Homework Assignment No. 14 - Solutions

Problem 1 - (10 points)

If the comparator used in Fig. 8.4-1 has a dominant pole at 10^4 radians/sec and a gain of 10^3, how long does it take C_{AZ} to charge to 99% of its final value, V_{OS}? What is the final value that the capacitor, C_{AZ}, will charge to if left in the configuration of Fig. 8.4-1(b) for a long time?

Solution

The output voltage for the circuit shown can be expressed as,

$$V_{out}(s) = (-V_{OS} - V_{out}(s))\left(\frac{A_v(0)}{1+\frac{s}{|p_1|}}\right)$$

This can be solved for the transfer $V_{out}(s)/V_{OS}$ as follows,

$$\frac{V_{out}(s)}{V_{OS}(s)} = \frac{\frac{A_v(0)}{1+\frac{s}{|p_1|}}}{1+\frac{A_v(0)}{1+\frac{s}{|p_1|}}} = \frac{A_v(0)}{1+A_v(0)+\frac{s}{|p_1|}} = \frac{A_v(0)|p_1|}{s+(1+A_v(0))\,|p_1|}$$

Assuming $V_{OS}(s)$ is a step function then,

$$V_{out}(s) = -\frac{V_{OS}}{s}\left(\frac{A_v(0)|p_1|}{s+(1+A_v(0))\,|p_1|}\right) = -\frac{A_v(0)V_{OS}}{1+A_v(0)}\left[\frac{1}{s} - \frac{1}{s+(1+A_v(0)|p_1|)}\right]$$

Taking the inverse Laplace transform gives,

$$v_{out}(t) = -\frac{A_v(0)V_{OS}}{1+A_v(0)}\left[1 - e^{-[1+A_v(0)]|p1|t}\right]$$

Let $v_{out}(t) = -0.99V_{OS}$ and solve for the time T.

$$v_{out}(t) = -0.99V_{OS} = -\frac{1000V_{OS}}{1000+1}\left[1 - e^{-1001\cdot 10^4 T}\right]$$

$$1-\frac{1001}{1000}\cdot\frac{99}{100} = 0.0090 = e^{-1001\cdot 10^4 T} \Rightarrow \qquad 110.99 = e^{1001\cdot 10^4 T}$$

$\therefore \qquad T = 0.9990\text{x}10^{-7}\, ln(110.99) = \underline{\underline{0.47\mu s}}$

As $t \to \infty$, $v_{out}(t) \to -\frac{1000V_{OS}}{1000+1} = \underline{\underline{0.999V_{OS}}}$

Problem 2 – (10 points)

What is the gain and -3dB bandwidth (in Hz) of Fig. P8.6-3 if C_L = 1pF? Ignore reverse bias voltage effects on the pn junctions and assume the bulk-source and bulk-drain areas are given by W x5µm. The W/L ratios for M1 and M2 are 10µm/1µm and for the remaining PMOS transistors the W/L ratios are all 2µm/1µm.

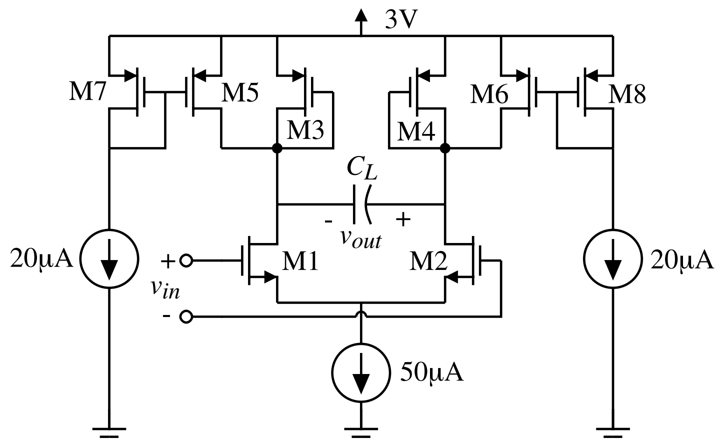
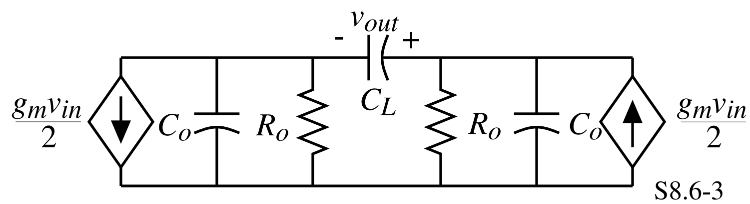


Fig. P8.6-3

Solution

A small-signal model which can be used to solve this problem is shown.



S8.6-3

The voltage gain and the –3dB bandwidth can be expressed as,

$$\frac{v_{out}}{v_{in}} = g_m R_o \quad \text{and} \quad \omega_{\text{-3dB}} = \frac{1}{(C_L+0.5C_o)2R_o}$$

The various values in the above relationships are:

$$g_m = \sqrt{2 \cdot K_N(W_1/L_1)I_{D1}} = \sqrt{2 \cdot 110 \cdot 10 \cdot 25}\ \mu\text{S} = 234.5\mu\text{S}$$

$$R_o \approx \frac{1}{g_{m3}} \| r_{ds1} \| r_{ds3} \| r_{ds5}, \quad g_{m3} = \sqrt{2 \cdot K_P(W_1/L_1)I_{D3}} = \sqrt{2 \cdot 50 \cdot 2 \cdot 5}\ \mu\text{S} = 31.62\mu\text{S}$$

$$r_{ds1} = \frac{1}{0.04 \cdot 25\mu\text{A}} = 1\text{M}\Omega,\ r_{ds3} = \frac{1}{0.05 \cdot 5\mu\text{A}} = 4\text{M}\Omega \text{ and } r_{ds5} = \frac{1}{0.04 \cdot 20\mu\text{A}} = 0.8\text{M}\Omega$$

$$\therefore\ R_o = 31.623\text{k}\Omega \| 1\text{M}\Omega \| 4\text{M}\Omega \| 0.8\text{M}\Omega = 29.31\text{k}\Omega$$

$$C_o \approx C_{gs3}+C_{bd1}+C_{bd3}+C_{bd5} \qquad C_{gs3} = CGSO \cdot W_5 + 0.67 \cdot C_{ox} \cdot W_5 \cdot L_5$$

$$= 220\text{x}10^{-12}\text{F/m} \cdot 2\text{x}10^{-6}\text{m} + 0.67 \cdot 24.7\text{x}10^{-4}\text{F/m}^2 \cdot 2\text{x}10^{-12}\text{m}^2 = 3.73\text{fF}$$

$$C_{bd1} = CJ \cdot AS + CJSW \cdot PS = 770\text{x}10^{-6}\text{F/m}^2 \cdot 50\text{x}10^{-12}\text{m}^2 + 380\text{x}10^{-12}\text{F/m} \cdot 30\text{x}10^{-6}\text{m}$$

$$= 38.5\text{fF} + 11.4\text{fF} = 49.9\text{fF}$$

$$C_{bd3} = C_{bd5} = 560\text{x}10^{-6}\text{F/m}^2 \cdot 10\text{x}10^{-12}\text{m}^2 + 350\text{x}10^{-12}\text{F/m} \cdot 14\text{x}10^{-6}\text{m} = 10.5\text{fF}$$

$$\therefore \qquad C_o = 74.6\text{fF} \rightarrow \omega_{\text{-3dB}} = \frac{1}{(1.073\text{pF})58.62\text{k}\Omega} = 16.445\text{x}10^6\text{rads/sec}$$

Finally, $f_{\text{-3dB}} = \underline{\underline{2.62\text{MHz}}}$ and $A_v = \underline{\underline{6.873\text{V/V}}}$

Problem 3 – (10 points)

Assume that a comparator consists of an amplifier cascaded with a latch. Assume the amplifier has a gain of 5V/V and a -3dB bandwidth of $1/\tau_L$, where τ_L is the latch time constant and is equal to 10ns. Find the propagation time delay for the overall configuration if the applied input voltage is $0.05(V_{OH}\text{-}V_{OL})$ and the voltage applied to the latch from the amplifier is (a) $\Delta V_i = 0.05(V_{OH}\text{-}V_{OL})$, (b) $\Delta V_i = 0.1(V_{OH}\text{-}V_{OL})$, (c) $\Delta V_i = 0.15(V_{OH}\text{-}V_{OL})$ and (d) $\Delta V_i = 0.2(V_{OH}\text{-}V_{OL})$. Assume that the latch is enabled as soon as the output of the amplifier is equal to $0.05(V_{OH}\text{-}V_{OL})$. From your results, what value of ΔV_i would give minimum propagation time delay?

Solution

The transfer function of the amplifier is $$A_v(s) = \frac{A_v(0)}{s\tau_L+1}$$

The output voltage of the amplifier is $v_o(t) = A_v(0)[1\text{-}e^{-t/\tau_L}]\Delta V_i$

Let $\Delta V_i = x\cdot(V_{OH}\text{-}V_{OL})$, therefore the delay of the amplifier can be found as

$$x(V_{OH}\text{-}V_{OL}) = A_v(0)[1\text{-}e^{-t_1/\tau_L}]0.05(V_{OH}\text{-}V_{OL}) = 5[1\text{-}e^{-t_1/\tau_L}]0.05(V_{OH}\text{-}V_{OL})$$

or

$$x = 0.25[1\text{-}e^{-t_1/\tau_L}] \quad \rightarrow \quad t_1 = \tau_L\, ln\left(\frac{1}{1\text{-}4x}\right)$$

The delay of the latch can be found as

$$t_2 = \tau_L\, ln\left(\frac{V_{OH}\text{-}V_{OL}}{2x(V_{OH}\text{-}V_{OL})}\right) = \tau_L\, ln\left(\frac{1}{2x}\right)$$

The propagation time delay of the comparator can be expressed in terms of x as,

$$t_p = t_1+t_2 = \tau_L\, ln\left(\frac{1}{1\text{-}4x}\right) + \tau_L\, ln\left(\frac{1}{2x}\right) = \tau_L\, ln\left(\frac{1}{2x\text{-}8x^2}\right)$$

Thus,

$x = 0.05 = 1/20 \quad \Rightarrow \quad \tau_p = t_1 + t_2 = 2.23\text{ns}+2.30\text{ns} = \underline{\underline{25.26\text{ns}}}$

$x = 0.1 = 1/10 \quad \Rightarrow \quad \tau_p = t_1 + t_2 = 5.11\text{ns}+16.09\text{ns} = \underline{\underline{21.20\text{ns}}}$

$x = 0.15 \quad \Rightarrow \quad \tau_p = t_1 + t_2 = 9.16\text{ns}+12.04\text{ns} = \underline{\underline{21.20\text{ns}}}$

$x = 0.2 = 1/5 \quad \Rightarrow \quad \tau_p = t_1 + t_2 = 16.09\text{ns}+9.16\text{ns} = \underline{\underline{25.26\text{ns}}}$

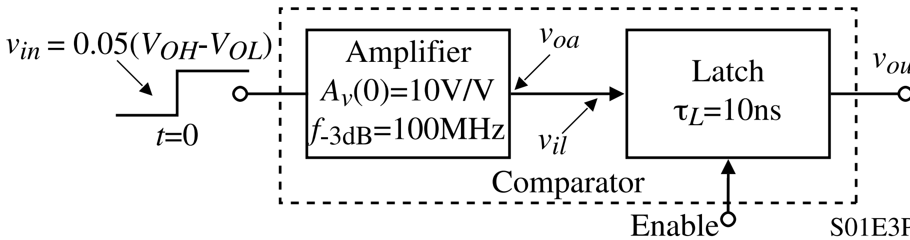
Note that differentiating t_p with respect to x and setting to zero gives

$x_{min} = 1/8 = 0.125$

Therefore, minimum delay of $\underline{\underline{20.08\text{ns}}}$ is achieved when $\underline{\underline{x = 1/8}}$.

Problem 4 - (10 points)

A comparator consists of an amplifier cascaded with a latch as shown below. The amplifier has a voltage gain of 10V/V and f_{-3dB} = 100MHz and the latch has a time constant of 10ns. The maximum and minimum voltage swings of the amplifier and latch are V_{OH} and V_{OL}. When should the latch be enabled after the application of a step input to the amplifier of $0.05(V_{OH}-V_{OL})$ to get minimum overall propagation time delay? What is the value of the minimum propagation time delay? It may useful to recall that the propagation time delay of the latch is given as $t_p = \tau_L \, ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right)$ where v_{il} is the latch input (ΔV_i of the text).



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Solution

The solution is based on the figure shown.
We note that,

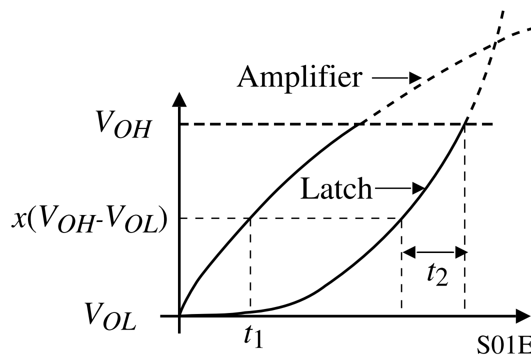
$$v_{oa}(t) = 10[1-e^{-\omega_{-3dB}t}]0.05(V_{OH}-V_{OL}).$$

If we define the input voltage to the latch as,

$$v_{il} = x\cdot(V_{OH}-V_{OL})$$

then we can solve for t_1 and t_2 as follows:

$$x\cdot(V_{OH}-V_{OL}) = 10[1-e^{-\omega_{-3dB}t_1}]0.05(V_{OH}-V_{OL}) \rightarrow x = 0.5[1-e^{-\omega_{-3dB}t_1}]$$



S01E3S1

This gives,

$$t_1 = \frac{1}{\omega_{-3dB}} ln\left(\frac{1}{1-2x}\right)$$

From the propagation time delay of the latch we get,

$$t_2 = \tau_L \, ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right) = \tau_L \, ln\left(\frac{1}{2x}\right)$$

$$\therefore \quad t_p = t_1 + t_2 = \frac{1}{\omega_{-3dB}} ln\left(\frac{1}{1-2x}\right) + \tau_L \, ln\left(\frac{1}{2x}\right) \quad \rightarrow \quad \frac{dt_p}{dx} = 0 \text{ gives } x = \frac{\pi}{1+2\pi} = 0.4313$$

$$t_1 = \frac{10\text{ns}}{2\pi} ln\,(1+2\pi) = 1.592\text{ns}\cdot 1.9856 = \underline{\underline{3.16\text{ns}}} \text{ and } t_2 = 10\text{ns}\, ln\left(\frac{1+2\pi}{2\pi}\right) = 1.477\text{ns}$$