LECTURE 070 – SINGLE-STAGE FREQUENCY RESPONSE - I
(READING: GHLM – 488-504)

Objective

The objective of this presentation is:

1.) Illustrate the frequency analysis of single stage amplifiers

2.) Introduce the Miller technique and the approximate method of solving for two poles

Outline

- Differential and Common Frequency Response of the Differential Amplifier
- Emitter/Source Follower Frequency Response
- Common Base/Gate Frequency Response
- Summary

FREQUENCY RESPONSE OF THE DIFFERENTIAL AMPLIFIER

Differential Mode

Differential Amplifiers:

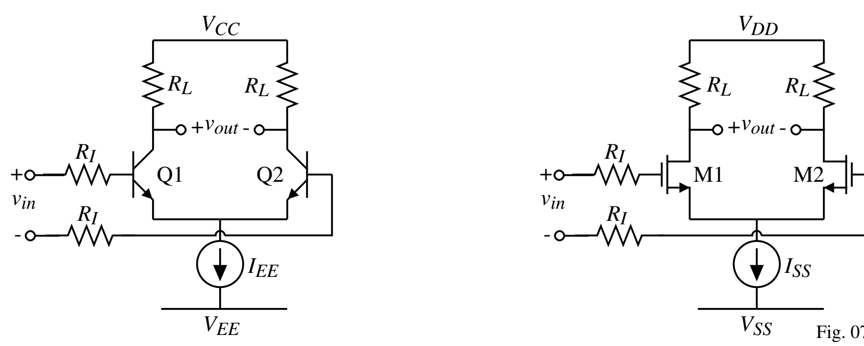

Fig. 070-01

Half-Circuit Concept:

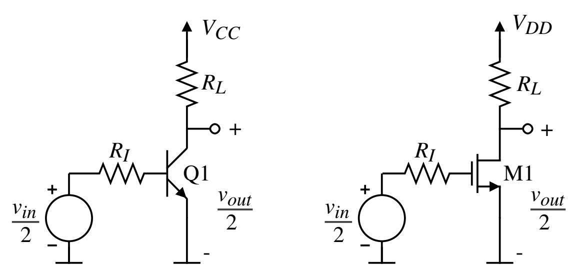

Fig. 070-02

Note that the following analysis is applicable to the CE and CS configurations.

Differential Mode Analysis – Miller Approach

Small Signal Model:

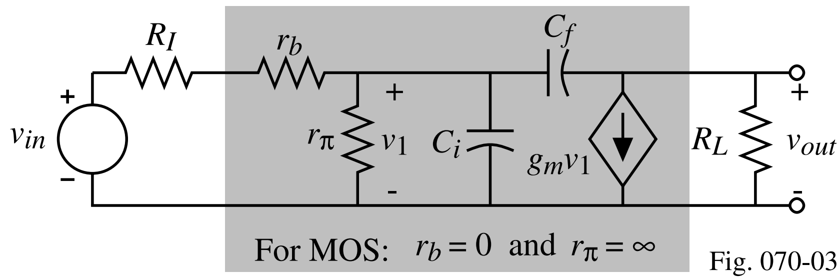

Fig. 070-03

Miller Approach:

Assume that $R_L < (1/\omega C_f)$, then $v_{out} \approx -g_m R_L v_1$

Therefore, the small-signal model can be approximated as,

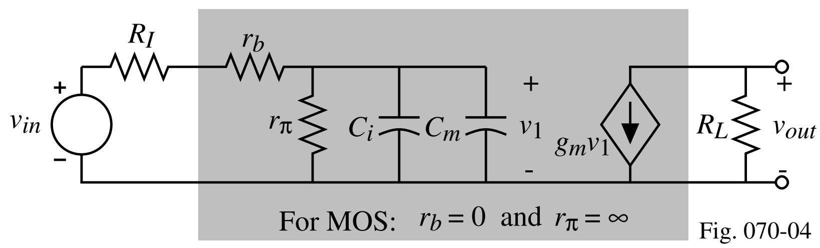

Fig. 070-04

where

$$C_m = C_f(1+g_m R_L)$$

Differential Mode Analysis – Continued

The small-signal analysis of the previous circuit defining $C_t = C_i + C_m$ is,

$$\frac{v_{out}}{v_{in}} = \left(\frac{v_{out}}{v_1}\right)\left(\frac{v_1}{v_{in}}\right) = (-g_m R_L)\left(\frac{\frac{r_\pi}{1+r_\pi C_t s}}{\frac{r_\pi}{1+r_\pi C_t s} + R_I + r_b}\right) = -g_m R_L\left(\frac{r_\pi}{r_\pi + R_I + r_b}\right)\left(\frac{1}{1+\frac{s r_\pi C_t(R_I+r_b)}{r_\pi + R_I + r_b}}\right)$$

Therefore we see that the gain (K), pole (p_1), and -3dB frequency (ω_{-3dB}) is given as,

	K	p_1	ω_{-3dB}
BJT	$-g_m R_L\left(\frac{r_\pi}{r_\pi + R_I + r_b}\right)$	$-\frac{r_\pi + R_I + r_b}{r_\pi C_t(R_I+r_b)}$	$\frac{r_\pi + R_I + r_b}{r_\pi C_t(R_I+r_b)}$
MOS	$-g_m R_L$	$-\frac{1}{C_t R_I}$	$\frac{1}{C_t R_I}$

Example 1

If R_I = 1kΩ, r_b=200Ω, I_C=I_D=1mA, β_o=100, K_N'=100μA/V^2, f_T = 400MHz (I_C = I_D =1mA), C_μ=0.5pF, C_{gd}=0.5pF, W/L = 1000, R_L = 5kΩ, find the gain and –3dB frequency of the BJT and MOS differential amplifier.

Solution

BJT:

$$r_\pi = \frac{\beta_o}{g_m} = 100(26) = 2.6\text{k}\Omega, \quad \tau_T = \frac{1}{2\pi f_T} = 398\text{ps} \Rightarrow C_\pi = g_m\tau_T - C_\mu = 15.3\text{pF} - 0.5\text{pF} = 14.8\text{pF}$$

$$\therefore \quad K = \frac{-5000}{26}\left(\frac{2.6}{1+0.2+2.6}\right) = -131.6\text{V/V}$$

$$C_t = C_\pi + C_\mu(1+g_mR_L) = 14.8\text{pF} + 0.5\text{pF}\left(1+\frac{5000}{26}\right) = 14.8\text{pF} + 96.7\text{pF} = 111.5\text{pF}$$

$$\therefore \quad \omega_{-3\text{dB}} = \frac{r_\pi + R_I + r_b}{r_\pi C_t(R_I+r_b)} = \frac{2600+1000+200}{2600(1000+200)111.5\text{pF}} = 10.92\text{x}10^6 \rightarrow f_{-3\text{dB}} = 1.74\text{MHz}$$

MOS:

$$g_m = \sqrt{2\cdot 1000\cdot 100\cdot 1000} = 14.1\text{mS} \text{ and } C_{gd}+C_{gs} = g_m/\omega_T = 14.1\text{x}10^{-3}/800\pi\text{MHz} = 5.6\text{pF}$$

$$\therefore \quad C_{gs} = 5.6\text{pF}-0.5\text{pF} = 5.1\text{pF}, \; C_t = 5.1\text{pF}+0.5\text{pF}(1+14.1\cdot 5) = 5.1\text{pf}+35.7\text{pF} = 40.8\text{pF}$$

$$\therefore K = -14.1\cdot 5 = -70.5\text{V/V and } \omega_{-3\text{dB}} = \frac{1}{40.8\text{pF}(1000)} = 24.5\text{x}10^6 \rightarrow f_{-3\text{dB}} = 3.90\text{MHz}$$

Differential Amplifier – Exact Frequency Response

The second method solves for the poles without using the Miller approximation.
Small-signal model:

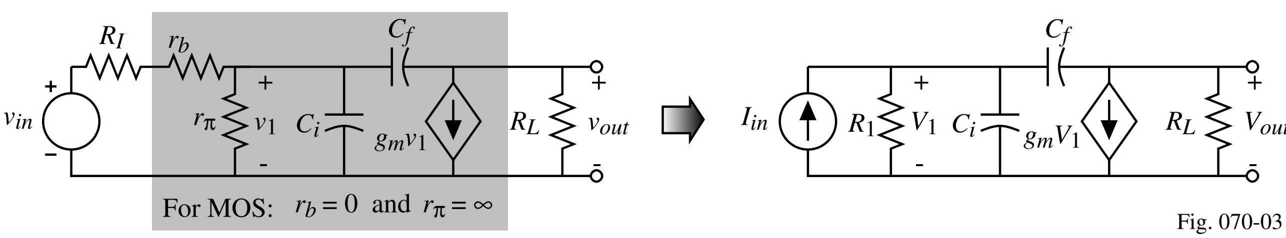

Fig. 070-03

where $\quad I_{in} = \dfrac{V_{in}}{R_I+r_b}$ and $R_1 = r_\pi||(R_I+r_b)$

Nodal Equations:

$$I_{in} = [G_1 + s(C_i + C_f)]V_1 - [sC_f]V_{out} \quad \text{and} \quad 0 = [g_m - sC_f]V_1 + [G_L + sC_f]V_{out}$$

Solving using Cramer's rule gives,

$$\frac{V_{out}(s)}{I_{in}(s)} = \frac{-(g_m - sC_f)}{G_1G_L+s\,[G_1C_f+G_LC_i+G_LC_f+g_mC_f]+s^2[C_fC_i]}$$

or

$$\frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{-g_mR_LR_1}{R_I+r_b}\right)\frac{[1 - s\,(C_f/g_m)]}{1+s\,[R_LC_f+R_1C_i+R_1C_f+g_mR_1R_LC_f]+s^2(R_1R_LC_iC_f)}$$

Note that the gain is $\dfrac{V_{out}(0)}{V_{in}(0)} = -g_mR_L\left(\dfrac{r_\pi}{R_I+r_b+r_\pi}\right)$

Differential Amplifer – Exact Frequency Response

In general, $D(s) = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right) = 1 - s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2} \rightarrow D(s) \approx 1 - \frac{s}{p_1} + \frac{s^2}{p_1 p_2}$, if $|p_2|>>|p_1|$

$$\therefore \quad p_1 = \frac{-1}{R_L C_f + R_1 C_i + R_1 C_f + g_m R_1 R_L C_f} \approx \frac{-1}{g_m R_1 R_L C_f} = \frac{r_\pi + R_I + r_b}{g_m R_L C_f (R_I + r_b) r_\pi}, \qquad z = \frac{g_m}{C_c}$$

$$p_2 = \frac{-[R_L C_f + R_1 C_i + R_1 C_f + g_m R_1 R_L C_f]}{R_1 R_L C_i C_f} \approx \frac{-g_m C_f}{C_i C_f} \approx \frac{-g_m}{C_i} \quad \text{where } g_m R_L > 1$$

The Miller approximation gave,

$$p_1(\text{BJT}) = -\frac{r_\pi + R_I + r_b}{r_\pi C_t (R_I + r_b)} \approx \frac{r_\pi + R_I + r_b}{r_\pi\, g_m R_L C_f (R_I + r_b)} \quad \text{and } p_1(\text{MOS}) = -\frac{1}{C_t R_I} \approx \frac{1}{g_m R_L C_f R_I}$$

which verifies the two methods.

Common-Mode Analysis of the Differential Amplifier

Assumptions: Tail capacitance is dominant and self-resistance is negligible.

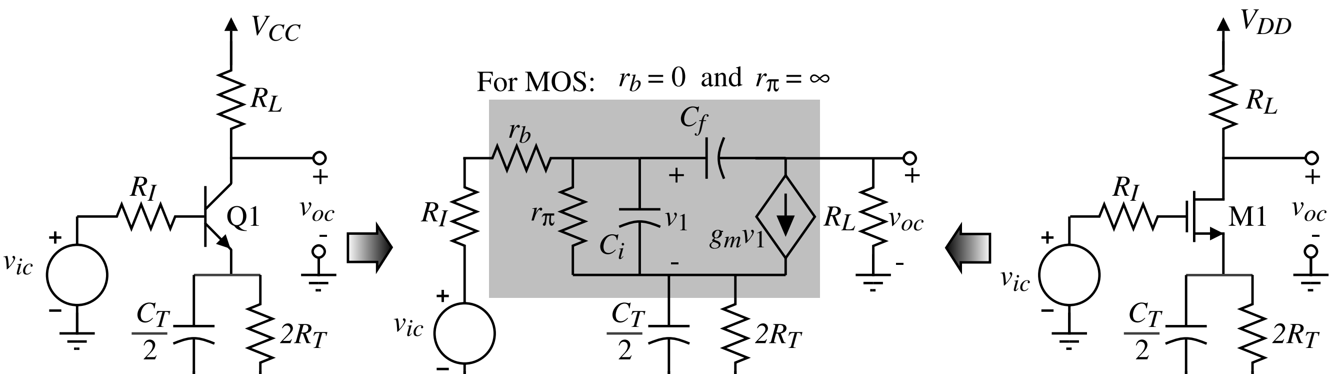

Fig. 070-06

$$\therefore \quad A_{cm} = \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{Z_T} \quad \text{where} \quad Z_T = \frac{2R_T}{1 + sR_T C_T} \quad \Rightarrow \quad A_{cm} = \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{2R_T}(1 + sR_T C_T)$$

This zero at $\omega = 1/R_T C_T$ causes the CM gain increases, resulting in a *CMRR* decrease.

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{\left(\frac{g_m 2R_T r_\pi}{r_\pi + R_I + r_b}\right)\left(\frac{1}{1 + \frac{s r_\pi C_t (R_I + r_b)}{r_\pi + R_I + r_b}}\right)}{(1 + sR_T C_T)} = \left(\frac{g_m 2R_T r_\pi}{r_\pi + R_I + r_b}\right)\left(\frac{1}{(1 + s/\omega_T)(1 + s/p_1)}\right)$$

CMRR Frequency Response

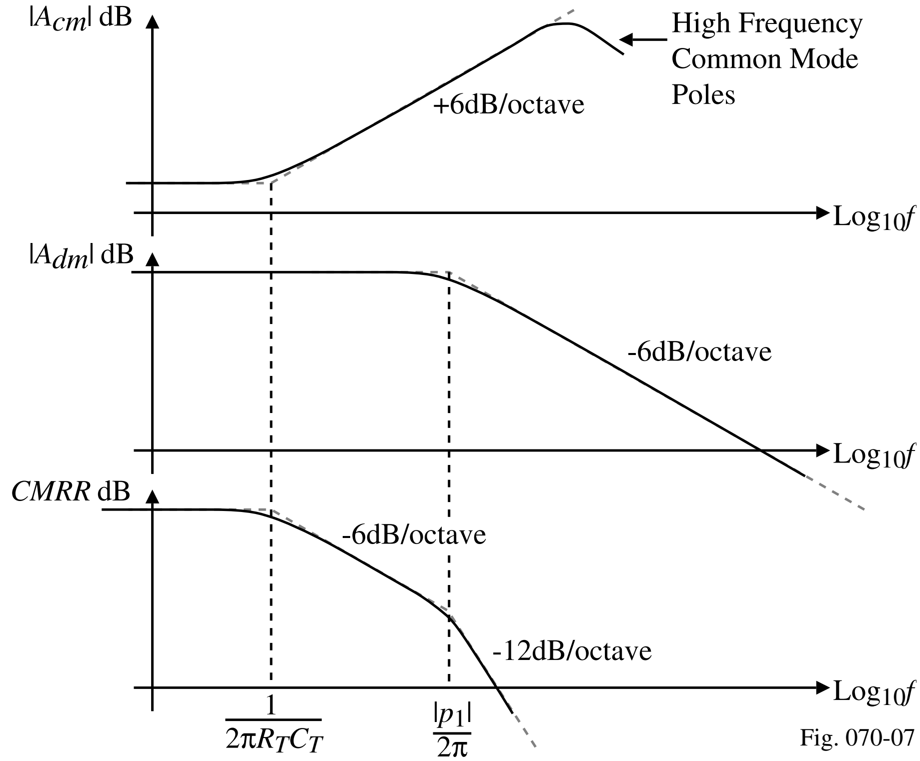

Fig. 070-07

Voltage Buffers

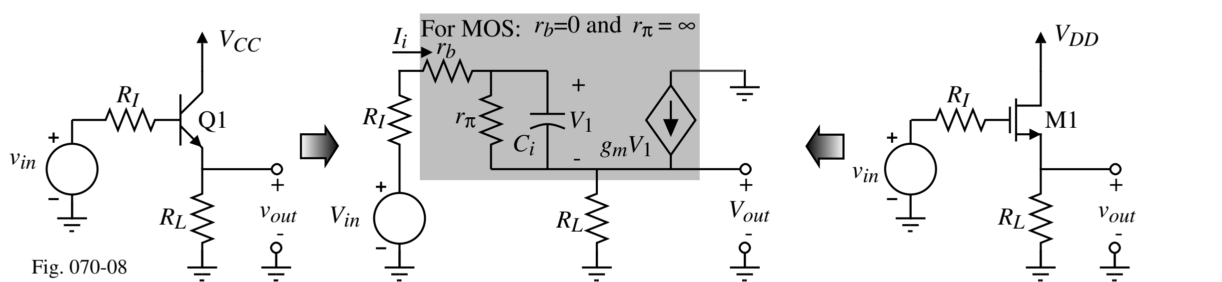

Fig. 070-08

Define $R'_I = R_I + r_b$ and write,

$$V_{in} = I_i R'_I + V_1 + V_{out}, \qquad I_i = \frac{V_1}{z_\pi}, \quad \text{and} \quad I_i + g_m V_1 = \frac{V_{out}}{R_L} \quad \text{where } z_\pi = \frac{r_\pi}{1+sC_i r_\pi}$$

Combining the last three terms gives,

$$\frac{V_1}{r_\pi}(1+sC_i r_\pi) + g_m V_1 = \frac{V_{out}}{R_L} \qquad \rightarrow \qquad V_1 = \frac{V_{out}}{R_L}\,\frac{1}{g_m + \frac{1}{r_\pi}(1+sC_i r_\pi)}$$

Finally, $$V_{in} = \left(\frac{R'_I}{z_\pi}+1\right)\frac{V_{out}}{R_L}\,\frac{1}{g_m+\frac{1}{r_\pi}(1+sC_i r_\pi)} + V_{out} \qquad \rightarrow \qquad \frac{V_{out}}{V_{in}} = \frac{g_m R_L + \frac{R_L}{r_\pi}}{1+g_m R_L + \frac{R'_I + R_L}{r_\pi}}\left[\frac{1-\frac{s}{z_1}}{1-\frac{s}{p_1}}\right]$$

where $z_1 = -\frac{g_m + (1/r_\pi)}{C_i}$, $p_1 = -\frac{1}{R_1 C_i}$ and $R_1 = r_\pi || \frac{R'_I + R_L}{1+g_m R_L}$

Frequency Response of the Emitter Follower

Assume that $g_mR_L >> 1$ and $g_mR_L >> (R'_I + R_L)/r_\pi$, then

$$z_1 = -\frac{g_m+(1/r_\pi)}{C_i} \approx -\frac{g_m}{C_\pi} \quad \text{and} \quad p_1 = -\frac{1}{R_1C_\pi}$$

Example

Calculate the transfer function for an emitter follower with $C_\pi = 10\text{pF}$, $C_\mu = 0$, $R_L = 2\text{k}\Omega$, $R_I = 50\Omega$, $r_b = 150\Omega$, $\beta = 100$, and $I_C = 1\text{mA}$.

From the data, $g_m = 1/26\text{S} = 38.5\text{mS}$, $r_\pi = 2.6\text{k}\Omega$, and $R'_I = R_I + r_b = 200\Omega$.

$$z_1 \approx -\frac{g_m}{C_\pi} = -\frac{38.5\text{x}10^3}{10^{-11}} = -3.85\text{x}10^9 \text{ rad/s} = |\omega_T|$$

$$R_1 = r_\pi || \frac{R'_I+R_L}{1+g_mR_L} = 2.6\text{k}||\frac{2.2\text{k}}{1+76.9} = 27.9\Omega$$

$$p_1 = -\frac{1}{R_1C_\pi} = -\frac{1}{27.9\cdot 10^{-11}} = -3.58\text{x}10^9 \text{ rad/s (570MHz)}$$

Note the pole and zero are closely spaced. Should consider the influence of C_μ for $\omega_{-3\text{dB}}$.

$$\frac{v_{out}}{v_{in}} = \frac{g_mR_L + \frac{R_L}{r_\pi}}{1 + g_mR_L + \frac{R'_I+R_L}{r_\pi}} = \frac{76.9 + \frac{2000}{2600}}{1 + 76.9 + \frac{2200}{2600}} = 0.986$$

Emitter Follower Frequency Response-Continued

Include the influence of C_μ.

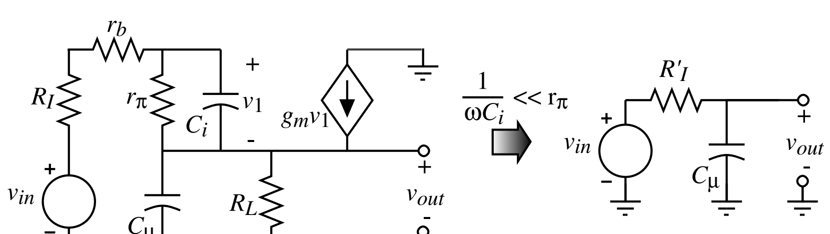

Fig. 070-08

The pole due to C_μ is approximately, $p_2 = -1/R'_IC_\mu$. For $C_\mu = 1\text{pF}$, $p_2 \approx 2\pi(795\text{MHz})$

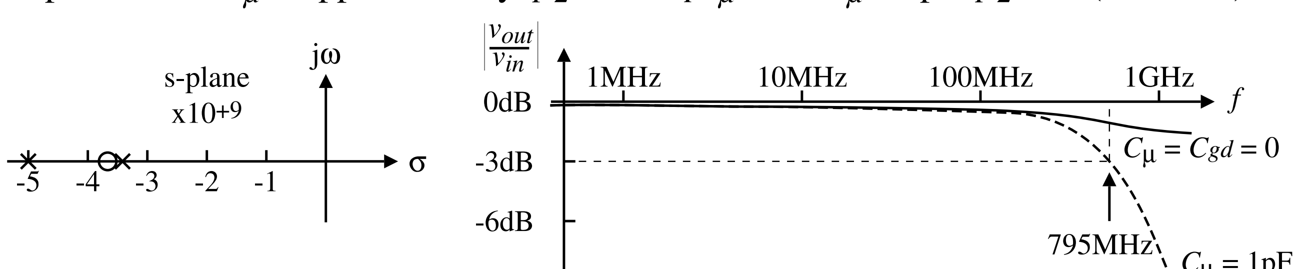

Fig. 070-10

The emitter follower bandwidth is still quite good even considering C_μ.

Next, we will consider the input and output impedances of the emitter follower in the next lecture.

