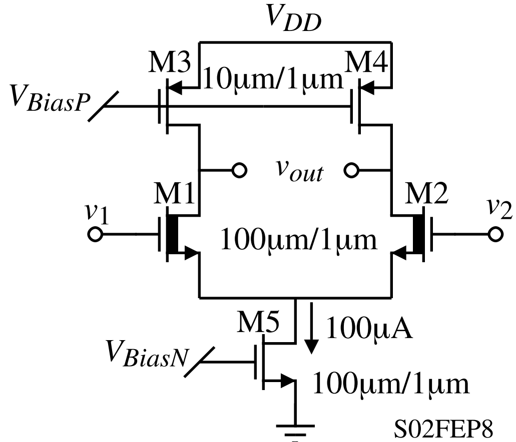
Homework Assignment No. 13 - Solutions

Problem 1 - (10 points)

A differential CMOS amplifier using depletion mode input devices is shown. Assume that the normal MOSFETs parameters are K_N' =110V/μA^2, V_{TN} = 0.7V, λ_N =0.04V^{-1} and for the PMOS transistors are K_P' =110V/μA^2, V_{TP} = 0.7V, λ_P =0.04V^{-1}. For the depletion mode NMOS transistors, the parameters are the same as the normal NMOS except that V_{TN} = -0.5V. (a.) What is the maximum input common-mode voltage, V_{icm}^+(max)? (b.) What is the minimum input common-mode voltage, V_{icm}^-(min)? (c.) What value of V_{DD} gives an $ICMR = 0.5V_{DD}$?



Solution

(a.) $V_{icm}^+(\text{max}) = V_{DD} - V_{SD3}(\text{sat}) - V_{DS1}(\text{sat}) + V_{GS1}(50\mu A)$

$$i_D = \frac{\beta}{2}(V_{GS1} - V_{T1})^2 \rightarrow \qquad V_{GS1} = \sqrt{\frac{2i_D}{\beta}} + V_{T1} = V_{DS1}(\text{sat}) + V_{T1}$$

$\therefore$ $$V_{icm}^+(\text{max}) = V_{DD} - V_{SD3}(\text{sat}) + V_{T1} = V_{DD} - \sqrt{\frac{2I_{D3}}{\beta_3}} + V_{T1}$$

$$V_{icm}^+(\text{max}) = V_{DD} - 0.4472 - 0.5 = \underline{\underline{V_{DD} - 0.9472}}$$

(b.) $V_{icm}^-(\text{min}) = V_{DS5}(\text{sat}) + V_{GS1}(50\mu A) = V_{DS5}(\text{sat}) + V_{DS1}(\text{sat}) + V_{T1}$

$$V_{icm}^-(\text{min}) = \sqrt{\frac{2I_{D5}}{\beta_5}} + \sqrt{\frac{2I_{D1}}{\beta_1}} + V_{T1} = 0.1348 + 0.0953 - 0.5 = \underline{\underline{-0.2698\text{V}}}$$

(c.) $ICMR = V_{icm}^+(\text{max}) - V_{icm}^-(\text{min}) = V_{DD} - 0.9472 + 0.2698 = V_{DD} - 0.6774$

$\therefore$ $V_{DD} - 0.6774 = 0.5V_{DD} \qquad \rightarrow \qquad V_{DD} = 2(0.6774) = \underline{\underline{1.355\text{V}}}$

Problem 2 – (10 points)

If the poles of a two-stage comparator are both equal to -10^7 radians/sec., find the maximum slope and the time it occurs if the magnitude of the input step is $10V_{in}$(min) and $V_{OH}-V_{OL}$ = 1V. What must be the *SR* of this comparator to avoid slewing?

Solution

The response to a step response to the above comparator can be written as,

$$v_{out}' = 1 - e^{-tn} - t_n e^{-tn} \qquad \text{where } v_{out}' = \frac{v_{out}}{A_v(0)V_{in}} \text{ and } t_n = tp_1$$

To find the maximum slope, differentiate twice and set to zero.

$$\frac{dv_{out}'}{dt_n} = e^{-tn} + t_n e^{-tn} - e^{-tn} = t_n e^{-tn}$$

$$\frac{d^2v_{out}'}{dt_n^2} = -t_n e^{-tn} + e^{-tn} = 0 \quad \Rightarrow \quad (1-t_n)e^{-tn} = 0 \quad \Rightarrow \quad t_n(\text{max}) = tp_1 = 1$$

$\therefore$ $t_n(\text{max}) = 1\text{sec}$ and $t(\text{max}) = \frac{t_n}{|p_1|} = \frac{1}{10^7} = \underline{\underline{0.1\mu s}}$

$$\frac{dv_{out}'(\text{max})}{dt_n} = e^{-1} = 0.3679 \text{ V/sec} \quad \text{or} \quad \frac{dv_{out}'(\text{max})}{dt_n} = 3.679\text{V}/\mu\text{s}$$

$$\frac{dv_{out}'(\text{max})}{dt} = 10(V_{OH}-V_{OL})\cdot\frac{dv_{out}'(\text{max})}{dt_n} = \underline{\underline{36.79\text{V}/\mu\text{s}}}$$

$\therefore$ Therefore, the slew rate of the comparator should be greater than 36.79V/µs to avoid slewing.

Problem 3 – (10 points)

Repeat Ex. 8.2-5 with v_{G2} constant and the waveform of Fig. 8.2-6 applied to v_{G1}.

Solution

Output fall time, t_r:

The initial states are $v_{o1} \approx$ -2.5V and $v_{out} \approx$ 2.5V. The reasoning for v_{o1} is interesting and should be understood. When V_{G1} = -2.5V and V_{G2} = 0V, the current in M1 is zero. This means the current is also zero in M4. Therefore, v_{o1} goes very negative and as M2 acts like a switch with $V_{DS} \approx 0$. Since the only current for M3 comes through M2 and from C_I, the voltage across M3 eventually collapses and I_3 becomes zero which causes $v_{o1} \approx$ -2.5V.

From Example 8.2-5, the trip point of the second stage is 1.465V, therefore the rise time of the first stage is,

$$t_{r1} = 0.2\text{pF}\left(\frac{1.465+2.5}{30\mu\text{A}}\right) = 26.4\text{ns}$$

The fall time of the second stage is found in Example 8.2-5 and is t_{f2} = 53.4ns. The total output fall time is

$$\therefore \quad t_r = t_{r1} + t_{f2} = \underline{\underline{79.8\text{ns}}}$$

Output rise time, t_r:

The initial states for this analysis are $v_{o1} \approx$ 2.5V and $v_{out} \approx$ -2.5V.

The input stage fall time is,

$$t_{f1} = 0.2\text{pF}\left(\frac{2.5-1.465}{30\mu\text{A}}\right) = 6.9\text{ns}$$

The output stage rise time is found by determining the best guess for V_{G6}. Since V_{G6} is going from 1.465 to –2.5V, let us approximate V_{G6} as

$$V_{G6} \approx 0.5(1.465\text{-}2.5) = -0.5175 \qquad \Rightarrow \qquad V_{SG6} = 2.5\text{-}(-0.5175) = 3.0175\text{V}$$

$$\therefore \quad I_6 = \frac{1}{2}K_{P}'\left(\frac{W_6}{L_6}\right)(V_{SG6}\text{-}|V_{TP}|)^2 = 0.5\cdot 50\text{x}10^{-6}\cdot 38(3.0175\text{-}0.7)^2 = 5102\mu\text{A}$$

$$t_{r2} = 5\text{pF}\left(\frac{2.5}{5102\mu\text{A}\text{-}234\mu\text{A}}\right) = 2.6\text{ns}$$

The total output rise time is,

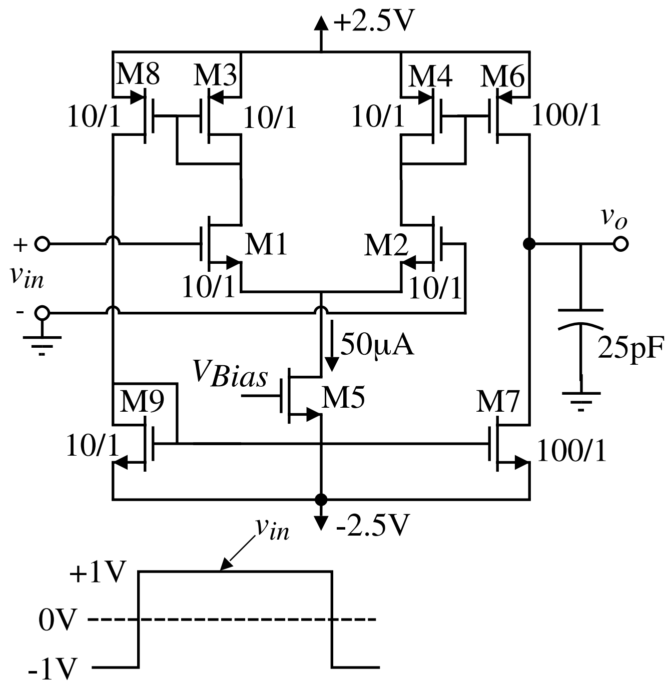
$$\therefore \quad t_r = t_{f1} + t_{r2} = \underline{\underline{9.5\text{ns}}}$$

The propagation time delay of the comparator is,

$$t_p = t_r + t_r = \underline{\underline{44.7\text{ns}}}$$

Problem 4 - (10 points)

The comparator shown has an input applied as shown. Assuming the the pulse width is wide enough, calculate the propagation delay time for this comparator. Assume that the trip point of the output is at 0V.



Solution

When v_{in} = -1V, $i_{D1} < i_{D2}$, which gives $i_{D6} > i_{D7}$. Therefore $v_o(0^-)$ = +2.5V. When v_{in} switches from –1V to +1V all of the 50µA flows through M1 and is mirrored via M3-M8 and M9-M7 and multiplied by 10 to give i_7 = 500µA. Thus, the falling propagation delay time is found as,

$$t_p^- = \frac{C\cdot \Delta V}{i_7} = \frac{25\text{pF}\cdot 2.5\text{V}}{500\mu\text{A}} = 125\text{ns}$$

Similarily, when v_{in} = +1V, v_o is at –2.5V. When v_{in} switches back to –1V, all of the 50µA of M5 flows through M2 giving i_6 = 500µA and i_7 =0. The rising propagation delay time is

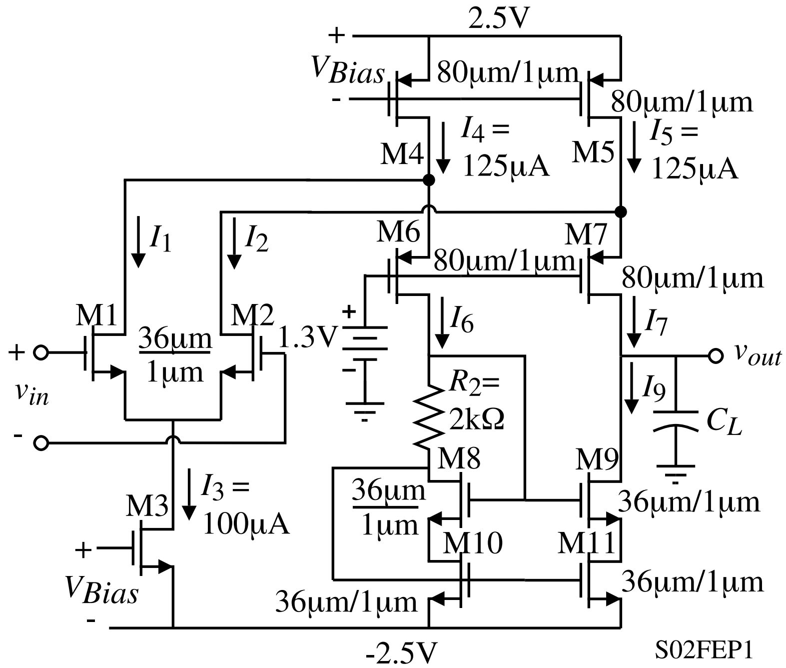
$$t_p^+ = \frac{C\cdot \Delta V}{i_6} = \frac{25\text{pF}\cdot 2.5\text{V}}{500\mu\text{A}} = 125\text{ns}$$

Consequently, the propagation delay time for this comparator is

$$\underline{\underline{t_p = 0.5(t_p^- + t_p^+) = 125\text{ns}}}$$

Problem 5

If the folded-cascode op amp shown having a small-signal voltage gain of 7464V/V is used as a comparator, find the dominant pole if C_L = 5pF. If the input step is 10mV, determine whether the response is linear or slewing and find the propagation delay time. Assume the parameters of the NMOS transistors are K_N'=110V/μA^2, V_{TN} = 0.7V, λ_N=0.04V^{-1} and for the PMOS transistors are K_P'=50V/μA^2, V_{TP} = -0.7V, λ_P=0.05V^{-1}.



Solution

V_{OH} and V_{OL} can be found from many approaches. The easiest is simply to assume that V_{OH} and V_{OL} are 2.5V and –2.5V, respectively. However, no matter what the input, the values of V_{OH} and V_{OL} will be in the following range,

$$(V_{DD}\text{-}2V_{ON}) < V_{OH} < V_{DD} \quad \text{and} \quad V_{DD} < V_{OH} < (V_{SS}+2V_{ON})$$

The reasoning is as follows, suppose V_{in} >0. This gives I_1>I_2 which gives I_6<I_7 which gives I_9<I_7. V_{out} will increase until I_7 equals I_9. The only way this can happen is for M5 and M7 to leave saturation. The same reasoning holds for V_{in} <0.

Therefore assuming that V_{OH} and V_{OL} are 2.5V and –2.5V, respectively, we get

$$V_{in}(\min) = \frac{5\text{V}}{7464} = 0.67\text{mV} \quad \rightarrow \quad k = \frac{10\text{mV}}{0.67\text{mV}} = 14.93$$

Problem 5 – Continued

The folded-cascode op amp as a comparator can be modeled by a single dominant pole. This pole is found as,

$$p_1 = \frac{1}{R_{out}C_L} \text{ where } R_{out} \approx g_{m9}r_{ds9}r_{ds11}||[g_{m7}r_{ds7}(r_{ds2}||r_{ds5})]$$

$g_{m9}=\sqrt{2\cdot 75\cdot 110\cdot 36}=771\mu S$, $g_{ds9}=g_{ds11}=75\text{x}10^{-6}\cdot 0.04 = 3\mu S$, $g_{ds2}=50\text{x}10^{-6}(0.04)=2\mu S$

$g_{m7}=\sqrt{2\cdot 75\cdot 50\cdot 80}=775\mu S$, $g_{ds5}=125\text{x}10^{-6}\cdot 0.05 = 6.25\mu S$, $g_{ds7}=50\text{x}10^{-6}(0.05)=3.75\mu S$

$$g_{m9}r_{ds9}r_{ds11}= (771\mu S)\left(\frac{1}{3\mu S}\right)\left(\frac{1}{3\mu S}\right) = 85.67M\Omega$$

$$g_{m7}r_{ds7}(r_{ds2}||r_{ds5} \approx (775\mu S)\left(\frac{1}{3.75\mu S}\right)\left(\frac{1}{2\mu S}||\frac{1}{6.25\mu S}\right) = 25.05M\Omega\ ,$$

$$R_{out} \approx 85.67M\Omega||25.05M\Omega = 19.4M\Omega$$

The dominant pole is found as, $p_1 = \frac{1}{R_{out}C_L} = \frac{1}{19.4\text{x}10^6 5\text{pF}} = 10{,}318$ rps

The time constant is $\tau_1 = 96.9\mu s$.

For a dominant pole system, the step response is, $v_{out}(t) = A_{vd}(1-e^{-t/\tau_1})V_{in}$

The slope is the largest at $t = 0$. Evaluating this slope gives,

$$\frac{dv_{out}}{dt} = \frac{A_{vd}}{\tau_1}e^{-t/\tau_1}V_{in}$$ For $t = 0$, the slope is $\frac{A_{vd}}{\tau_1}V_{in} = \frac{7464}{96.9\mu s}(10\text{mV}) = 0.77\text{V}/\mu s$

The slew rate of this op amp/comparator is $SR = \frac{I_3}{C_L} = \frac{100\mu A}{5\text{pF}} = 20\text{V}/\mu s$

Therefore, the comparator does not slew and its propagation delay time is found from the linear response as,

$$t_P = \tau_1 \ln\left(\frac{2k}{2k-1}\right) = 96.9\mu s\cdot\ln\left(\frac{2\cdot 14.93}{2\cdot 14.93-1}\right) = (96.9\mu s)(0.0341) = \underline{\underline{3.3\mu s}}$$