

## LECTURE 140 – SIMPLE OP AMPS

(READING: Text-GHLM – 425-434, 453-454, AH – 249-253)

### INTRODUCTION

The objective of this presentation is:

- 1.) Illustrate the analysis of BJT and CMOS op amps
- 2.) Prepare for the design of BJT and CMOS op amps

### Outline

- Simple CMOS Op Amps
  - Two-stage
  - Folded-cascode
- Simple BJT Op Amps
  - Two-stage
  - Folded-cascode
- Summary

## SIMPLE TWO-STAGE CMOS OP AMPS

### Two-Stage CMOS Op Amp

Circuit:

DC Conditions:

$$I_5 = I_{bias}, \quad I_1 = I_2 = 0.5I_5 = 0.5I_{bias},$$

$$I_7 = I_6 = nI_{Bias}$$

$$V_{icm}(\max) = V_{DD} - V_{SG3} + V_{T1}$$

$$V_{icm}(\min) = V_{SS} + V_{DS5}(\text{sat}) + V_{GS1}$$

$$V_{out}(\max) = V_{DD} - V_{SD6}(\text{sat})$$

$$V_{out}(\min) = V_{SS} + V_{DS7}(\text{sat})$$

Notice that the output stage is class A

$$\therefore I_{sink} = I_7 \text{ and } I_{source} = \frac{K_N'W_6}{2L_6} (V_{DD} - V_{SS} - V_T)^2 - I_7$$

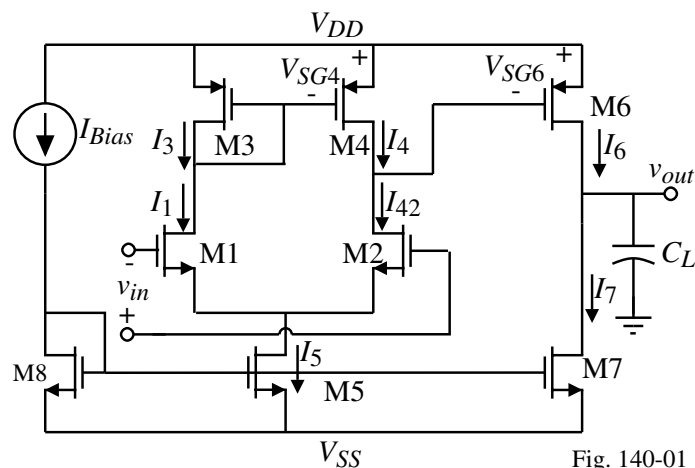


Fig. 140-01

### DC Balance Conditions for the Two-Stage Op Amp

For best performance, keep all transistors in saturation.

M4 is the only transistor that cannot be forced into saturation by internal connections or external voltages.

Therefore, we develop conditions to force M4 to be in saturation.

1.) First *assume* that  $V_{SG4} = V_{SG6}$ . This will cause “proper mirroring” in the M3-M4 mirror. Also, the gate and drain of M4 are at the same potential so that M4 is “guaranteed” to be in saturation.

2.) Let  $S_i \equiv \frac{W_i}{L_i}$ , if  $V_{SG4} = V_{SG6}$ , then  $I_6 = \left(\frac{S_6}{S_4}\right)I_4$

3.) However,  $I_7 = \left(\frac{S_7}{S_5}\right)I_5 = \left(\frac{S_7}{S_5}\right)(2I_4)$

4.) For balance,  $I_6$  must equal  $I_7 \Rightarrow \frac{S_6}{S_4} = \frac{2S_7}{S_5}$  which is called the “balance conditions”

5.) So if the balance conditions are satisfied, then  $V_{DG4} = 0$  and M4 is saturated.

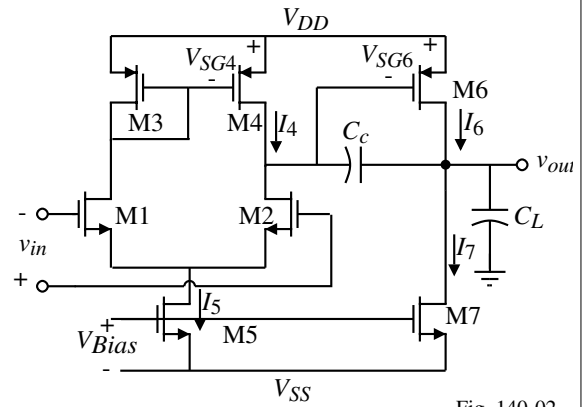


Fig. 140-02

### Small-Signal Performance of the Two-Stage CMOS Op Amp

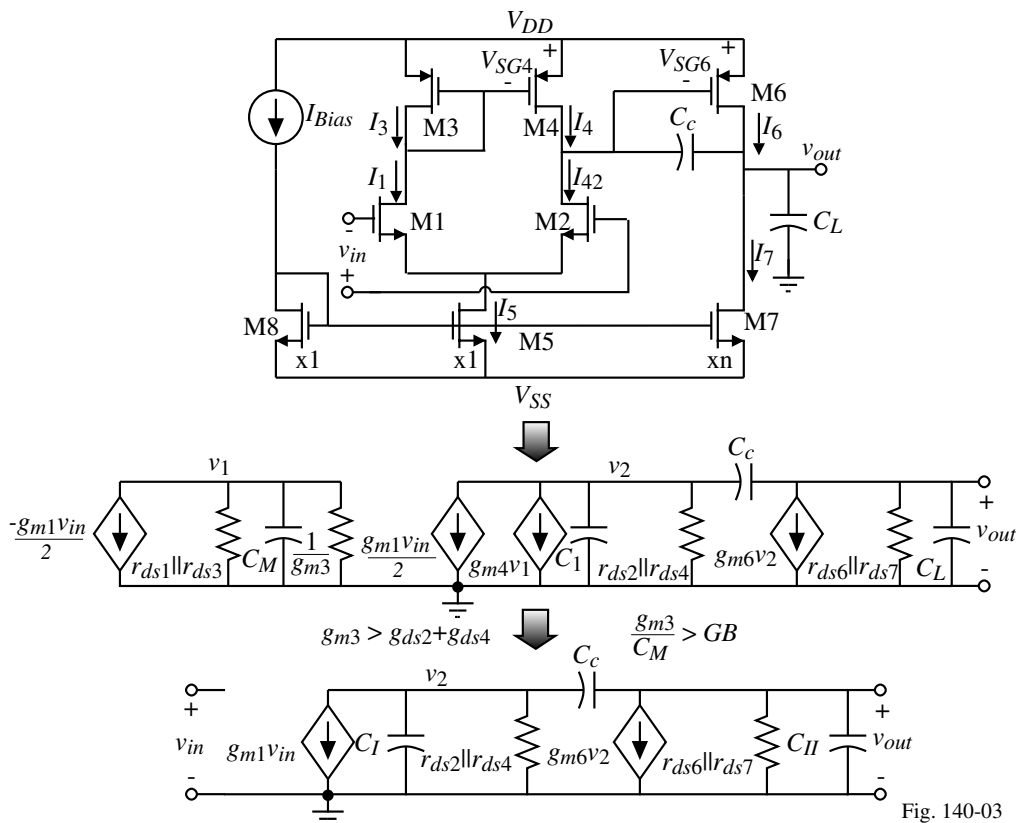


Fig. 140-03

### Small-Signal Performance of the Two-Stage CMOS Op Amp

Summary of the small signal performance:

Midband performance-

$$A_o = g_{m1}g_{mII}R_I R_{II} \approx g_{m1}g_{m6}(r_{ds2} \parallel r_{ds4})(r_{ds6} \parallel r_{ds7}), \quad R_{out} = r_{ds6} \parallel r_{ds7}, \quad R_{in} = \infty$$

Roots-

$$\text{Zero} = \frac{g_{mII}}{C_c} = \frac{g_{m6}}{C_c}$$

$$\text{Poles at } p_1 \approx \frac{-1}{g_{mII}R_I R_{II}C_c} = \frac{-(g_{ds2}+g_{ds4})(g_{ds6}+g_{ds7})}{g_{m6}C_c} \quad \text{and} \quad p_2 \approx \frac{-g_{mII}}{C_{II}} \approx \frac{-g_{m6}}{C_L}$$

Assume that  $g_{m1} = 100\mu\text{S}$ ,  $g_{m6} = 1\text{mS}$ ,  $r_{ds2} = r_{ds4} = 2\text{M}\Omega$ ,  $r_{ds6} = r_{ds7} = 0.5\text{M}\Omega$ ,  $C_c = 5\text{pF}$  and  $C_L = 10\text{pF}$ :

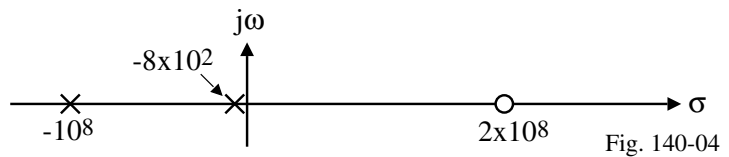
$$A_o = (100\mu\text{S})(1\text{M}\Omega)(1000\mu\text{S})(0.25\text{M}\Omega) = 25,000\text{V/V}, \quad R_{in} = \infty, \quad R_{out} = 250\text{k}\Omega$$

$$\text{Zero} = 1000\mu\text{S}/5\text{pF} = 2 \times 10^8 \text{ rads/sec or } 31.83\text{MHz},$$

$$p_1 = \frac{-1}{(1\text{mS})(1\text{M}\Omega)(0.25\text{M}\Omega)(5\text{pF})} = -800 \text{ rads/sec or } 127.3\text{Hz},$$

$$GB = 3.178\text{MHz}$$

$$\text{and } p_2 = (-1000\mu\text{S}/10\text{pF}) = 10^8 \text{ rads/sec or } 15.915\text{MHz}$$



### Slew Rate of a Two-Stage CMOS Op Amp

Remember that slew rate occurs when currents flowing in a capacitor become limited and is given as

$$I_{lim} = C \frac{dv_C}{dt} \text{ where } v_C \text{ is the voltage across the capacitor } C.$$

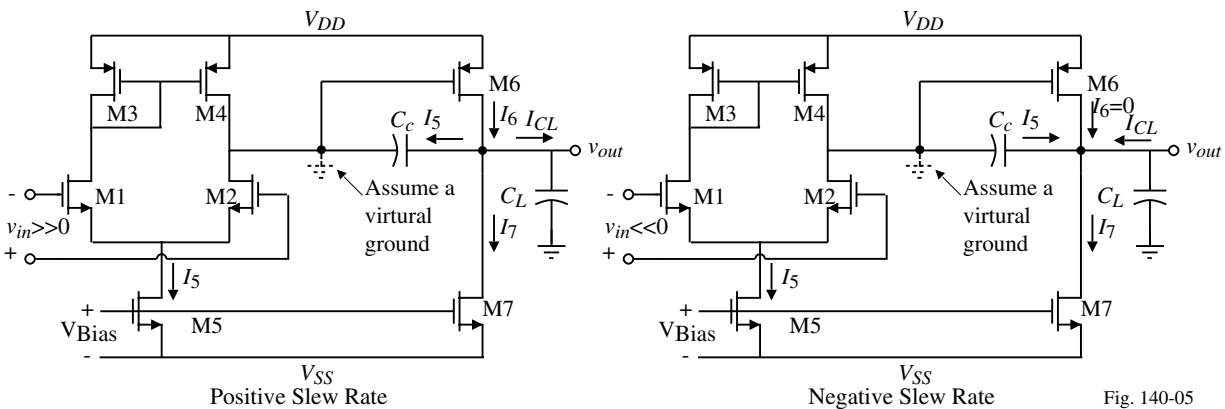


Fig. 140-05

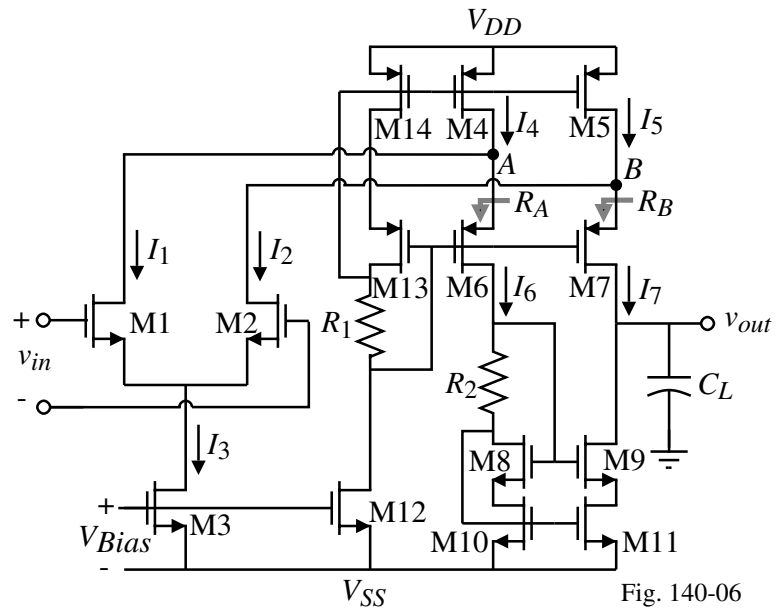
$$SR^+ = \min\left[\frac{I_5}{C_c}, \frac{I_6 - I_5 - I_7}{C_L}\right] = \frac{I_5}{C_c} \text{ because } I_6 \gg I_5 \quad SR^- = \min\left[\frac{I_5}{C_c}, \frac{I_7 - I_5}{C_L}\right] = \frac{I_5}{C_c} \text{ if } I_7 \gg I_5.$$

Therefore, if  $C_L$  is not too large and if  $I_7$  is significantly greater than  $I_5$ , then the slew rate of the two-stage op amp should be,

$$SR = \frac{I_5}{C_c}$$

### Folded Cascode, CMOS Op Amp

Circuit:



Comments:

- The bias currents,  $I_4$  and  $I_5$ , should be designed so that  $I_6$  and  $I_7$  never become zero (i.e.  $I_5=I_6=1.5I_3$ )
- This amplifier is nearly balanced (would be exactly if  $R_A$  was equal to  $R_B$ )
- Self compensating
- Poor noise performance, the gain occurs at the output so all intermediate transistors contribute to the noise along with the input transistors. (Some first stage gain can be achieved if  $R_A$  and  $R_B$  are greater than  $g_{m1}$  or  $g_{m2}$ .)

### Small-Signal Analysis of the Folded Cascode Op Amp

Model:

Recalling what we learned about the resistance looking into the source of the cascode transistor;

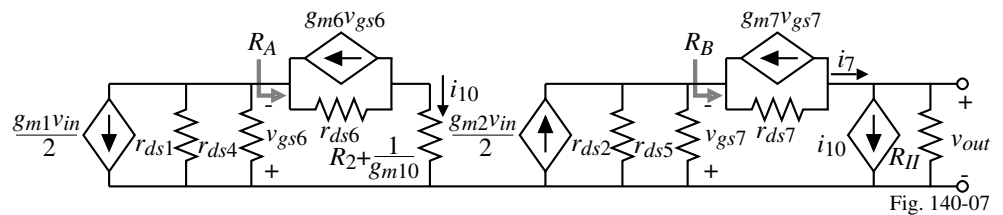


Fig. 140-07

$$R_A = \frac{r_{ds6} + R_2 + (1/g_{m10})}{1 + g_{m6}r_{gs6}} \approx \frac{1}{g_{m6}} \quad \text{and} \quad R_B = \frac{r_{ds7} + R_{II}}{1 + g_{m7}r_{ds7}} \approx \frac{R_{II}}{g_{m7}r_{ds7}} \quad \text{where} \quad R_{II} \approx g_{m9}r_{ds9}r_{ds11}$$

The small-signal voltage transfer function can be found as follows. The current  $i_{10}$  is written as

$$i_{10} = \frac{-g_{m1}(r_{ds1} \parallel r_{ds4})v_{in}}{2[R_A + (r_{ds1} \parallel r_{ds4})]} \approx \frac{-g_{m1}v_{in}}{2}$$

and the current  $i_7$  can be expressed as

$$i_7 = \frac{\frac{g_{m2}(r_{ds2} \parallel r_{ds5})v_{in}}{2} \cdot \frac{R_{II}}{g_{m7}r_{ds7}}}{2 \left[ \frac{R_{II}}{g_{m7}r_{ds7}} + (r_{ds2} \parallel r_{ds5}) \right]} = \frac{g_{m2}v_{in}}{2 \left( 1 + \frac{R_{II}(g_{ds2} + g_{ds5})}{g_{m7}r_{ds7}} \right)} = \frac{g_{m2}v_{in}}{2(1+k)} \quad \text{where} \quad k = \frac{R_{II}(g_{ds2} + g_{ds4})}{g_{m7}r_{ds7}}$$

The output voltage,  $v_{out}$ , is equal to the sum of  $i_7$  and  $i_{10}$  flowing through  $R_{out}$ . Thus,

$$\frac{v_{out}}{v_{in}} = \left( \frac{g_{m1}}{2} + \frac{g_{m2}}{2(1+k)} \right) R_{out} = \left( \frac{2+k}{2+2k} \right) g_{m1}R_{out}$$

## Frequency Response of the Folded Cascode Op Amp

The frequency response of the folded cascode op amp is determined primarily by the output pole which is given as

$$p_{out} = \frac{-1}{R_{out}C_{out}}$$

where  $C_{out}$  is all the capacitance connected from the output of the op amp to ground.

All other poles must be greater than  $GB = g_{m1}/C_{out}$ . The approximate expressions for each pole is

- 1.) Pole at node A:  $p_A \approx -1/R_A C_A$
- 2.) Pole at node B:  $p_B \approx -1/R_B C_B$
- 3.) Pole at drain of M6:  $p_6 \approx \frac{-1}{(R_2+1/g_{m10})C_6}$
- 4.) Pole at source of M8:  $p_8 \approx -g_{m8}/C_8$
- 5.) Pole at source of M9:  $p_9 \approx -g_{m9}/C_9$
- 6.) Pole at gate of M10:  $p_{10} \approx -g_{m10}/C_{10}$

where the approximate expressions are found by the reciprocal product of the resistance and parasitic capacitance seen to ground from a given node. One might feel that because  $R_B$  is approximately  $r_{ds}$  that this pole might be too small. However, at frequencies where this pole has influence,  $C_{out}$ , causes  $R_{out}$  to be much smaller making  $p_B$  also non-dominant.

## Example 1 - Folded Cascode, CMOS Op Amp

Assume that all  $g_{mN} = g_{mP} = 100\mu\text{S}$ ,  $r_{dsN} = 2\text{M}\Omega$ ,  $r_{dsP} = 1\text{M}\Omega$ , and  $C_L = 10\text{pF}$ . Find all of the small-signal performance values for the folded-cascode op amp.

$$R_{II} = 0.4\text{G}\Omega, R_A = 10\text{k}\Omega, \text{ and } R_B = 4\text{M}\Omega \therefore k = \frac{0.4 \times 10^9 (0.3 \times 10^{-6})}{100} = 1.2$$

$$\frac{v_{out}}{v_{in}} = \left( \frac{2+1.2}{2+2.2} \right) (100)(57.143) = 4,354\text{V/V}$$

$$R_{out} = R_{II} \parallel [g_{m7}r_{ds7}(r_{ds5} \parallel r_{ds2})] = 400\text{M}\Omega \parallel [(100)(0.667\text{M}\Omega)] = 57.143\text{M}\Omega$$

$$|p_{out}| = \frac{1}{R_{out}C_{out}} = \frac{1}{57.143\text{M}\Omega \cdot 10\text{pF}} = 1,750 \text{ rads/sec.} \Rightarrow 278\text{Hz} \Rightarrow GB = 1.21\text{MHz}$$

Comments on the Folded Cascode, CMOS Op Amp:

- Good PSRR
- Good ICMR
- Self compensated
- Can cascade an output stage to get extremely high gain with lower output resistance (use Miller compensation in this case)
- Need first stage gain for good noise performance
- Widely used in telecommunication circuits where large dynamic range is required

## SIMPLE TWO-STAGE BJT OP AMPS

### BJT Two-Stage Op Amp

Circuit:

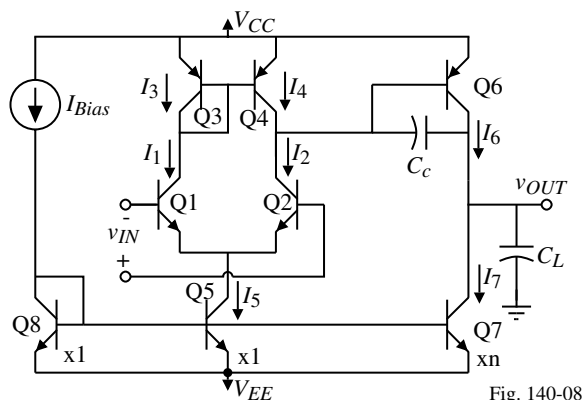


Fig. 140-08

DC Conditions:

$$I_5 = I_{bias}, \quad I_1 = I_2 = 0.5I_5 = 0.5I_{bias}, \quad I_7 = I_6 = nI_{Bias}$$

$$V_{icm(max)} = V_{CC} - V_{EB3} - V_{CE1(sat)} + V_{BE1}$$

$$V_{icm(min)} = V_{EE} + V_{CE5(sat)} + V_{BE1}$$

$$V_{out(max)} = V_{CC} - V_{EC6(sat)}$$

$$V_{out(min)} = V_{EE} + V_{CE7(sat)}$$

Notice that the output stage is class A  $\Rightarrow I_{sink} = I_7$  and  $I_{source} = \beta F I_5 - I_7$

### Two-Stage BJT Op Amp - Continued

Small Signal Performance:

Assuming differential mode operation, we can write the small-signal model as,

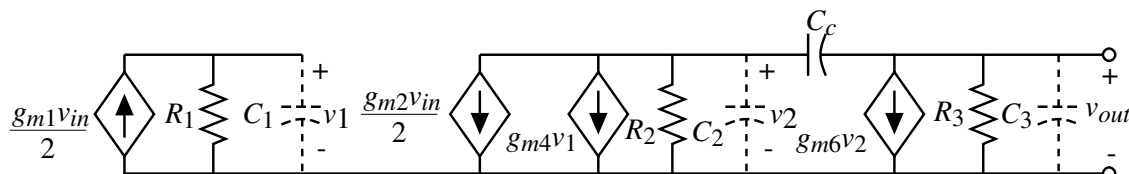


Fig. 140-09

where,

$$R_1 = \frac{1}{g_{m3}} \parallel r_{\pi 3} \parallel r_{\pi 4} \parallel r_{o3} \approx \frac{1}{g_{m3}} \quad R_2 = r_{\pi 6} \parallel r_{o2} \parallel r_{o4} \approx r_{\pi 6} \quad \text{and} \quad R_3 = r_{o6} \parallel r_{o7}$$

$$C_1 = C_{\pi 3} + C_{\pi 4} + C_{cs1} + C_{cs3} \quad C_2 = C_{\pi 6} + C_{cs2} + C_{cs4} \quad \text{and} \quad C_3 = C_L + C_{cs6} + C_{cs7}$$

Note that we have ignored the base-collector capacitors,  $C_{\mu}$ , except for M6, which is called  $C_c$ .

Assuming the pole due to  $C_1$  is much greater than the poles due to  $C_2$  and  $C_3$  gives

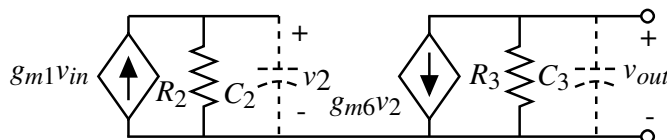


Fig. 140-10

### Two-Stage BJT Op Amp - Continued

Summary of the small signal performance:

Midband performance-

$$A_o = g_{m1}g_{mII}R_I R_{II} \approx g_{m1}g_{m6}r_{\pi6}(r_{o6}||r_{o7}) = g_{m1}\beta_F6(r_{o6}||r_{o7}), R_{out} = r_{o6}||r_{o7}, R_{in} = 2r_{\pi1}$$

Roots-

$$\text{Zero} = \frac{g_{mII}}{C_c} = \frac{g_{m6}}{C_c}$$

$$\text{Poles at } p_1 \approx \frac{-1}{g_{mII}R_I R_{II}C_c} = \frac{-1}{g_{m6}r_{\pi6}(r_{o6}||r_{o7})C_c} = \frac{-g_{m1}}{A_o C_c} \quad \text{and} \quad p_2 \approx \frac{-g_{mII}}{C_{II}} \approx \frac{-g_{m6}}{C_L}$$

Assume that  $\beta_F=100$ ,  $g_{m1}=1\text{mS}$ ,  $g_{m6}=10\text{mS}$ ,  $r_{o6}=r_{o7}=0.5\text{M}\Omega$ ,  $C_c=5\text{pF}$  and  $C_L=10\text{pF}$ :

$$A_o=(1\text{mS})(100)(250\text{k}\Omega)=25,000\text{V/V}, R_{in} 2(\beta_F/g_{m1})2(100\text{k}\Omega)=200\text{k}\Omega, R_{out}=250\text{k}\Omega$$

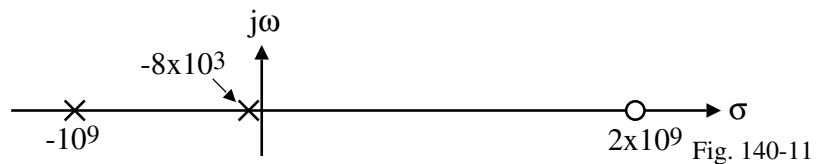
$$\text{Zero} = \frac{10\text{mS}}{5\text{pF}} = 2 \times 10^9 \text{ rads/sec}$$

or 318.3MHz,

$$p_1 = \frac{-1\text{mS}}{(25,000)5\text{pF}} = \frac{-2 \times 10^8}{25,000}$$

= -8000 rads/sec or 1273Hz,

and  $p_2 = -10\text{mS}/10\text{pF} = -10^9 \text{ rads/sec}$  or 159.15MHz



### Slew Rate of the Two-Stage BJT Op Amp

Remember that slew rate occurs when currents flowing in a capacitor become limited and is given as

$$I_{lim} = C \frac{dv_C}{dt} \text{ where } v_C \text{ is the voltage across the capacitor } C.$$

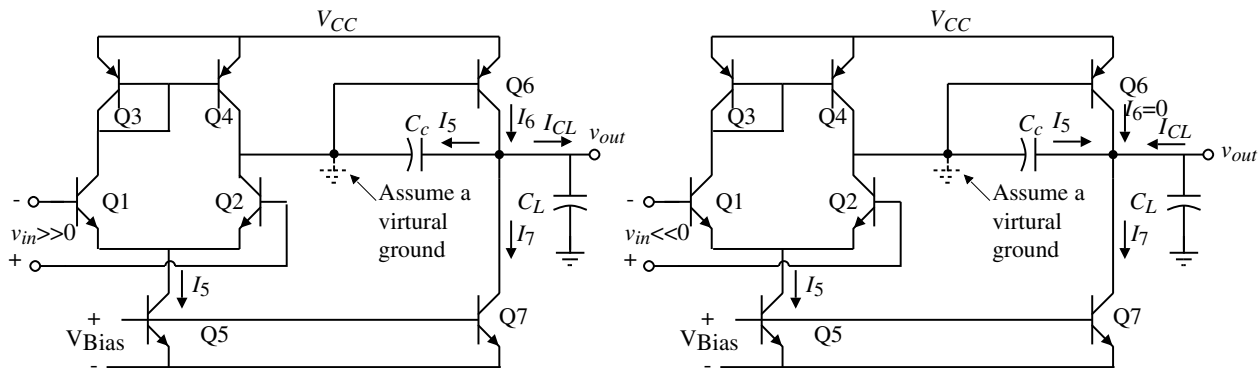


Fig. 140-12

$$SR^+ = \min \left[ \frac{I_5}{C_c}, \frac{I_6 - I_5 - I_7}{C_L} \right] = \frac{I_5}{C_c} \text{ because } I_6 \gg I_5$$

$$SR^- = \min \left[ \frac{I_5}{C_c}, \frac{I_7 - I_5}{C_L} \right] = \frac{I_5}{C_c} \text{ if } I_7 \gg I_5.$$

Therefore, if  $C_L$  is not too large and if  $I_7$  is significantly greater than  $I_5$ , then the slew rate of the two-stage op amp should be,

$$SR = \frac{I_5}{C_c}$$

## Folded-Cascode BJT Op Amp

### Circuit

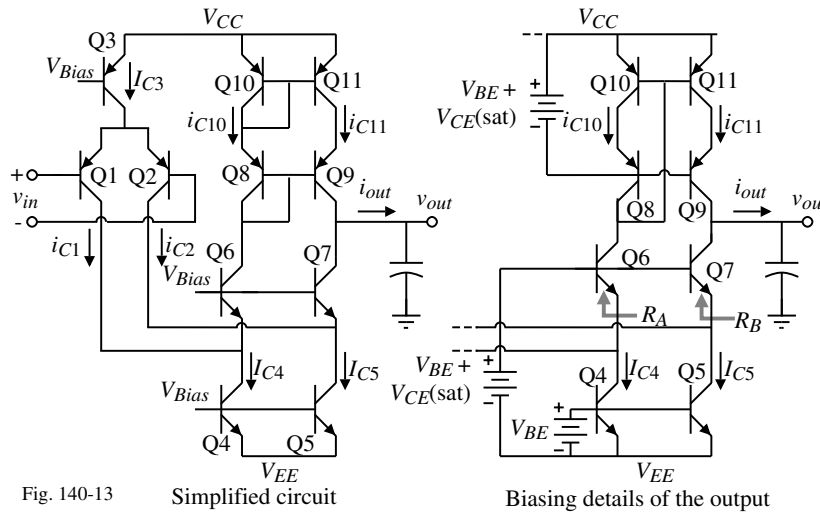


Fig. 140-13

Simplified circuit

Biasing details of the output

### DC Conditions:

$$I_3 = I_{bias}, I_1 = I_2 = 0.5I_5 = 0.5I_{bias}, I_4 = I_5 = kI_{Bias}, I_{10} = I_{11} = kI_{Bias} - 0.5I_{bias} \quad (k > 1)$$

$$V_{icm(max)} = V_{CC} - V_{CE3(sat)} + V_{EB1} \quad V_{icm(min)} = V_{EE} + V_{CE4(sat)} + V_{EC1(sat)} - V_{BE1}$$

$$V_{out(max)} = V_{CC} - V_{EC9(sat)} - V_{EC11(sat)} \quad V_{out(min)} = V_{EE} + V_{CE5(sat)} + V_{CE7(sat)}$$

Notice that the output stage is push-pull  $\Rightarrow I_{sink}$  and  $I_{source}$  are limited by the base current.

## Folded-Cascode BJT Op Amp - Continued

### Small-Signal Analysis:

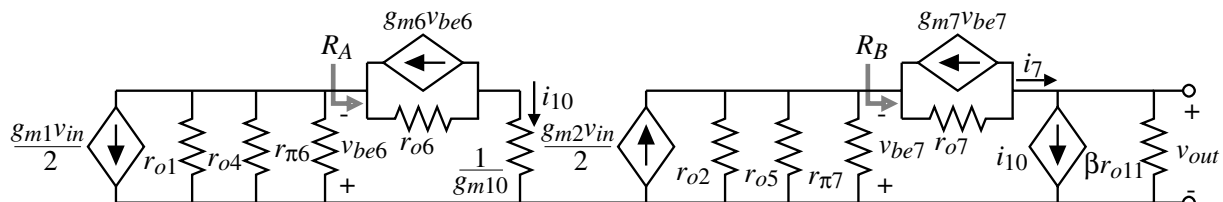


Fig. 140-14

where  $R_A \approx 1/g_{m6}$  and  $R_B \approx \frac{r_{o7} + \beta P r_{o11}/2}{1 + g_{m7} r_{o7}} \approx \frac{r_{\pi 7}}{2}$  if  $r_{o7} \approx r_{o11}$

$$i_{10} \approx \frac{-g_{m1} r_{\pi 6} v_{in}}{2(r_{\pi 6} + R_A)} \approx \frac{-g_{m1} v_{in}}{2} \quad i_7 \approx \frac{g_{m2} r_{\pi 7} v_{in}}{2(r_{\pi 7} + R_B)} \approx \frac{g_{m2} r_{\pi 7} v_{in}}{2(r_{\pi 7} + 0.5r_{\pi 7})} = \frac{g_{m2} v_{in}}{3}$$

$$\therefore v_{out} = (i_7 - i_{10}) \beta P R_{out} v_{in} = \frac{5}{6} (g_{m1} \beta P R_{out}) v_{in} \quad \text{if } g_{m1} = g_{m2} \Rightarrow \frac{v_{out}}{v_{in}} = \frac{5}{6} (g_{m1} \beta P R_{out})$$

$$R_{out} = \beta P r_{o11} \parallel [\beta_N (r_{o5} \parallel r_{o2})] \quad \text{and} \quad R_{in} = 2r_{\pi 1}$$

Assume that  $\beta_{FN} = 100$ ,  $\beta_{FP} = 50$ ,  $g_{m1} = g_{m2} = 1\text{mS}$ ,  $r_{oN} = 1\text{M}\Omega$ , and  $r_{oP} = 0.5\text{M}\Omega$ :

$$\frac{v_{out}}{v_{in}} = 14,285\text{V/V} \quad R_{out} = 14.285\text{M}\Omega \quad \text{and} \quad R_{in} = 100\text{k}\Omega$$



## Folded-Cascode BJT Op Amp - Continued

Frequency response includes only 1 dominant pole at the output (self-compensation),

$$p_1 = \frac{-1}{R_{out} C_L}$$

There are other poles but we shall assume that they are less than  $GB$

If  $C_L = 25\text{pF}$ , then  $|p_1| = 2800\text{ rads/sec. or } 446\text{Hz} \Rightarrow GB = 6.371\text{ MHz}$

Checking some of the nondominant poles gives:

$$|p_A| = \frac{1}{R_A C_A} = \frac{g_{m6}}{C_A} \Rightarrow 159\text{MHz if } C_A = 1\text{pf}$$

(the capacitance to ac ground at the emitter of Q6)

$$|p_B| = \frac{1}{R_B C_B} = \frac{2}{r_{\pi 7} C_B} \Rightarrow 6.37\text{MHz if } C_B = 1\text{pf (the capacitance to ac ground at the$$

emitter of Q7)

This indicates that for small capacitive loads, this op amp will suffer from higher poles with respect to phase margin. Capacitive loads greater than  $25\text{pF}$ , will have better stability (and less  $GB$ ).

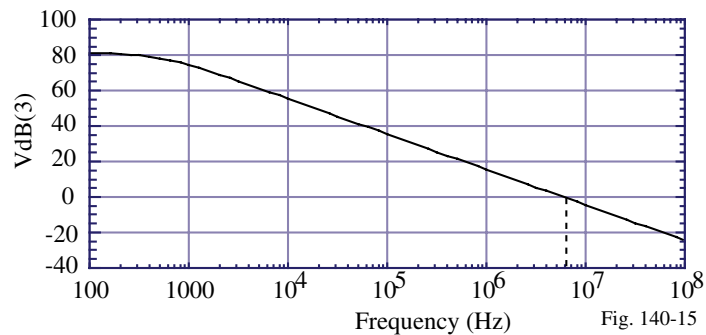


Fig. 140-15

## SUMMARY

- Two stage op amp gives reasonably robust performance as an “on-chip” op amp
- DC balance conditions insure proper mirroring and all transistors in saturation
- Slew rate of the two-stage op amp is  $I_5/C_c$
- Folded cascode op amp offers wider input common voltage range
- Folded cascode op amp is a self-compensated op amp because the dominant pole at the output and proportional to the load capacitor