} = \frac{S_7}{10} = 80 \rightarrow \boxed{W_9 = W_{10} = 80\mu m}$$

and

$S_{11} = S_{12} = \frac{1.5\cdot S_7}{10} = 120 \rightarrow W_{11} = W_{12} = 120\mu m$. We saw in step 5 that W_{11} and W_{12} had to be greater than $120\mu m$ to satisfy $V_{ic}(max)$. $\therefore$ $\boxed{W_{11}=W_{12}=120\mu m}$

11.) $P_{diss} = 15I\cdot 1.5V = 300\mu A\cdot 1.5V = 450\mu W$

C_c	I	$W_1=W_2$	$W_3=W_4$	$W_5=W_8$	W_6	W_7	$W_9=W_{10}$	$W_{11}=W_{12}$	P_{diss}
2pF	20μA	33μm	7μm	202μm	165μm	800μm	80μm	120μm	450μW