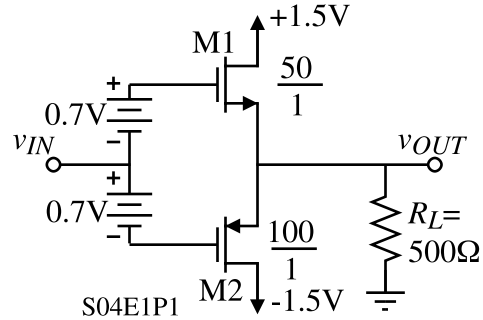
EXAMINATION NO. 1 - SOLUTIONS

(Average Score = 66.2/100)

Problem 1 - (25 points)

A push-pull follower is shown with a 500Ω load. Assume that the MOSFETs have the following model parameters: . $K_N' = 100\mu A/V^2$, $V_{TN} = 0.5V$, and $K_P' = 50\mu A/V^2$, $V_{TP} = -0.5V$. Ignore the bulk effects and assume $\lambda = 0$.

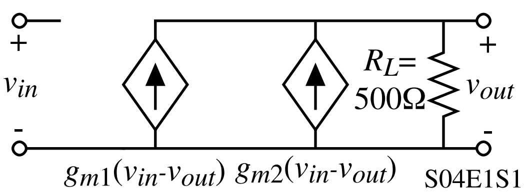


a.) Find the small signal voltage gain and the output resistance (not including R_L) for the conditions of part a.) if the dc current in M1 and M2 is 100μA.

b.) What is the output voltage when $v_{IN} = 0.5V$?

Solution

a.) The small-signal model is given as shown



where $g_{m1} = \sqrt{2K_N'(W_1/L_1)I_{D1}} = \sqrt{2\cdot100\cdot50\cdot100}$

$g_{m1} = 1mS$, $g_{m2} = \sqrt{2\cdot50\cdot100\cdot100} = 1mS$

Summing currents at the output node gives,

$$g_{m1}(v_{in} - v_{out}) + g_{m2}(v_{in} - v_{out}) = G_L v_{out}$$

$$\therefore \quad \frac{v_{out}}{v_{in}} = \frac{g_{m1}+g_{m2}}{g_{m1}+g_{m2}+G_L} = \frac{1+1}{1+1+2} = \underline{\underline{0.5V/V}} \quad \text{and} \quad R_{out} = \frac{1}{g_{m1}+g_{m2}} = \frac{1}{2mS} = \underline{\underline{500\Omega}}$$

b.) Under the condition of $v_{IN} = 0.5V$, the gate voltages are

$V_{G1} = 0.5V+0.7V = 1.2V$ and $V_{G1} = 0.5V-0.7V = -0.2V$

We know that the output voltage can be expressed as $V_{OUT} = (I_1\text{-}I_2)0.5k\Omega$ where I_1andI_2 are the dc currents in M1 and M2.

Next we need to make an assumption about the operating region of the two transistors. Let us assume that M1 is saturated and M2 is cutoff. Therefore, $I_2 = 0$ and

$$I_1 = 0.5(100)(50)[1.2\text{-}V_{OUT} - 0.5)]^2\ (\mu A) = 2.5(0.7\text{-}V_{OUT})^2\ (mA)$$

$$\therefore \quad V_{OUT} = (I_1)0.5k\Omega = 1.25(0.7\text{-}V_{OUT})^2 \rightarrow 0.8V_{OUT} = 0.49 - 1.4V_{OUT} + V_{OUT}^2$$

The resulting quadratic, $V_{OUT}^2 - 2.2V_{OUT} + 0.49 = 0$ gives

$$V_{OUT} = 1.19\pm0.5\sqrt{2.2^2\text{-}4(0.49)} = 1.1 \pm 0.5(2.880) = 1.1\pm0.845 = \underline{\underline{0.252V}}$$

Check the regions of M1 and M2.

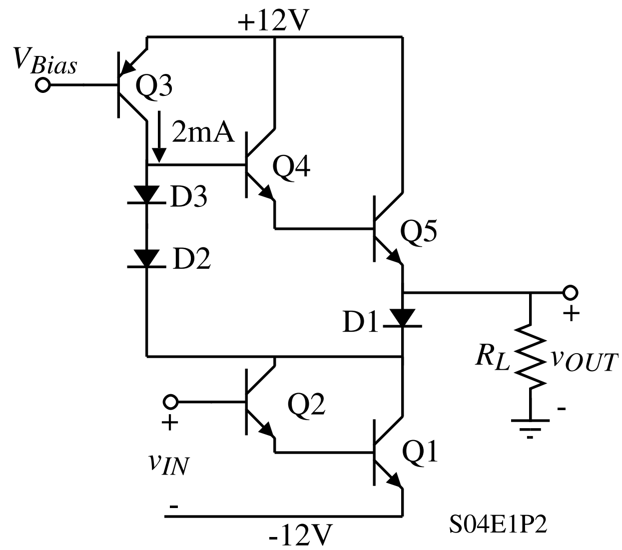
M1: $V_{DS1} = 1.5\text{-}0.252 = 1.25V > V_{GS1}\text{-}V_{TN} = 1.2\text{-}0.5 = 0.7$ $\therefore$ M1 is saturated.

M2: $V_{SG2} = 0.252 - (-0.2) = 0.452 < |V_{TP}| = 0.5$ $\therefore$ M2 is cutoff

(Note: Many of you multiplied the gain of (a.) by 0.5 to get the answer which happened to be close. If you did this you lost 3 points because the current in M1 is no longer 100μA but 506μA and the current in M2 is 0μA. Your answer may be right but the approach is wrong. PA)

Problem 2 - (25 points)

An all-*npn* Darlington output stage is shown. For all devices assume that $V_{BE(\text{on})}$ = 0.7V, $V_{CE(\text{sat})}$ = 0.2V, and β_F = 100. The magnitude of the collector current in $Q3$ is 2mA.



a.) If R_L = 8Ω, calculate the maximum positive and negative limits of v_{OUT}.

b.) Calculate the power dissipated in the circuit for v_{OUT} = 0V.

c.) Calculate the maximum average power that can be delivered to R_L = 8Ω before clipping occurs and the corresponding efficiency of the *complete* circuit. Assume that feedback is used around the circuit so that v_{OUT} is approximately a sinusoidal.

Solution

a.) Assume that current to R_L is not limited. Then,

$$V_o^+ = V_{CC} - V_{CE3}(\text{sat}) - V_{BE5} - V_{BE4} = 12 - 0.2 - 0.7 - 0.7 = \underline{\underline{10.4\text{V}}}$$

$$V_o^- = -V_{CC} + V_{CE2}(\text{sat}) + V_{BE1} + V_{D1} = -12 + 0.2 + 0.7 + 0.7 = \underline{\underline{-10.4\text{V}}}$$

b.) For v_{OUT} = 0V, the 2mA flowing through Q3 flows through D3, D2 and Q1-Q2. The voltage drop across D2 and D3 is not enough to turn on D1, Q4, or Q5. Thus, the power dissipation is

$$P_Q = [+12-(-12)](2\text{mA}) = \underline{\underline{48\text{mW}}}$$

c.) From part b.) we observe that the amplifier is operating in class B. The maximum load power is

$$P_L(\max) = \frac{\left(\frac{V_o(\text{peak})}{\sqrt{2}}\right)^2}{R_L} = \frac{(10.4)^2}{2\cdot 8} = 6.76\text{W}$$

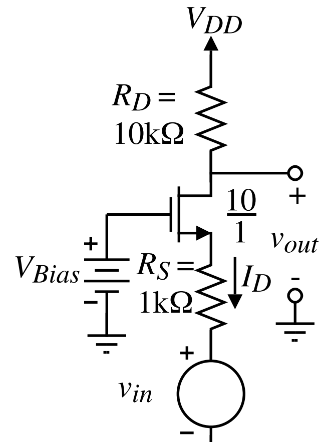
$$I_{Supply} = \frac{1}{\pi} I\,(\text{peak}) = \frac{1}{\pi}\frac{10.4\text{V}}{8\Omega} = 0.4138\text{A}$$

$$\therefore \quad P_{Supply} = [12-(-12)](0.4138) = 9.93\text{W}$$

$$\therefore \quad \text{Efficiency} = \eta = \frac{P_L(\max)}{P_Q + P_{Supply}} = \frac{6.76\text{W}}{0.048\text{W} + 9.93\text{W}} = \underline{\underline{67.7\%}}$$

Problem 3 - (25 points)

Find the voltage transfer function of the common-gate amplifier shown. Identify the numerical values of the small-signal voltage gain, v_{out}/v_{in}, and the poles and zeros. Assume that I_D = 500μA, K_N' = 100μA/V^2, V_{TN} = 0.5V, and K_P' = 50μA/V^2, V_{TP} = -0.5V, $\lambda \approx 0$V^{-1}, C_{gs} = 0.5pF and C_{gd} = 0.1pF.



S04E1P3

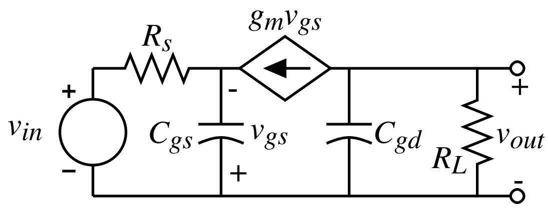
Solution

The small signal transconductance is,

$$g_m = \sqrt{2 \cdot K_N' (\mathrm{W/L}) I_D} = \sqrt{2 \cdot 100 \cdot 10 \cdot 500} = 1\mathrm{mS}$$

$$r_{ds} = \infty$$

The small signal model is,



S04E1S3

The voltage gain can be expressed as follows,

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_{in}}\right), \qquad \frac{V_{out}}{V_{gs}} = -g_m\left(\frac{R_L(1/sC_{gd})}{R_L + (1/sC_{gd})}\right)$$

Sum currents at the source to get,

$$\frac{V_{in} + V_{gs}}{R_s} + g_m V_{gs} + sC_{gs}V_{gs} = 0 \quad \rightarrow \quad \frac{V_{gs}}{V_{in}} = \frac{-G_s}{G_s + g_m + sC_{gs}}$$

$$\therefore \quad \frac{V_{out}}{V_{in}} = \left(\frac{g_m R_L}{1 + g_m R_L}\right)\left(\frac{1}{sC_{gd}R_L + 1}\right)\left(\frac{1}{\frac{sC_{gs}}{g_m + G_s} + 1}\right)$$

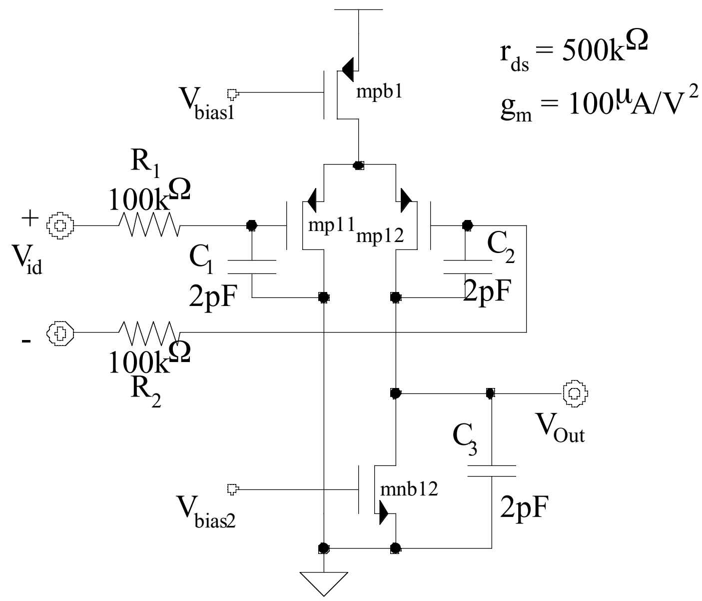
The various values are,

$$\text{Voltage gain} = \frac{g_m R_L}{1 + g_m R_L} = \frac{1 \cdot 10}{1+1} = \underline{\underline{5\text{V/V}}}$$

$$p_1 = \frac{-1}{C_{gd}R_L} = \frac{-1}{10^{-13} \cdot 10^4} = \underline{\underline{-10^9 \text{ radians/sec.}}}$$

$$p_2 = \frac{-(g_m + G_s)}{C_{gs}} = \frac{-10^{-3} + 10^{-3}}{0.5\text{x}10^{-12}} = \underline{\underline{-4\text{x}10^9 \text{ radians/sec.}}}$$

Problem 4 - (25 points)



For the circuit shown, assume all transistors are saturated and that their parasitic capacitors are negligible when compared with the ones shown. Assume that their respective g_m's and r_{ds}' are equal.

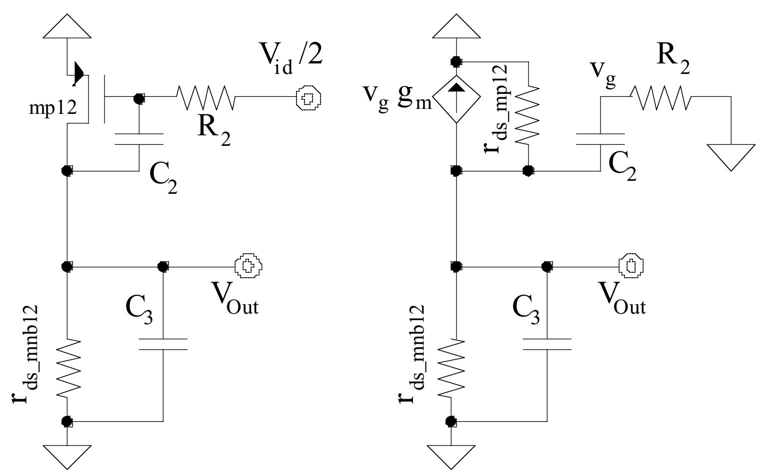
a.) Using either the open-circuit or short-circuit time-constant test, determine the 3-dB bandwidth ($f_{3\text{-dB}}$) of the circuit.

b.) Which capacitor is predominant in determining this frequency?

c.) Using either the open-circuit or short-circuit time-constant test, determine the location of the highest frequency pole, assuming all the poles are more than an order of magnitude away from each other.

d.) Which capacitor determines this high-frequency pole?

Solution:



a.) O.C.T.C. Tst:

$Roc2 = r_{ds\_mp12}//r_{ds\_mnb12} + R_2 + g_m(r_{ds\_mp12}//r_{ds\_mnb12})R_2$

$= 250k + 100k + 2.5M \approx 2.8M$

$Roc3 = r_{ds\_mp12}//r_{ds\_mnb12} = 250k$

$W_{3\text{-dB}} \approx (\Sigma\ RC)^{-1} \approx 1 / (2.8M)(2pF)$

$= 180k$ rad/s

b.) C_2

c) S.C.T.C. Tst:

$Rsc2 = R_2 = 100k$

$Rsc3 = r_{ds\_mp12}//r_{ds\_mnb12}//R_2//(1/g_m) = (250k)//(100k)//(10k) \approx 9k$

$W_{hf} \approx \Sigma\ (1/RC) = 1/(100k)(2p) + 1/(9k)(2p) \approx 60M$ rad/s

d) C_3